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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

OBSERVED EFFECTS OF GEOMETRIC DISTORTION IN HYDRAULIC MODELS

By Kenneth D. Nichols,[1] Jun. Am. Soc. C. E.

Synopsis

Geometric distortion in hydraulic models is a systematic change from geometric similitude involving the use of exaggerated depths or slopes. In many cases the advantages of distorted models more than compensate for their disadvantages; hence distorted models are commonly used in many countries. Questions of costs and benefits, size of prototype, laboratory facilities, time, and requirements of turbulent flow and adequate tractive force in the model may make geometric distortion necessary or desirable in hydraulic laboratory models. For both technical and economic reasons the degree of geometric distortion is usually greater for large than for small prototypes.

In this paper, studies of specific models are summarized and analyzed for effects of geometric distortion in the behavior of the models. Comparative data are presented for an undistorted and a distorted model of the same prototype showing effects of slope distortion and depth distortion upon velocity conversion factors, from model to prototype. Data are presented for other models showing effects on bed formations of variations in bed materials (including light-weight materials), and slope and depth distortions. Selected studies of flood control and river navigation in the Mississippi River System are summarized and analyzed. Brief reference is also made to observations, pertinent to effects of geometric distortion, in many other hydraulic studies.

Analysis of the data showed that a lesser degree of distortion may be required in movable bed models if light-weight materials are used to simulate the stream bed. The analysis also outlined representative methods of compensating for distortion and effecting hydraulic similarity in specific details when over-all similarity cannot be expected.

Note.—Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **October 15, 1938.**

[1] Lieut., Corps of Engrs., U. S. Army, West Point, N. Y. (Formerly Asst. to Director, U. S. Waterways Experiment Station, Vicksburg, Miss.)

On the basis of this study recommendations are made concerning the use, design, and operation of distorted models.

INTRODUCTION

Definition of Geometric Distortion.—Geometric distortion in hydraulic models is a systematic change from geometric similitude involving the use of exaggerated depths or slopes.

The general term, "geometric distortion," should be sub-divided into "depth distortion" and "slope distortion." In depth-distorted models the depth scale, $d_r = \frac{d_m}{d_p}$, is greater than the length or horizontal scale, $L_r = \frac{L_m}{L_p}$. The degree of depth distortion, $b = \frac{d_r}{L_r}$, is the ratio of the depth scale to the length scale. The degree of slope distortion is the slope scale, $S_r = \frac{S_m}{S_p}$. In slope-distorted models the slope scale is other than unity, and S_r may be greater than or less than b. Depth distortion and slope distortion may be used independently, or in combination. In practice, depth distortion is usually accompanied by slope distortion of equal or greater degree. Other types of geometric distortion have been proposed, but their use is not common enough to warrant further discussion.

Advantages and Disadvantages of Distorted Models.—The advantages of distorted models may be listed as follows:

(1) In order to satisfy the minimum requirements of tractive force, turbulent flow, or measurable depths for a model of any prototype, a larger horizontal scale is necessary if the model is undistorted than if distorted. (Comparisons between a distorted model and an undistorted model apply also to models with a high degree of distortion and with a low degree, respectively.) The distorted model will have the following advantages over the undistorted model: (*a*) It will occupy a smaller space; (*b*) less water will be needed; (*c*) the laboratory facilities and personnel required will be less; (*d*) a shorter time may be required for the model study; and (*e*) the cost of the model study will undoubtedly be less.

(2) For very large prototypes the cost of the smallest practical undistorted model may be prohibitive, whereas a smaller distorted model of lesser cost may be economically justifiable and may serve the desired purpose.

The disadvantages and limitations of distorted models are:

(1) A smaller distorted model usually will depart from true similarity in more details of model performance than the smallest practical undistorted model of the same prototype;

(2) Owing to the lack of all-around similarity, a distorted model must be designed to obtain hydraulic similarity in specified details; hence, it cannot be used for all-around studies;

(3) A distorted model usually needs more extensive field data for verification purposes than an undistorted model; and,

(4) Because of the uncertainty of many of the effects of distortion the dangers of a model study being a failure are greater for a distorted model than for an undistorted model.

Degree of Geometric Distortion Used in Practice.—For any prototype the questions of costs and benefits, size of prototype, laboratory facilities, time, and requirements of turbulent flow and adequate tractive force in the model may make geometric distortion necessary or desirable. These factors combined with certain natural characteristics of rivers usually require a greater degree of distortion for large prototypes than for small prototypes. This is borne out in experience as follows: Considering chiefly costs and benefits, in practice the length scale used in hydraulic models decreases as the size of the prototype increases. For example, in hydraulic laboratories in Prague and Zurich one finds a length scale of 1 : 50 used and suitable for the small rivers in Czecho-Slovakia and Switzerland; at Karlsruhe Laboratory, in Germany, a scale of 1 : 170 or 1 : 200 for a Middle Rhine River model; at Berlin, Germany, 1 : 200 for a Middle Elbe River model and 1 : 500 for a Lower Elbe River model; at Budapest, Hungary, 1 : 500 for the Danube River; and at Vicksburg, Miss., 1 : 450 to 1 : 2 000 for the Mississippi River.

Due mainly to technical reasons and to a change in the characteristics of rivers with size, the depth scale for these models does not fluctuate over such a wide range. In the model a certain minimum depth and slope are usually necessary to insure turbulent flow, measurable depths, or sufficient tractive force. Moreover, for most rivers, as the size of the river increases, the slope and the grain size of the bed material decrease, and the ratio of width to depth increases. Consequently, for larger prototypes the depth scale must be proportionately greater than the length scale.

In the foregoing models, the range in depth scales was only 1 : 50 to 1 : 200, compared to a range in length scales of 1 : 50 to 1 : 2 000. The range in depth distortion was from 1 to 20, and the range in slope distortion was from 1 to 26. Models of the larger prototypes were built with the greater degree of geometric distortion.

DESIGN OF DISTORTED MODELS

Design of River Models.—The design of a geometrically distorted model, like most engineering design, involves the careful balancing of many conflicting factors, and requires much sound judgment if the finished product is to be useful. The procedure in hydraulic model studies is as follows:

I. Preliminary Work.—

(*A*) Study all aspects of the field problem and determine those phases for which model tests may be of value. Consider the limitations of distorted models. Decide on the general type, scope, and purpose of the model study.

(*B*) Secure and assemble all necessary field data. Special data may be necessary for the verification of a distorted model.

II. Design of Models.—

(*A*) Tentatively, select the extent of model area, and the horizontal and vertical scale ratios. If a movable bed model is considered, also tentatively select the bed material.

(*B*) Modify this tentative design on the basis of: Laboratory facilities available (particularly the water supply and space available); turbulent flow requirements; the limits of accuracy of measurements; roughness requirements to obtain the desired velocity scale (for movable bed models substitute tractive force requirements to move bed material); the probable effects of geometric distortion; and a comparison of the costs of the model with the value of benefits to be derived from the model.

(*C*) Prepare adequate plans for the verification and operation of the model. In the preparation of these plans particular attention must be given to the probable effects of distortion.

(*D*) Complete the design of the model.

III. Construction of Model.—

(*A*) Use construction methods that will permit subsequent necessary alterations to be made. In movable bed distorted models it may be desirable to modify the degree of distortion after the model is built.

IV. Operation of Model.—

(*A*) Conduct adjustment and verification tests. Additional field data may be necessary to complete the verification tests. Special observations should be made to determine the effects of geometric distortion.

(*B*) Conduct formal model tests pertaining to the field problem. For these tests the model should be operated in accordance with special operating conditions determined during the verification tests. During all the tests make observations to determine the effects of geometric distortion, particularly the effects on the comparative powers of the model.

V. Report.—

(*A*) Report and interpret the model test data and observations. The interpretation of the effects of distortion on model results is of major importance.

Other Fundamental Principles.—In every step of the foregoing outline the importance of the effects of geometric distortion is emphasized. Evidently, the design of a distorted model cannot be outlined independent of the necessary preliminary work and subsequent construction, operation, and report. The designer of a distorted model must consider all phases of the study during the design phase; particularly, he must anticipate the effects of distortion in the subsequent verification and operation of the model and in the interpretation of the test data.

In addition to the foregoing procedure the following four fundamental principles of distorted model design and operation are presented:

(1) Field data must be available for a model verification, and definite plans must be made for this verification. The model operator must assure himself by appropriate comparisons of model and prototype behavior that the model faithfully simulates the prototype in the particular details being studied.

(2) A distorted model should be designed for a particular problem. A given model can only be used to solve problems closely related to the one for which it was designed. If the study involves several different problems, two or more models or a larger model may be necessary.

(3) The effects of scale, and of depth and slope distortion, should be analyzed carefully in conjunction with the fundamental laws of hydraulics in order to avoid costly mistakes in the construction and operation of the model, and the interpretation of model results. In considering these effects it should be remembered that the primary aim of most distorted models is to effect hydraulic similarity in specific details when over-all similarity cannot be expected.

(4) Light-weight bed materials may be used in movable bed models in order to reduce the degree of geometric distortion necessary.

It is evident that a knowledge of the effects of geometric distortion is of major importance in the design, operation, and interpretation, of distorted models. Because most knowledge concerning the effects of distortion is obtained from specific model studies, the experience of the laboratory personnel must be considered in model design in some such way as are the laboratory facilities. The experience of the designer undoubtedly influences his design.

Abstracts of Model Studies

Fourteen hydraulic studies pertaining to the observed effects of geometric distortion are summarized in Table 1 and further described in the following text. Items Nos. 4 and 13 refer to studies in Berlin, Germany (Preussische Versuchsanstalt für Wasserbau und Schiffbau); Item No. 7 was from the University of Manchester, Manchester, England; and the remainder were tests made at the United States Waterway Experiment Station, at Vicksburg, Miss., Item No. 14 being in part from Cornell University, at Ithaca, N. Y. Items Nos. 4, 7, 10, 11, and 13 had sand beds; Item No. 12 had a light-weight material for the bed; Item No. 14 had both fixed and sand beds; and the remainder were all fixed beds.

Item No. 1, Table 1.—Tests of the St. Clair River sills were conducted in a tilting flume, supplementary flume tests being conducted in conjunction with the model study to determine the back-water effect of submerged sills (21) (33).[2] The conclusions reached were: (1) The back-water produced by any given sill varies as the square of the velocity of approach; (2) tests conducted with equal Froude's number gave approximately similar results regardless of scale; (3) the shape of the sill, particularly that of the up-stream face, influences the back-water produced by the sill, the back-water being greater for a vertical face than for a sloping face; and, (4) for the large spacings proposed, the effect of one sill is not reduced by the addition of more sills.

Items Nos. 2 and 3, Table 1.—Models were constructed of a 3-mile stretch of the St. Clair River (just below Lake St. Clair) to determine the back-water effects of a series of submerged sills (21) (33). The relation between depth scale and length scale for the distorted model is expressed by:

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[2] Numbers in parentheses refer to the Bibliography in the Appendix.

$$d_r = L_r^{0.75} \qquad (1)$$

In the distorted model the sills were built to an undistorted scale of 1 : 30. It was impossible to roughen the bed surface sufficiently to obtain the required

TABLE 1.—DATA PERTAINING TO MODEL STUDIES

Study No.	Type of study	Remarks*	MODEL CHARACTERISTICS			
			Length ratio, L_r	Depth ratio, d_r	Degree of depth distortion, b	Degree of slope distortion, S_r
1.....	Flume tests; St. Clair River	Average sill was 12 ft high in 42 ft of water; (21) and (33)	1 : 16 1 : 60 1 : 100 1 : 120	1 : 16 1 : 60 1 : 100 1 : 120	1.0 1.0 1.0 1.0	¶ ¶ ¶ ¶
2.....	Model tests; St. Clair River	Approximately 3 miles of river reproduced in both models; (21 and (33)	1 : 100	1 : 30	3.3	3.3 : 1
3.....	Model tests; St. Clair River		1 : 100	1 : 100	1.0	1.0 : 1
4.....	Flume tests; Elbe River	Active bed movement but stable banks; (27) and (43)	1 : 200 1 : 200	1 : 40 1 : 200	5.0 1.0	8 : 1†† 43 : 1‡‡
5.....	Model study; Ohio River	Coney Island model study, Dam No. 36; (21) and (44)	1 : 250	1 : 60	4.2	4.2 : 1
6.....	Model study; Mississippi River	Helena-Donaldsonville model; (21) and (44)	1 : 2 000	1 : 100	20.0	20 : 1
7.....	Model study; Severn Estuary	Tidal model of the Severn Estuary; (12)	1 : 8 500 1 : 8 500	1 : 100 1 : 200	85.0 42.5	85.0 : 1 42.5 : 1
8.....	Model study; Mississippi River at Chicot Landing, Miss.	Effect of distortion on the distribution of velocity in a model stream; (21) and (34)	1 : 1 000 1 : 1 000 1 : 1 000	1 : 100 1 : 167 1 : 250	10.0 6.0 4.0	10 : 1 6 : 1 4 : 1
9.....	Model study, Mississippi River, at Vicksburg, Miss.	Effect of distortion on the distribution of velocity in a model stream; (21) and (34)	1 : 1 000 1 : 1 000 1 : 1 000 1 : 1 000	1 : 100 1 : 125 1 : 167 1 : 250	10.0 8.0 6.0 4.0	10 : 1 8 : 1 6 : 1 4 : 1
10.....	Model study; Mississippi River	Model of Fitler Bend; (21) and (44)	1 : 500 1 : 500 1 : 500	1 : 150 1 : 150 1 : 150	3.3 3.3 3.3	26.2 : 1 19.0 : 1 15.7 : 1
11.....	Model study; Mississippi River	Robinson Crusoe Island model; (21) and (44)	1 : 1 000	1 : 125	8.0	15 : 1
12.....	Model study; Mississippi River	Memphis (Tenn.) Depot model; (21) and (44)	1 : 450 1 : 450	1 : 150 1 : 150	3.0 3.0	20.0 : 1 12.5 : 1
13.....	Flume test; Berlin, Germany	Effect of geometric distortion on bed load movement in a bend; (27)	† †	‡ §	*X*¶ 3*X*¶	§§ ‖‖
14.....	Flume test	Bifurcated flume tests; (4), (21), (36), and (44)	†ᵃ	‖	**	¶¶

* Numbers in parentheses refer to corresponding items in Appendix. † Width of flume = 6.6 ft. †ᵃ Width of flume = 2.0 ft. ‡ d_m = 0.16 ft. § d_m = 0.49 ft. ‖ d_m = 0.25 ft to 0.58 ft. ¶ Variable. ** Varies. †† S_m = 1 : 800. ‡‡ S_m = 1 : 150. §§ S_m = 1 : 400. ‖‖ S_m = 1 : 1 200. ¶¶ Velocity varied from 0.47 ft per sec to 0.99 ft per sec.

Froudian Velocity Scale, the relation between the velocity scale and the depth scale being:

$$v_r = 1.15\, d_r^{0.5} \qquad (2a)$$

instead of,

$$v_r = d_r^{0.5} \qquad (2b)$$

This shows that the so-called Law of Compatibility, based on equal roughness coefficient in model and prototype (that is, when d_r is expressed by Equation (1) the velocity scale is expressed by Equation (2*b*)), is of little practical value.

The undistorted model, operated in accordance with Equation (2*b*) indicated less back-water than the distorted model. When operated in accordance with Equation (2*a*), two conclusions were indicated:

(*a*) Sills with vertical faces, or with slopes of 1 on 1, indicated approximately the same back-water as the distorted model, thus supporting the contention that when velocity and roughness requirements can be satisfied a model with a low degree of distortion may be used for this type of problem; and,

(*b*) Sills with faces sloped at 1 on 3 indicated about twice as much back-water as the distorted model. For these latter tests, in the distorted model, the toes of the sills were almost touching and the sills no longer retained their individual action; hence, the back-water was reduced.

Conclusion (*a*) confirms the belief that, in a depth-distorted model, an overfall structure should be built undistorted to the depth scale; but Conclusion (*b*) illustrates that there are limits to the application of this rule.

Item No. 4, Table 1.—Supplementary flume tests were made in the Prussian Experiment Institute for Hydraulic Engineering and Ship Building, in Berlin, Germany, for a model study of the Elbe River (27) (43). These tests were made to determine the relative effects of depth and slope distortion, and the effect of change in cross-section of an impermeable training dike on the scour resulting from the dike. In the first tests, the flume was considered to be a depth-distorted model; but the cross-sections of the dikes were built to various scales $\left(L_r = \frac{1}{40};\ d_r = \frac{1}{40};\ L_r = \frac{1}{60};\ d_r = \frac{1}{40};\ L_r = \frac{1}{80};\ d_r = \frac{1}{40};\ \text{etc.}\right)$, representing depth distortions, b, of 1, 1.5, 2, 3, 4, 5 and ∞.

Two findings pertaining to geometric distortion were:

(1) The change in cross-section of a dike such as would be caused by depth distortion will increase the resulting scour around the dike. The change in slope of the dike from the shore end to the outer end such as caused by depth distortion will also increase the relative scour. The tests did not indicate that the increase of scour would be in the correct ratio to the increase in depth scale; and,

(2) An increase in slope distortion will increase the scour resulting from a dike.

Item No. 5, Table 1.—The purpose of the so-called "Coney Island Model Study" was to find a method of eliminating the back-lash eddy along the lock approach below Dam No. 36 in the Ohio River (21) (44). Similarly, to Items Nos. 2 and 3, the relation between the depth scale and the length scale is expressed by Equation (1). In the model, the dam was built to an undistorted scale of 1 : 60. Two findings are pertinent:

(1) As in Items Nos. 2 and 3 it was determined that sufficient roughness could not be applied in order to reduce the velocity scale to that of the Froudian requirement. By applying roughness, n_m was increased from 0.013 to 0.021. The resulting velocity scale was approximately 20% to 25% too great.

(2) In spite of excess velocities a good reproduction of the back-lash eddy was obtained, and the model was suited for the study.

Item No. 6, Table 1.—A model of a 600-mile stretch of the Mississippi River (see Fig. 1) was constructed in order to determine the comparative value of various flood-control measures (21) (44). The model was particularly effective in giving a vivid picture of the flood problem for the entire Lower Mississippi Valley, by the reproduction of past major floods. Considering this aspect, the model might be termed to have an educational value, which is often under-estimated when evaluating the benefits to be derived from a model study. The conclusions were:

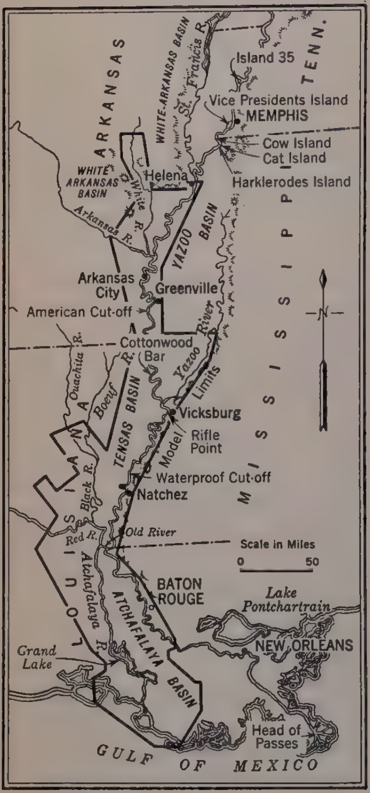

FIG. 1.—MODEL OF THE MISSISSIPPI RIVER FROM HELENA, ARK., TO DONALDSVILLE, MISS. (MODEL SCALES: HORIZONTAL, 1 : 2 000; AND VERTICAL, 1 : 100).

(1) Before the application of roughness it was determined that the model discharge scale varied with stage from a value of 1.484×10^{-6} to 0.823×10^{-6}. This variation could not be tolerated because the length of the model was so great that the corresponding varying time scale introduced serious errors into the form of a flood wave.

(2) Computations for selected sections of the model revealed that the scale for hydraulic radius varied from 1 : 118 to 1 : 527.

(3) In order to obtain similarity of hydraulic capacity for all parts of the model, variable intensities of roughness were applied and a fixed discharge scale of 1 : 1 500 000 resulted. This discharge scale is in excess of the Froudian scale of 1 : 2 000 000.

(4) In spite of extreme distortion and the resulting difficulties of adjustment, it was possible to verify the model by reproducing recent major floods accurately. The wealth of field data which are available for the Lower Mississippi River System permitted this verification.

Item No. 7, Table 1.—The purpose of this study was to test the effects of a barrage in the Severn River (12). Various sizes of sand were used in an effort to improve the verification. Three findings are to be noted:

(1) The type of sand used as a bed material influences the bed configuration. The height of banks formed in the model was found to be a function of the diameter of the sand grains.

(2) The model having a vertical scale of 1 : 200 was in closer agreement with the prototype than the model having a vertical scale of 1 : 100.

(3) In spite of extreme distortion a good verification was obtained.

Items Nos. 8 and 9, Table 1.—These studies were part of a general research program to determine the effects of geometric distortion (21) (34). They were conducted in existing models, changes in distortion being made by remolding the channel. No effort was made to adjust the roughness to obtain any particular velocity scale. The results showed that:

(1) The degree of distortion affects the distribution of velocity and, therefore, would affect the relative horizontal distribution of energy and the effective tractive force; and,

(2) There appeared to be greater discrepancy between the prototype and the model for a high degree of geometric distortion than for a low degree.

Item No. 10, Table 1.—This was a navigation study to determine methods of improving one or the other of the two channels at Fitler Bend, at Point Lookout, La. (Fig. 2 is a river survey made from September 2 to October 6, 1933). A low degree of depth distortion was considered necessary in order to test various dredging proposals. Light-weight bed materials were not being used at U. S. Waterways Experiment Station at the time of this study.

Three general conclusions may be drawn from this study (21) (44):

(1) For a slope distortion of $\frac{26.2}{1}$ the adjustment tests indicated that the resulting velocities were too high. The resulting thalweg was very definitely on the outside of the bend, whereas a survey of the prototype showed greater development of the inside channel.

(2) For a decrease in slope distortion the model verification was improved. Verification tests were run with a slope distortion of $\frac{19.0}{1}$ and $\frac{15.7}{1}$. The latter slope furnished the better verification of natural conditions. In both cases the movement of sand was decreased, but there still remained sufficient tractive force to provide adequate movement of the bed materials.

(3) These tests show that a change in slope distortion may result in changed bed configuration particularly in a change of thalweg alignment. They also indicate the importance of adequate field data for supporting verification tests.

Items Nos. 11 and 12, Table 1.—The Robinson Crusoe Island model (Item No. 11) was designed and operated for a study at, and up stream from, the City of Memphis, Tenn. (21) (44). Because the model included the Memphis Depot area an attempt was made to use this existing model for the Memphis Depot Study (Item No. 12); but it was soon determined that the existing model was not suited for the study, and a new model was designed for the latter case. Items Nos. 11 and 12 were both navigation studies for channel improvements at Memphis, Item No. 11 being up stream from Memphis, and Item No. 12 in the approach channel to the dock of the United States Engineer Office (known as Memphis Depot). Fig. 3(*a*) shows the river survey made of the Memphis Depot area in July and August, 1935. Five conclusions are offered:

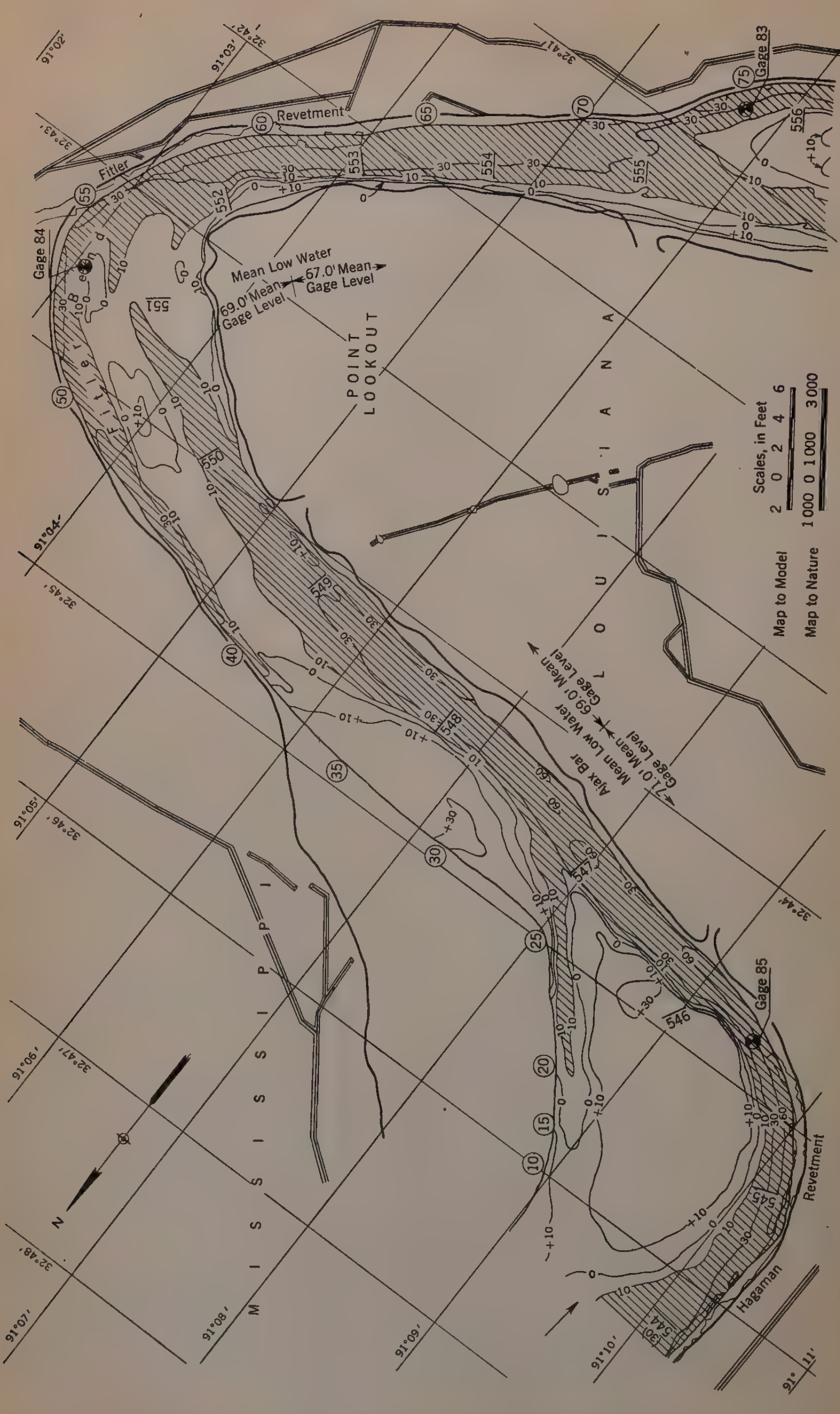

FIG. 2.—1933 SURVEY OF FITLER BEND (MISSISSIPPI RIVER), FOR CHANNEL IMPROVEMENT (MILES 543.7 TO 557.2) BELOW CAIRO, ILL.

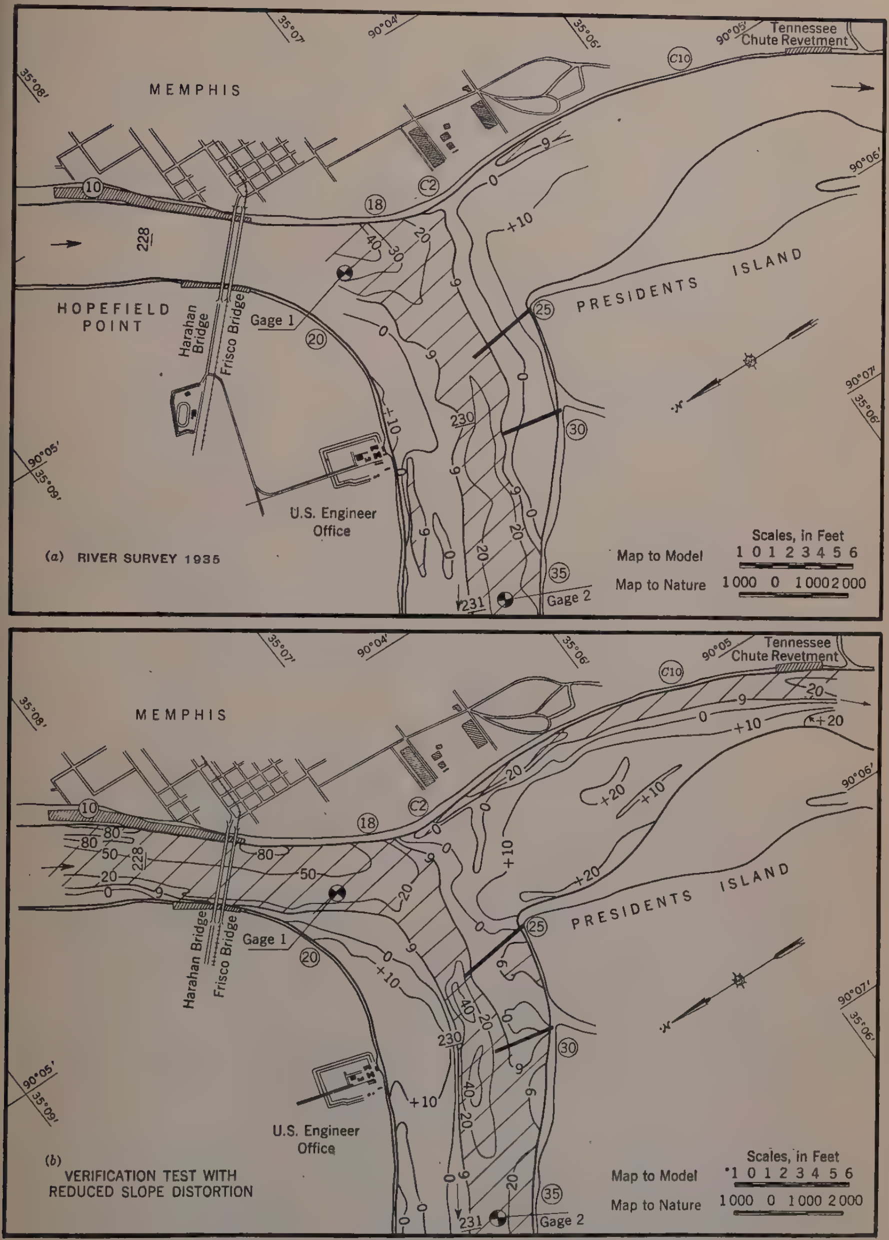

FIG. 3.—MEMPHIS DEPOT MODEL STUDIES FOR CHANNEL IMPROVEMENTS

(1) The Robinson Crusoe Island model was not suited for studying the area in the vicinity of the head of President's Island. Excessive bar building occurred on the left bank, forcing the thalweg to the center of the left-hand channel; and an excessive attack on the head of President's Island resulted.

(2) The Memphis Depot model, as first constructed, was also unsatisfactory due to excessive slope distortion $\left(S_r = \frac{20.0}{1}\right)$. (See Fig. 4.)

(3) Fig. 3(*b*) shows the improvement resulting in the verification tests due to a reduction in slope distortion. The scale was reduced, successively, from $\frac{20.0}{1}$ to $\frac{12.5}{1}$. Fig. 3(*b*) is for a slope distortion of $\frac{12.5}{1}$. A comparison of Figs. 3(*b*) and 4 with Fig. 3(*a*) shows the improvement resulting in verification tests due to a reduction in slope distortions.

Other data are:

Duration of test (Test No. 13, Run 5), in hours	27.5
Slope in prototype	0.00008
Total slope in model	0.0010
Horizontal scale of model	1 : 450
Vertical scale of model	1 : 150
Actual discharge scale	1 : 317 000
Actual velocity scale	1 : 4.7
Theoretical velocity scale, v_r	1 : 12.2
Ratio: $\frac{\text{Theoretical velocity scale}}{\text{Actual velocity scale}}$	1 : 2.6
Mean grain size of bed material, in millimeters	1.040
Specific gravity of bed material	1.85

(4) For this study excellent year-by-year field surveys were available for the area involved, and a more accurate verification thereby resulted.

(5) These tests show the advantages of using a light-weight bed material in combination with moderate depth and slope distortion instead of sand in combination with greater depth and slope distortion.

Item No. 13, Table 1.—In Berlin (27), a study of two models was conducted to determine the relative merits of depth and slope distortion in a model in which the radius of curvature of the channel was 6 m (approximately 20 ft). The following were the most significant findings:

(1) Although the tractive force available in the two models was equal (tractive force expressed as the product of depth times slope), greater bed movement occurred in the model having the greater depth.

(2) Shape and alignment of sand riffles indicated that greater depth created a greater tendency for diagonal currents or helicoidal flow.

(3) The size of the sand riffles increased with the increased depth.

(4) In the two models the location of the thalweg remained approximately the same; the depths of scour varied approximately with the depth scale. The secondary channel on the inside of the bend was not so apparent in the depth distorted model. This latter is probably due to the increased movement of sand to the inside of the bend, caused by the increase of diagonal currents.

Item No. 14, Table 1.—In the bifurcated flume tests of Item No. 14, tests were conducted with a fixed bed and with a sand bed, various types of sand being tested (4) (21) (36) (44). The purpose was to study the bed-load action at the fork of a stream. One branch of the flume model was a continuation of the approach channel and the other, or side channel, made an angle 30° with the main channel. The approach and two branch channels were of equal rectangular cross-section. (See references given for other types of channels and variation in the angle between the channels.) Pertinent results of these investigations are:

(1) The tests indicated clearly that for equal distribution of flow the water entering the side channel is mainly that flowing along the bottom and one side (nearer the side channel) of the approach channel. The zone of top currents turning into the side channel usually extends over less than 50% of the top width; the zone of bottom currents entering the side channel extends over the greater percentage of the bottom width.

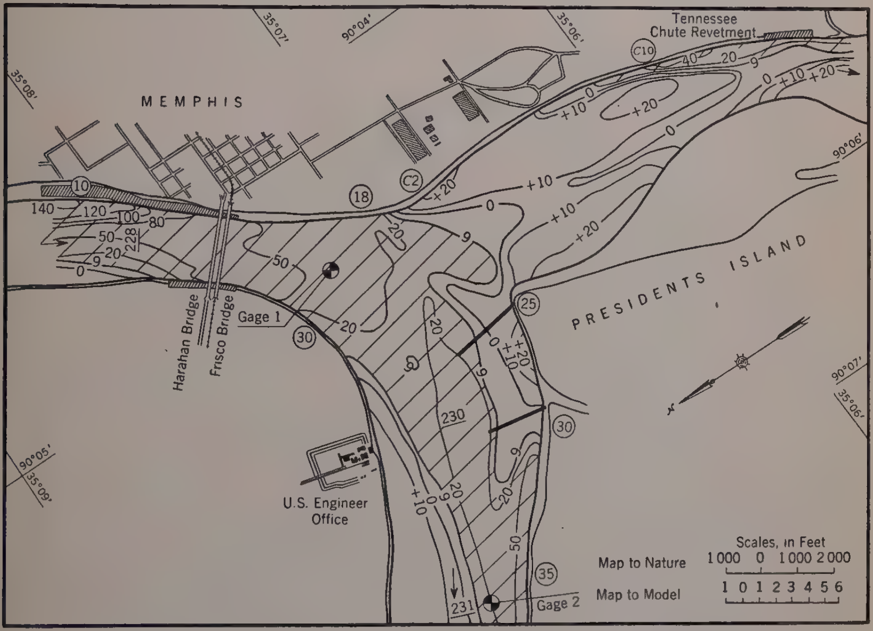

FIG. 4.—EFFECT OF EXCESSIVE SLOPE DISTORTION, MEMPHIS DEPOT MODEL STUDY

(2) The data show clearly that for a given depth of flow, as the velocity (or slope distortion) increases, the zone of bottom currents turning into the side channel increases in width. For example, for one velocity the zone of bottom currents entering the side channel may cover 70% of the bottom width of the approach channel, whereas for a higher velocity the zone may cover 80% or 90% of the width.

(3) For equal distribution of water flowing down the two channels, the percentage of bed load entering the side channel was greater than that entering

the straight channel. The percentage of bed load diverted increased as the velocity increased.

(4) For otherwise like conditions, it was shown that the bed material mixture influences the percentage distribution of bed load entering the two channels. As the bed material mixture becomes finer, and a greater percentage of the material moved is in suspension, the percentage entering the side channel decreases.

Analysis of Data and Observations

The foregoing observations, and the data in Table 1, together with some other observations not included, may be classified and analyzed under two major headings: (1) Effects of geometric distortion; and (2) behavior of bed materials. Under the first classification are discussed changes in form, changes in hydraulic capacity, changes in flow characteristics, secondary currents, and miscellaneous effects. Under the second classification are discussed the general behavior of bed materials, and the use of light-weight bed materials.

Changes in Form.—The most obvious effect of depth distortion is in the change in the form of the cross-section of the channel and of the structures. Change in form influences the back-water effect of structures and also, in movable bed models, varies the scour pattern caused by such structures. The ratio of width to depth in model and prototype is not the same, and the relative area of cross-section affected by side-wall influence on hydraulic flow is thereby altered. In movable-bed, depth-distorted models the side slope of the bed material is steepened, which may increase the rate of scour of the material; or, in cases where the angle of repose of the submerged material is exceeded it may result in an undesirable sloughing. Depth distortion changes the shape of the channel in the model, thereby causing the scale for the hydraulic radius to be a variable. The hydraulic capacity of the channel is thus affected. All the foregoing effects of depth distortion must be considered in model design, operation, and interpretation.

Hydraulic Capacity of Channel.—Both depth distortion and slope distortion influence the hydraulic capacity of an open channel. The hydraulic capacity of a channel is its capacity to carry water, as measured by the rate of flow through a given reach for a given stage. Based on Manning's formula for velocity the capacity of a channel in model and prototype is similar only when,

$$Q_r = \left[\frac{L\, d\, R^{0.67}\, S^{0.50}}{n}\right]_r \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

in which Q_r = discharge scale $= \frac{Q_m}{Q_p}$; R_r = hydraulic radius scale $= \frac{R_m}{R_p}$; and n_r = coefficient of roughness scale $= \frac{n_m}{n_p}$.

In depth-distorted models it has been shown that R_r varies from cross-section to cross-section and also in the same cross-section for different stages; hence, if Q_r is to remain constant, n_r must vary within each cross-section and from cross-section to cross-section. It is common practice to vary n_r by roughening a model by trial and error.

In the Helena-Donaldsonville model (Item No. 6, Table 1) the roughness varied from rough stuccoed concrete to extremely smooth concrete. In fact, in a few isolated sections it was impossible to make the concrete smooth enough and the section was enlarged arbitrarily.

If the slope of the model is also distorted, the roughness must be further increased to compensate for the greater slope, or the discharge scale will increase. In many studies the actual value of the discharge scale is relatively unimportant as long as a similarity of hydraulic capacity is obtained; but in other studies it is desirable to comply with Froude's requirement for the velocity scale. (Supplementary flume tests showed that compliance with Froude's law was desirable for the St. Clair River model tests.) For a distorted model this requirement is met only when Equation (2*b*) holds. Combining Manning's formula and Froude's law, it can easily be shown that this formula applies only when,

$$\frac{R_r^{0.67}\, S_r^{0.50}}{d_r^{0.5}\, n_r} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

(The criterion (35), $v_r = d_r^{0.5}$ when $d_r = L_r^{0.75}$, is based on the assumption that $n_r = 1$, which is seldom true.)

When the degrees of depth and slope distortion are equal, it follows that greater roughness must be used as the degree of distortion increases. For such a distorted model of a rectangular channel the roughness scale should increase as the width-depth ratio of the prototype, the degree of distortion, or the horizontal scale of the model, increases (21). Consequently, the velocity scale must often exceed the value indicated by Equation (2*b*). Where this requirement must be satisfied, one must roughen the model to extremes, reduce the depth and slope distortion, or reduce the slope distortion.

Changes in Flow Characteristics.—Slope distortion and depth distortion will cause changes in the characteristics of flow. Variation of the depth or slope distortion will affect the magnitude of velocity and also the velocity distribution, such as to change the form of the vertical and horizontal velocity distribution curves. Another effect of distortion is to alter the direction of currents. Particularly, the relative divergence of surface and sub-surface currents may be changed. Distortion may cause eddies to occur in a model where none exists in the prototype; or it may affect the size, shape, location, and intensity of existing eddies or rollers. The intensity of turbulence or the pulsating character of the flow may be altered. Another common effect of distortion is an undesired increase in the transverse slope of the water surface. Many experimental data are available to show specific instances of such changes, but insufficient data are at hand to provide a basis for the formulation of definite laws concerning such changes.

Of particular interest is the effect of geometric distortion on the shape of the vertical velocity curve. Experimental data and field observations on natural streams show definitely that depth, slope, form of channel, and roughness affect the form or shape of the vertical velocity curve. For example, the following statements have been fairly well substantiated by field and laboratory observations:

(*a*) The vertical velocity curve approximates a parabola, the axis of which is horizontal and passes through the point of maximum velocity.

(*b*) As the roughness of the bottom increases, the depth of the filament of maximum velocity and that of mean velocity in any vertical, is less. The ratio of surface to bottom velocities also increases.

(*c*) For the range of model slopes and model depths commonly used in hydraulic laboratories, a change in slope or depth distortion will result in a change in the vertical velocity distribution. The exact nature of the law of variation has not yet been determined conclusively.

(*d*) Side-wall effect on the vertical velocity curve is apparent to a point about 2.5 times the depth from a side wall.

(*e*) A rapidly rising channel bottom—that is, a bottom of adverse slope—tends to make the velocity distribution more uniform, whereas a rapidly falling bottom tends to make the ratio of upper to bottom velocities increase.

Hence, a change in the combination of depth and slope distortion will change the vertical velocity curve in some unpredictable manner. The change in distribution of velocity becomes of particular importance when a bend section or divided flow around an island or obstruction is considered.

Secondary Currents.—When water flows in an open channel secondary currents usually exist. From the standpoint of hydraulic models the most important of these are the cross-currents near the bottom of a stream, which are set up in a channel whenever there is a change in direction of flow such as that caused by a curved channel, a divided channel, a large obstruction, deflecting dikes, etc.

One explanation for the cross-currents is as follows: Usually in an open channel the surface currents are moving faster than the bottom currents. Hence, when a change in direction occurs, it takes less force to change the direction of the bottom, than the surface, currents. At a bend in the channel a transverse slope is set up due to the change in direction. This slope acts as an unbalanced force on each filament of current and deflects it so that the radius of curvature of each filament is approximately proportional to the square of its velocity. In a bend section the radius of curvature is generally less for the bottom than for the top currents and this phenomenon is a part of helicoidal flow.

In general, such cross-currents exist for any change of direction. For example, in Item No. 14, Table 1, the bulk of the water entering the side channel was that water which had been flowing near the bottom of the approach channel. In these tests the horizontal angle between the direction of the top and bottom currents varied from zero to 180 degrees.

In movable bed studies the engineer is particularly interested in the direction of the bottom currents because they are the principal determining factor in the direction and amount of movement of the bed.

Inasmuch as geometric distortion affects the magnitude of velocity and also the form of the vertical velocity curve, it is logical to assume that the nature of the cross-currents will also be influenced. Items Nos. 10, 12, 13, and 14, in Table 1, and other studies, illustrate, directly or indirectly, the effect of a change in distortion on the secondary currents.

Analysis of these studies indicates that usually greater depth or slope distortion will increase the divergence of surface and bottom currents, and that the divergence is more marked for an increase in depth distortion than for an equal increase in slope distortion. Other factors, such as horizontal velocity distribution, relation between wave velocity and mean velocity, roughness, effect of one bend on a lower bend, etc., greatly affect the nature of flow. Additional data are needed before conclusive relationships can be established.

Miscellaneous Effects.—In addition to the specific examples referred to in Table 1, countless other examples of discrepancies introduced by geometric distortion could be presented. Many of these discrepancies are unimportant or may be properly weighed in interpreting model results. Among the more important, especially in movable bed models, are the following effects which have been observed at the U. S. Waterways Experiment Station (in all cases the name of one representative model or reach of a Mississippi River model is given in parentheses, see Fig. 1 for the geographic location of many of these studies):

I. Excessive Scour:
    (*A*) At a bend (Hotchkiss Bend);
    (*B*) At the end of deflecting dikes (Island 35);
    (*C*) At the head of sand-bars, tow-heads, and islands (Cat Island);
    (*D*) At relatively narrow sections (American Cut-Off); and,
    (*E*) At a point of junction of two streams (Vice-President's Island).
II. Improper Location of Scour:
    (*A*) At a bend (Head of Passes); and,
    (*B*) Displacement down stream below a sill or submerged dike (Head of Passes).
III. Unstable Side Slopes:
    (*A*) Steep bank occurring in prototype (Memphis);
    (*B*) Dredging holes or cuts (Cat Island); and,
    (*C*) Sand dikes (Rifle Point).
IV. Other Effects:
    (*A*) Excessive bar building at a relatively wide section (Waterproof Cut-Off);
    (*B*) Displacement down stream and relative straightening of crossing (Cow Island);
    (*C*) Introduction of eddy and a resultant bar at a point of sudden widening of the channel (Cottonwood Bar);
    (*D*) Improper thalweg alignment and sand deposits in a bend (Cat Island);
    (*E*) Improper division of flow and excessive development of one channel and excessive deterioration of the other in cases of divided flow around an island (Harklerodes Island).

Behavior of Bed Materials.—In most model studies the movement of the bed material is by rolling or sliding along the bottom; hence, the movement of material and the resulting bed configuration is largely influenced by the bottom currents. Inasmuch as the direction of bottom currents is influenced by distortion, the bed configuration will also be affected.

A change in the bed material may change the bed configuration. This is illustrated in the Severn Estuary model (Item No. 7, Table 1). An explanation is as follows: A change in the grain size or the specific weight of the bed material will affect the rate of movement and may affect the type of movement (that is, movement by riffles to movement by banks, or true bed-load movement to movement in suspension, etc.). Any change in the nature of movement will increase or decrease the roughness, which, in turn, will affect the magnitude of velocity and the cross-currents. Hence, the bed configuration may be changed by merely substituting one bed material for another. One may take advantage of this factor during the verification of a model. If one bed material will not provide a successful verification, a change of bed material may materially improve the verification.

Light-Weight Bed Materials.—For a given hydraulic model study, the degree of geometric distortion necessary may be reduced by the use of light-weight bed materials instead of natural sand (see Fig. 5 and Table 2). Many laboratories have resorted to the use of such materials. In Europe, broken briquettes of coal, pumice, and amber are being used successfully. The U. S. Waterways Experiment Station has operated models with haydite, coal, and gilsonite and the use of (I) limed rosin has been considered.

TABLE 2.—CHARACTERISTICS OF MODEL BED MATERIAL, CURVES IN FIG. 5

Curve No. (see Fig. 5)	Material	Specific gravity, ρ	Mean grain size, D_g, in millimeters	Slope of test, S
1	Gilsonite	1.028	3.112	0.0003
2	Gilsonite	1.028	1.165	0.0003
3	Gilsonite	1.052	3.526	0.0003
4	Gilsonite	1.070	3.553	0.0003
5	Gilsonite	1.052	1.249	0.0003
6	Gilsonite	1.070	0.899	0.0003
7	Wood rosin (5.5% lime)	1.111	1.317	0.0003
8	Wood rosin (5.5% lime)	1.111	2.780	0.0003
9	"Slack" bituminous coal	1.36	1.081	0.0005
10	"Slack" bituminous coal	1.36	1.081	0.0010
11	"Semi-anthracite" coal (briquettes)	1.30	1.168	0.0010
12	"Carona" coal (bituminous)	1.31	1.097	0.0010
13	"Carona" coal (bituminous)	1.31	1.097	0.0005
14	"Semi-anthracite" coal (briquettes)	1.30	1.168	0.0005
15	"Semi-anthracite" coal (briquettes)	1.30	2.610	0.0005
16	"Semi-anthracite" coal (briquettes)	1.30	2.610	0.0010
17	"Slack" bituminous coal	1.36	2.481	0.0010
18	"Slack" bituminous coal	1.36	2.481	0.0005
19	"Carona" coal (bituminous)	1.31	2.008	0.0010
20	"Haydite" (⅛ in.)	1.85	0.908	0.0005, 0.0010, and 0.0015
21	"Haydite" (No. 6)	1.85	1.040	0.0005, 0.0010, and 0.0015
22	Fitler sand	2.65	0.485	0.0010

Light-weight materials have the advantage over heavier materials in that, for a given grain-size, movement occurs at a lower tractive force and that coarser light-weight materials may be used instead of heavier sand to reduce riffling and the degree of distortion. However, the apparent specific gravity

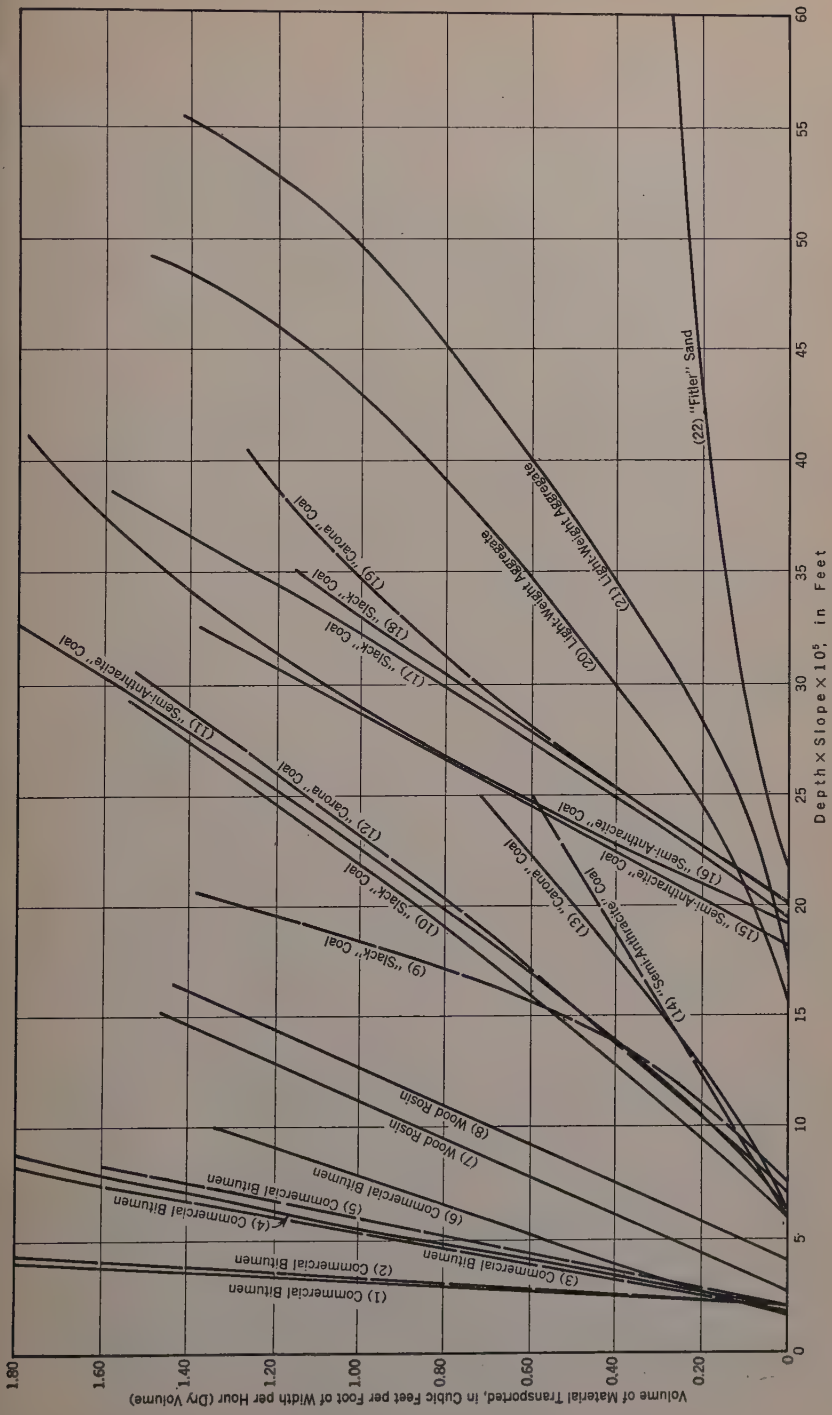

FIG. 5.—RESULTS OF FLUME TESTS ON LIGHT-WEIGHT BED MATERIALS

and size of coal, pumice, and haydite grains change after continued use. Amber and gilsonite are very costly and are extremely difficult to use. Because of their light weight these materials are not suitable for studying wave action on beaches or certain types of river problems where bars are exposed. Unless gilsonite is soaked in water, shaped rapidly to conform to the model bed, and flooded almost immediately, tiny air bubbles attach themselves to the grains and cause the material to float. However, the use of light-weight materials is feasible, and it is recommended as a substitute for extreme geometric distortion. Experience indicates that a combination of moderate depth distortion, moderate slope distortion, and light-weight bed material is better than the use of one extreme to the exclusion of the others.

CONCLUSIONS

Summary.—This study shows the following: (*a*) There is a definite need for constructing geometrically distorted models, particularly because such models are often suitable when a geometrically similar model is out of the question on account of cost or other considerations; (*b*) geometrically distorted models should be used with extreme caution; (*c*) arbitrary measures may sometimes be taken to compensate for some of the effects of distortion; (*d*) light-weight bed materials may be used to reduce the degree of distortion necessary; (*e*) in general, a change in the degree of depth or slope distortion, or in the bed material, may result in a change in model results; and finally (*f*) owing to the complex nature of many of the effects of distortion the importance of model verification cannot be over-emphasized.

Recommendations.—On the basis of this study the following recommendations are made: (1) Geometrically distorted river models should be avoided whenever suitable undistorted models can be constructed at reasonable cost; (2) distorted models should be designed for specific purposes and they should not be used for general studies; (3) the designer should weigh, intelligently, the probable effects of distortion in order to avoid failures or reconstruction costs; he should consider carefully the advisability of using light-weight bed materials instead of extreme depth or slope distortion; and (4) the operator should assure himself by appropriate comparisons of model and prototype behavior that the model faithfully simulates the prototype in the particular details being studied.

ACKNOWLEDGMENTS

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[4] The paper is based on data contained in "The Observed Effects of Geometric Distortion in Hydraulic Models," a thesis presented by the writer to the State University of Iowa in 1937, in partial fulfillment of the requirement for the degree of Doctor of Philosophy.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

THE THREE-POINT PROBLEM IN A CO-ORDINATED FIELD

By R. Robinson Rowe,[1] M. Am. Soc. C. E.

Synopsis

An improved method of solving the classical three-point problem, particularly adapted to orientation or location in a previously co-ordinated field, and to calculation by machine instead of by logarithms, is offered in this paper. The probable normal and position errors due to errors of observation are evaluated to measure the precision of the location and, in case more than three points are observed, to aid in the selection of the best three-point set or to weight the adjustment of all sets. A graphical adjustment of the "four-or-more-point" problem is outlined and a practical problem is solved to illustrate the use of the formulas.

The Problem

In Fig. 1, Points *O*, *A*, *B*, *C*, and *D* are triangulation stations with known co-ordinates in a plane system. If an observer at Point *P* measures the angle, $O\,P\,A = \alpha$, he has limited his position locus to the circle, $O\,P\,A$. If he then measures Angle $O\,P\,B = \beta$, he determines his position as the intersection of the circles, $O\,P\,A$ and $O\,P\,B$, unless these two circles coincide. Of course, the circles intersect at two points, but one will be Point *O*; and it can be shown that Angle $A\,P\,B = \beta - \alpha = \theta$ determines a third circle which passes through the intersection at Point *P*. Hence, the observer should measure all three angles and adjust the results before further computation; or he should measure the two angles defining the circles with the best intersection.

If one measures Angles θ and $C\,P\,D$, he defines two position-locus circles ordinarily intersecting at two points. His position is fixed if he can decide between them—by a compass bearing, for instance. Such data may give strong locations, but they will be weak if the circles are nearly tangent, or if they are coincident.

With four points, *A*, *B*, *C*, and *D*, visible, it is best to measure the three angles, θ, $B\,P\,C$, and $C\,P\,D$, and also Angle $A\,P\,D$ for check and adjustment.

Note.—Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted by **October 15, 1938.**

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Then, Angles $A\,P\,C$ and $B\,P\,D$ may be calculated, so that six position-locus circles are defined. Unless all data are exact, there will be four independent points of intersection, due to errors of observation at Point P, errors in the previous co-ordination of the signals, and to the plane projection distortion of the field. If the four intersections are tolerably approximate, considering the probable error of angular measurements, a mean position can be determined by least squares or by the graphical method outlined herein. Otherwise, it may be assumed that at least one of the signals was not accurately located; or, that it has since shifted in position. Such a shift should be anticipated when depending upon old co-ordinates of spires, due to uneven subsidence, translation by earthquake, or altered reconstruction after fires.

If but one signal is in error, a fifth signal can be used to determine which should be rejected. Then, a complete set would determine ten circles and ten independent intersections. A graph of these intersections would show four points tolerably approximate, the other six being erratic and dependent upon the signal which must be rejected.

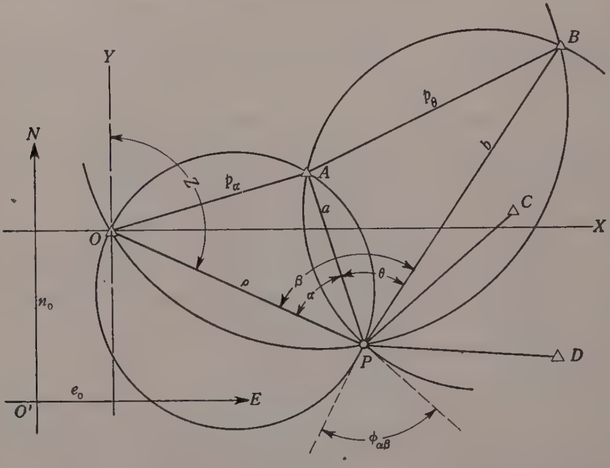


FIG. 1

Use of six or more signals increases the labor of solution, but there is the possibility that more than one station must be rejected or that some locus circles will intersect at very acute angles, too small for the desired precision. The field data to all visible signals should be obtained. In general, for observations on n signals, the number of locus circles, N_c, is:

$$N_c = 0.5\,n\,(n - 1) \ldots\ldots\ldots\ldots (1)$$

and the number of independent intersections, N_i, is:

$$N_i = \frac{n!}{6\,(n - 3)!} \ldots\ldots\ldots\ldots (2)$$

Along the coasts, many areas have been co-ordinated by the United States Coast and Geodetic Survey and the United States Engineer Office. Reestablishment of the instrument points is not always simple, but co-ordinated spires are usually abundant. The three-point method affords a practical device for: (1) Connecting to the U. S. Coast and Geodetic horizontal datum (plane and geographical co-ordinates); (2) determining azimuth without astronomical observations, particularly when the sun or the stars are obscured, or when they are not in favorable position; or (3) controlling a stub traverse by three-point ties near each end, which, incidentally, serves to check base lines or rate the tape used in the traverse. The solutions of connections required in a resurvey of a part of the mean high-tide line of Coronado, South Island, California, in 1929, led to the development of the methods described in this paper.

SOLUTIONS

The graphical solution on the plane-table is known to all topographers and is explained in most surveying texts. The methods in this paper are not applicable to such work. The usual trigonometric solution requires: (*a*) Computing closing courses and distances between the observed signals; (*b*) solving a series of three triangles which depend upon the locus circles (and two more triangles for a check); and (*c*) computing latitudes and departures from one signal to the instrument position. Reference is made to any surveying textbook for details of the operations.

A simpler method is derived as follows: In Fig. 1, the given co-ordinates of triangulation stations at Points O, A, and B, referred to rectangular Cartesian axes, $O'\,N$ and $O'\,E$, are (e_o, n_o), (e_a, n_a), etc. By the linear transformation, the formulas,

$$x = e - e_o \qquad (3a)$$

and,

$$y = n - n_o \qquad (3b)$$

refer all co-ordinates to the auxiliary origin, O, such that $O'\,N$ is parallel to $O\,Y$ and perpendicular to $O\,X$, and the co-ordinates are reduced by subtraction to $O = (0, 0)$, $A = (x_a, y_a)$, etc. To determine the position of Point P, Angles α and β have been measured. Let the azimuth of $O\,P$ be Z = Angle $Y\,O\,P$.

Determining the equations of Lines $O\,P$, $A\,P$, and $B\,P$, Fig. 1, from slope forms, and remembering that $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$,

Line $O\,P$:

$$\frac{y}{x} = \frac{y_p}{x_p} = \cot Z \qquad (4a)$$

Line $A\,P$:

$$\frac{y - y_a}{x - x_a} = \frac{y_p - x_p \tan\alpha}{x_p + y_p \tan\alpha} \qquad (4b)$$

and, Line $B\,P$:

$$\frac{y - y_b}{x - x_b} = \frac{y_p - x_p \tan\beta}{x_p + y_p \tan\beta} \qquad (4c)$$

and, these lines being concurrent at Point P, simultaneous solution of their equations will determine the position of P. Substituting Equation (4*a*) in

Equations (4*b*) and (4*c*) to eliminate x_p and y_p, then:

$$(x^2 + y^2) \tan \alpha = x\,(x_a \tan \alpha + y_a) + y\,(y_a \tan \alpha - x_a) \tag{5a}$$

and,

$$(x^2 + y^2) \tan \beta = x\,(x_b \tan \beta + y_b) + y\,(y_b \tan \beta - x_b) \tag{5b}$$

Dividing Equation (5*a*) by Equation (5*b*) and simplifying:

$$\frac{y}{x} = \frac{(x_b - x_a) \tan \alpha \tan \beta + y_b \tan \alpha - y_a \tan \beta}{(y_a - y_b) \tan \alpha \tan \beta + x_b \tan \alpha - x_a \tan \beta} \tag{6}$$

Dividing the third and fourth parts by tan α tan β and substituting Equation (4*a*):

$$\cot Z = \frac{x_a - x_b + y_a \cot \alpha - y_b \cot \beta}{- y_a + y_b + x_a \cot \alpha - x_b \cot \beta} \tag{7}$$

At Point P in Equation (5*a*), $x = x_p$; $y = y_p$; $x^2 + y^2 = x^2 \csc^2 Z$; and, $y = x \cot Z$. Substituting these values in Equation (7):

$$x_p^2 \csc^2 Z \tan \alpha = x_p\,(x_a \tan \alpha + y_a) + x_p \cot Z\,(y_a \tan \alpha - x_a) \tag{8a}$$

$$x_p = \sin^2 Z\,[x_a + y_a \cot \alpha + \cot Z\,(y_a - x_a \cot \alpha)] \tag{8b}$$

$$y_p = \cos^2 Z\,[\tan Z\,(x_a + y_a \cot \alpha) + y_a - x_a \cot \alpha] \tag{8c}$$

and,

$$l = O\,P = \sin Z\,(x_a + y_a \cot \alpha) + \cos Z\,(y_a - x_a \cot \alpha) \tag{8d}$$

For the most direct solution, solve Equation (7) for Z; then Equation (8*b*) for x_p; and then:

$$y_p = x_p \cot Z \tag{9}$$

If Z is not required, solve Equation (7) for cot Z; and, for Equation (8*b*), use

$$x_p = \frac{1}{1 + \cot^2 Z}\,[x_a + y_a \cot \alpha + \cot Z\,(y_a - x_a \cot \alpha)] \tag{10}$$

In Equation (7) there will be two solutions for Z differing by 180°; but if the wrong Z is selected, then l from Equation (8*d*) will be negative and the same point will be determined by either choice of Z. The other equations are free from ambiguity except in singular cases. In the insoluble case with O, A, B, and P lying on a circle, Equation (7) gives $\cot Z = \frac{0}{0}$. If $\alpha = 0°$ or $180°$, $\cot Z = \frac{\infty}{\infty}$, the apparent indeterminacy having been introduced by the division by $\tan \alpha \tan \beta\ (= 0)$. This case is best solved by a simple resection triangle; or, determining Equation (7) by limits:

$$\cot Z = \frac{y_a}{x_a} \tag{11}$$

and, substituting Equation (11) in Equation (5*b*),

$$x_p = \frac{x_a}{x_a^2 + y_a^2}\,[x_a\, x_b + y_a\, y_b + \cot \beta\,(x_a\, y_b - x_b\, y_a)] \tag{12}$$

To complete the solution, co-ordinates to the origin, O', can be computed by reversing the transformation of Equations (3) by:

$$e_p = x_p + e_o \tag{13a}$$

and,

$$n_p = y_p + n_o \tag{13b}$$

If true azimuth is desired, Z must be corrected for the difference in longitude represented by the distance, e_p.

RELATIVE WEIGHTS AND FIGURE STRENGTHS

In Fig. 1, with Angle θ observed between given Signals A and B, the equation of Circle $P\ A\ B$ is:

$$\begin{aligned} x^2 + y^2 - x\,[x_b + x_a + (y_b - y_a) \cot \theta] - y\,[y_b + y_a - (x_b - x_a) \cot \theta] \\ + x_b\, x_a + y_b\, y_a + (x_a\, y_b - x_b\, y_a) \cot \theta = 0 \end{aligned} \tag{14}$$

The tangent to this circle at Point P is:

$$\begin{aligned} & x\,[2\,x_p - x_a - x_b + (y_a - y_b) \cot \theta] \\ & \quad + y\,[2\,y_p - y_a - y_b - (x_a - x_b) \cot \theta] + 2\,x_a\,x_b \\ & \quad + 2\,y_a\,y_b - x_p\,(x_a + x_b) - y_p\,(y_a + y_b) \\ & \quad + \cot \theta\,[2\,x_a\,y_b - 2\,x_b\,y_a + x_p\,(y_a - y_b) - y_p\,(x_a - x_b)] = 0 \end{aligned} \tag{15}$$

The slope of the circle and tangent at Point P is:

$$\lambda_{p(ABP)} = - \frac{2\,x_p - x_a - x_b + (y_a - y_b) \cot \theta}{2\,y_p - y_a - y_b - (x_a - x_b) \cot \theta} \tag{16a}$$

and the slope for Circle $O\ A\ P$, substituting α for θ and $x_b = y_b = 0$:

$$\lambda_{p(OAP)} = - \frac{2\,x_p - x_a - y_a \cot \alpha}{2\,y_p - y_a + x_a \cot \alpha} \tag{16b}$$

In the general triangle, $A\ B\ P$, let the three angles be A, B, and θ, and let the opposite sides be b, a, and p_θ, respectively. If there was a small error, $\Delta\theta$, in the measurement of θ there will be little change in the slope from Equation (16a). Practically, the tangent is in error by a normal distance, ΔN, such that:

$$\begin{aligned} \Delta N &= \tfrac{1}{2}\, p_\theta\, [\Delta\,(\csc \theta) + \Delta\,(\cot \theta) \cos\,(A - B)] \\ &= - \frac{p_\theta\, \Delta\theta}{2 \sin^2 \theta}\, [\cos \theta + \cos\,(A - B)] \\ &= - p_\theta\, \Delta\theta\, \frac{\sin A \sin B}{\sin^2 \theta} = - \Delta\theta\, \frac{ab}{p_\theta} \end{aligned} \tag{17}$$

If the error in seconds of arc is expressed by δ'', and N is the absolute normal error:

$$N_\theta = \frac{(\delta'')^{ab}}{206\,265\, p_\theta} \tag{18}$$

In the use of Equation (18) in the solution of four-or-more-point systems, it is necessary to solve for N only to two or three significant figures. Having approximated P from one set of three points and λ for any pair of those points, a, b, and p can be scaled; δ'' can be determined as the probable error of that measurement; and, N can be computed. Now, the angle, $\theta \pm \delta''$, has limited the locus for P to a band of width, $2\,N$, slope, λ, centered on Point P, since a short part of the locus circle is essentially a straight line. Intersection of this band with bands pertinent to other combinations of points should further limit this locus area. If probable errors for all sets are then gradually reduced in the same ratio until only one point is common to all bands, there will result an acceptable solution which will generally be very close to the least squares solution. The reduction can be done rapidly by cut-and-try on a large-scale graph.

If the bands do not have a common area, at least one error will exceed the computed probable error. To obtain a solution, gradually increase the widths of all bands in the same ratio until there is just one point common to all. In either case, the common point will be the intersection of the edges of three bands. If two of these critical bands are nearly parallel, the solution can usually be improved by letting these two nearly parallel band edges determine their median as a line locus; then the width of all other bands can be decreased gradually in proportion to their normal errors until the common area of these remaining bands has only one point on the determined line locus. This adjustment will have increased the departure from the nearly parallel bands by a small amount, but will have decreased the departure from most of the other bands by a relatively large amount.

Equation (18) can be used reciprocally for weighting a series of angles observed in a four-or-more-point system. The relative strength of different such figures is found by holding δ'' constant; and, since constants are unnecessary in strength comparison, the relative figure strength may be expressed as:

$$s_\theta = \frac{p_\theta}{a\,b} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

To obtain the absolute position error from a three-point observation with only two angles measured (such as Angles α and θ) compute N_α and N_θ from Equation (18), measure or compute $\phi_{\alpha\theta}$, the acute angle between slope tangents, $\lambda_{p(OPA)}$ and $\lambda_{p(APB)}$, from Equation (16a) or Equation (16b). Then, the absolute position error, $\Delta_{\alpha\theta}$, is the long semi-diagonal of a parallelogram with one angle, $\phi_{\alpha\theta}$, and altitudes, N_α and N_θ; or,

$$\Delta_{\alpha\theta} = \csc \phi_{\alpha\theta} \sqrt{N_\alpha^2 + N_\theta^2 + 2\,N_\alpha\,N_\theta \cos \phi_{\alpha\theta}} \;\ldots\ldots\ldots (20a)$$

and, reciprocally, the relative strength of the three-point figure compared to others is:

$$S_{\alpha\theta} = \frac{p_\alpha\,p_\theta \sin \phi_{\alpha\theta}}{a\sqrt{l^2\,p_\theta^2 + 2\,b\,l\,p_\alpha\,p_\theta \cos \phi_{\alpha\theta} + b^2\,p_\alpha^2}} \;\ldots\ldots\ldots (20b)$$

If all three angles, α, β, and θ, were measured, the probable errors of each observation are interdependent and there would be three solutions for Equations (20). If the angles and probable error are balanced, the three normal-error bands will intersect with a hexaparallelogram for a common area, and the position error is the greatest of the three semi-diameters,

$$\Delta_{\alpha\beta\theta} \geqq N_\theta \csc \theta_{\beta\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

or, $N_\alpha \csc \phi_{\alpha\beta}$, etc. Reciprocally, the relative strength of the three-point figure with all three angles measured, is,

$$S_{\alpha\beta\theta} \leqq S_\alpha \sin \phi_{\alpha\theta} \text{ (etc.)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21b)$$

EXAMPLE

Fig. 2 shows positions of seven spires observed from Point P, which was taken arbitrarily on the roof of the Guymon Building, in San Diego, Calif.

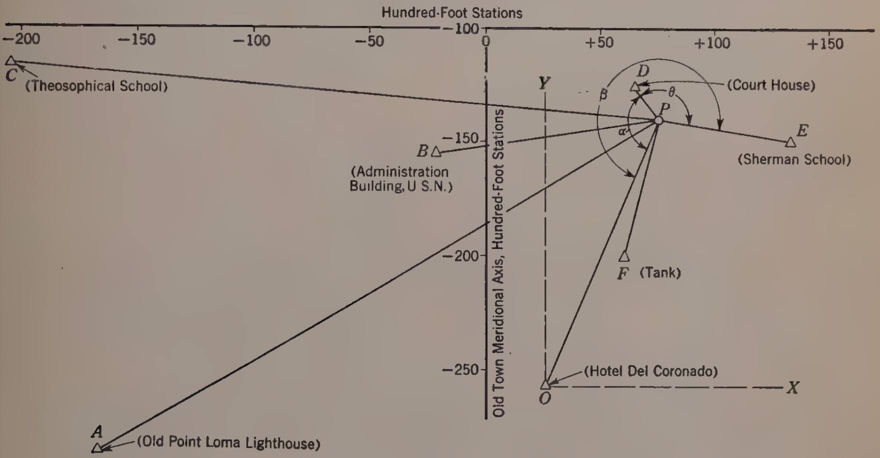


FIG. 2.—LOCATION OF PROBLEM STATIONS, SAN DIEGO COUNTY, CALIFORNIA

Plane co-ordinates were taken from the "Map of San Diego Harbor, Calif., Harbor Lines," of the U. S. Engineer Office, dated September 24, 1927, which data had been computed from the U. S. Coast and Geodetic Survey's triangulation of many years before. Angles were turned with an engineer's transit equipped with a 30″ vernier. To illustrate the method of weighting, repetitions were purposely varied in the range from 4 to 20. The basic data and the transformation of Equation (3a) are given in Table 1.

TABLE 1.—BASIC DATA AND SOLUTION OF EQUATION (3a)

No.	SIGNAL MARK		CO-ORDINATES				Observed angle, clockwise from 0
	Spire name	Point (see Fig. 2)	Old Town Origin		Transformed		
			East	North	X	Y	
63	Hotel del Coronado....	O	+ 2 469.5	−25 641.5	0.0	0.0	0° 0′ 0″
3	Point Loma Light House	A	−16 850.1	−28 687.8	−19 319.6	− 3 046.3	35° 31′ 52″
574	Administration Building	B	− 2 196.7	−14 417.6	− 4 666.2	+11 223.9	64° 02′ 40″
534	Theosophical School....	C	−19 543.7	−11 587.1	−22 013.2	+14 054.4	71° 43′ 05″
33	Court House..........	D	+ 6 466.4	−12 618.6	+ 3 996.9	+13 022.9	119° 18′ 17″
39	Sherman School........	E	+13 228.2	−14 985.0	+10 758.7	+10 656.5	256° 43′ 00″
65	Tank.................	F	+ 5 941.4	−20 033.7	+ 3 471.9	+ 5 607.8	351° 05′ 30″

A first approximation, p_1, is computed from the three-point Triangle $O\,D\,E$, and Equations (7), (8*b*), and (9):

Given: x_d, y_d, x_e, y_e, α, and β, from Table 1

$\alpha = 119° \, 18' \, 17''$; $\cot \alpha = -0.5612822$

$\beta = 256° \, 43' \, 00''$; $\cot \beta = +0.2360829$

$x_d =$	$+\ 3\,996.9$	
$y_d \cot \alpha =$	$-\ 7\,309.52$	$-\ 3\,312.62$
$-x_e =$		$-10\,758.7$
$-y_e \cot \beta$		$-\ 2\,515.82$
numerator		$-16\,587.14$
$-y_d$	$-13\,022.9$	
$x_d \cot \alpha$	$-\ 2\,243.39$	$-15\,266.29$
y_e		$+10\,656.5$
$-x_e \cot \beta$		$-\ 2\,539.95$
denominator		$-\ 7\,149.74$

$$\cot Z = \frac{-16\,587.14}{-\ 7\,149.74} = +2.3199641$$

$Z = 203° \, 19' \, 04.9''$

$\sin Z = -0.3958345$

$x_p = (-0.3958345)^2 \, (-3\,312.62 + 2.3199641 \times 15\,266.29) = +5\,030.31$

$y_p = 2.3199641 \times 5\,030.31 = +11\,670.14$

The foregoing outline would complete the solution if no other signals had been observed, and Equation (13*a*) would give P = East 7 499.81, South 13 971.36. From Equation (16*a*), $\lambda_{p(ODP)} = -\dfrac{2 \times 5\,030.31 + 3\,312.62}{2 \times 11\,670.14 + (-\,15\,266.29)} = -\,1.6563$; and, similarly, $\lambda_{p(OEP)} = +\,0.211$; and, $\lambda_{p(DEP)} = -\,0.945$. From field data not given, the probable angular errors were computed to be: $\delta_\alpha'' = 4.7$; $\delta_\beta'' = 5.9$; and $\delta_\theta'' = 7.6$. By scaling Fig. 2, $l = 12\,700$; $P\,D = 1\,700$; $P\,E = 5\,800$; and $O\,D = 13\,600$. Then, from Equations (18) and (19): $N_\alpha = \dfrac{4.7 \times 12\,700 \times 1\,700}{206\,265 \times 13\,600} = 0.036$ ft. Similarly, $N_\beta = 0.142$; $N_\theta = 0.051$; $S_\alpha = \dfrac{13\,600}{12\,700 \times 1\,700} = 0.00063$; $S_\beta = 0.00020$; and $S_\theta = 0.00071$. Hence, in this three-point set, Triangle $P\,D\,E$ is the strongest figure, but the greater accuracy used in measuring the angle in Triangle $P\,O\,D$ gives the latter a greater weight.

The angles between slope tangents were measured by protractor as $\phi_{\alpha\beta} = 70.8°$; $\phi_{\alpha\theta} = 15.5°$, and $\phi_{\beta\theta} = 55.3°$. From Equation (20*a*),

$$\Delta_{\alpha\beta} = \csc 70.8° \sqrt{0.036^2 + 0.142^2 + 2 \times 0.036 \times 0.142 \cos 70.8°} = 0.177$$

and, similarly, $\Delta_{\alpha\theta} = 0.322$; and, $\Delta_{\beta\theta} = 0.220$. Hence, if only two of the three angles are to be measured, α and β would give the greatest accuracy. With all three angles measured, the position error is reduced to $\Delta_{\alpha\beta\theta} = 0.142 \csc 70.8° = 0.151$. For comparison with other three-point sets, the relative strength of the figure is $S_{\alpha\beta\theta} = 0.00063 \sin 15.5° = 0.000166$.

In routine calculations, the experienced computer will usually be able to select the strongest figures by eye. It may be of interest, however, to compare calculated strengths for the great variety of figures shown in Fig. 2. With calculations similar to the foregoing, using a slide-rule or a graph where less than four significant figures are carried, the data in Table 2 were obtained for the

TABLE 2.—RELATIVE VALUES OF COMPUTED STRENGTH

Signals (1)	Probable error, in seconds of arc (2)	Slope (3)	Normal error for 1″ (4)	Probable normal error (5)	Strength, $10^5 S$ (6)
O A	2.6	+50.45	0.090	0.23	5.4
O B	3.4	− 0.941	0.050	0.17	9.8
O C	5.3	−13.64	0.065	0.34	7.5
O D	4.7	− 1.656	0.008	0.04	63
O E	5.9	+ 0.211	0.024	0.14	20
O F	4.2	+ 9.69	0.058	0.24	8.3
A B	4.2	− 0.185	0.066	0.28	7.4
A C	5.6	− 3.28	0.22	1.23	2.2
A D	5.2	− 1.48	0.008	0.04	58
A E	6.0	− 0.059	0.024	0.14	20
A F	4.8	+14.07	0.035	0.17	14
B C	6.2	+ 0.120	0.072	0.45	6.7
B D	5.7	− 1.86	0.009	0.05	54
B E	6.6	− 0.093	0.018	0.12	27
B F	5.4	− 2.37	0.030	0.16	16
C D	7.1	− 1.45	0.009	0.06	56
C E	7.9	− 0.161	0.023	0.18	21
C F	6.7	+51.5	0.030	0.20	16
D E	7.6	− 0.945	0.007	0.05	71
D F	6.3	− 1.93	0.007	0.04	70
E F	7.1	+ 0.597	0.020	0.14	24
....	....	....	....	....	....

21 position-locus circles. Note that two distant points (such as Points *A* and *C*) give low precision, but that either one combined with a near point gives great precision.

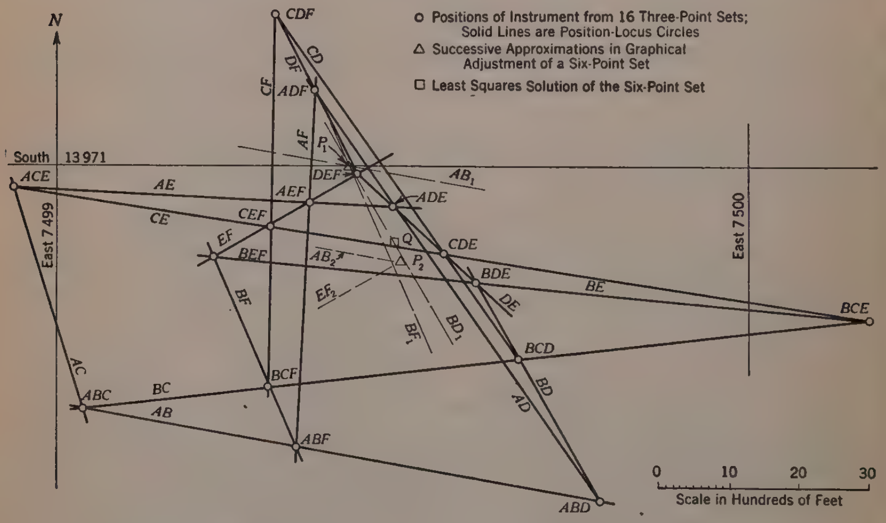
Data for the thirty-five independent intersections of these twenty-one circles are given in Table 3. Certain three-point sets may be rejected from further

TABLE 3.—COMPUTATIONS OF INDEPENDENT INTERSECTIONS

Three-point set	CO-ORDINATES OF P_1 FROM EAST 7 490, AND SOUTH 13 900		INTERSECTION ANGLES			POSITION	STRENGTH
	East	South	$\phi_{\alpha\beta}$	$\phi_{\alpha\theta}$	$\phi_{\beta\theta}$	Probable error, $\Delta_{\alpha\beta\theta}$	$10^5 S_{\alpha\beta\theta}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
O A B	10.85	71.69	47.9	80.7	32.8	0.38	5.3
O A C	10.75	76.95	5.4	18.2	12.8	5.57	0.5
O A D	10.82	73.04	32.3	35.2	2.9	0.79	3.2
O A E	10.86	71.14	76.9	87.8	15.3	0.53	5.3
O A F	11.46	41.45	4.7	2.9	1.8	7.65	0.3
O B C	10.33	71.20	42.5	50.1	87.3	0.59	5.1
O B D	8.91	69.87	15.6	18.4	2.8	0.82	3.0
O B E	10.37	71.24	55.2	38.0	17.2	0.47	6.0
O B F	8.55	69.53	52.6	23.8	28.8	0.50	4.0
O C D	10.42	72.37	26.9	30.4	3.5	0.67	3.8
O C E	10.33	71.25	82.3	76.7	21.0	0.39	7.2
O C F	9.53	60.15	10.1	5.3	5.8	3.69	0.8
O D E	9.81	71.36	70.8	15.5	55.3	0.17	16.6
O D F	8.57	69.31	37.0	3.7	33.3	0.62	4.3
O E F	8.33	71.67	72.2	18.9	53.3	0.43	6.6
A B C	9.04	71.36	62.5	17.3	79.8	1.25	2.2
A B D	9.79	71.49	45.5	51.2	5.7	0.40	5.8
A B E	6.50	70.88	7.1	5.2	1.9	4.22	0.7
A B F	9.35	71.41	83.6	56.6	27.0	0.37	6.3
A C D	8.50	69.60	17.0	17.6	0.6	4.07	0.6
A C E	8.94	71.03	69.6	63.9	5.7	1.40	2.0
A C F	9.29	72.18	21.1	18.1	3.0	3.95	0.7
A D E	9.49	71.06	52.6	12.6	40.0	0.22	13.0
A D F	9.38	70.89	38.1	6.6	31.5	0.35	7.0
A E F	9.37	71.05	89.3	34.2	55.1	0.25	11.0
B C D	9.67	71.28	68.5	62.2	6.3	0.51	5.9
B C E	10.17	71.22	12.1	15.9	3.8	1.81	1.8
B C F	9.31	71.32	73.9	82.1	24.0	0.46	6.6
B D E	9.61	71.17	56.4	18.3	38.1	0.19	16.8
B D F	7.97	68.14	5.4	0.9	4.5	3.18	1.3
B E F	9.23	71.13	61.8	36.1	82.1	0.20	15.8
C D E	9.56	71.13	46.3	12.0	34.3	0.32	11.8
C D F	9.32	70.78	35.7	7.2	28.5	0.48	7.3
C E F	9.32	71.08	82.0	39.9	58.1	0.28	13.5
D E F	9.44	71.01	19.2	74.2	86.6	0.15	23.0
....	....	....	....	....	....	....	....

consideration: (*a*) Because of large departure of co-ordinates in Columns (2) and (3), Table 3, from the approximate centroid at E. 7 499.5, S. 13 971.0 (note that *O A F* is 30 ft away); (*b*) because all three intersection angles are small (note $O\ A\ F$, $O\ C\ F$, $A\ B\ E$, and $B\ D\ F$); (*c*) because of the large position error in Column (7), Table 3, or the low figure strength in Column (8); or, (*d*) because of any combination of Causes (*a*), (*b*), and (*c*). If all angles are measured to nearly the same precision, figure strength is the easiest determinant; but if field precisions were quite varied, position error is a better comparison. First, however, one must determine the position more closely from the more precise sets (say, $O\ D\ E$, $B\ D\ E$, and $D\ E\ F$, giving a mean of E. 7 499.62, S. 13 971.18) and list any intersections that fall much farther from this position than their calculated position errors. In this example, all intersections seem tolerable except for twelve of the fifteen that are dependent upon Signal O, which is sufficient evidence for the rejection of all data depending upon O. A study of the large-scale plot of these intersections makes the conclusion more obvious.

Eliminating these intersections and four others that have probable position errors greater than 3 ft, the other sixteen intersections are plotted to a large scale in Fig. 3. Bands the width of the probable normal error on each side of the lines do not intersect with an area common to all. Increasing each band width by 45%, a common point, P_1, is found on the edges of bands of $B\ F$, $B\ D$, and $A\ B$, and within all other bands. As $B\ D$ and $B\ F$ are nearly parallel, draw the median to the edges of those bands. By diminishing the width of all other bands until the width of each side is 2% greater than the probable error, the diminished common area just touches the median at P_2. This point is accepted as the solution; its departure from position-locus circles is about 1.50 times the probable error for two circles; 1.02 times for two others; and, less than the probable error for the other twelve. The least squares solution, which is too long for inclusion herein, gives Point Q, distant 0.03 ft from Point P_2 and has a probable error of 0.13 ft.


N
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East 7 499
East 7 500
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BC
AB
BCF
BD
AD
ABF
ABD
○ Positions of Instrument from 16 Three-Point Sets; Solid Lines are Position-Locus Circles
△ Successive Approximations in Graphical Adjustment of a Six-Point Set
☐ Least Squares Solution of the Six-Point Set
0
10
20
30
Scale in Hundreds of Feet

FIG. 3.—GRAPHICAL SOLUTION OF A SIX-POINT SET

Before concluding, the rejected signal, O, must be disposed of. Computing closing azimuths from P_2 to each of the other signals, and then Azimuth $P_2\,O$ from these and the measured angles, the resulting six values may be expressed as 203° 19′ 13″.5 ± 4″.2, with a weighted mean of 203° 19′ 13″.7. The closing azimuth from P_2 to the given position of O is 203° 18′ 58″.9. The difference, 14″.8, is the angle subtended at P_2 by the position error of O. As the distance is about 12 708 ft, the normal projection of this position error is 0.92 ft. Measurement of the position-error projection from another instrument point would determine the absolute error, and, thereafter, the station could be used.

It should not be concluded that the other six signals have been proved correct. Any of them might have had a shift in such a direction that its normal projection from P is small; or a distant signal might have an error that would not be appreciable until the survey approached it.

SUMMARY AND CONCLUSIONS

The salient features of the paper may be summarized as follows:

(1) The three-point problem may be solved by formulas, without ambiguity;

(2) The use of spires, co-ordinated by responsible surveys, affords a simple method of connecting to other systems or controlling spur traverses;

(3) If such spires have not been observed recently, shifts due to several causes should be anticipated (use of a fourth spire affords a check; and, a fifth spire will serve to eliminate one which is in error);

(4) Methods of analysis of four-or-more-point systems have been outlined fully (the experienced computer may substitute his judgment for formulas expressing relative strengths of figures); and,

(5) The graphical method of compromising intersection errors by representing each line by a band of width proportional to its probable error is also applicable to ordinary triangulation, particularly in the convergence of second-order surveys to locate third-order stations.

ACKNOWLEDGMENTS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

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A SYMPOSIUM

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JOINT RESPONSIBILITY OF THE AUTOMOTIVE AND THE CIVIL ENGINEER

BY C. F. KETTERING,[1] M. AM. SOC. C. E.

SYNOPSIS

A number of years ago the writer was making a motor trip in Kentucky running some tests, and had gone far off the main road. The map did not show any roads leading away from the place it was so small. It was late in the afternoon and he wanted to return home before dark. Driving along a little stream bed (which was the only road there was) he overtook an old gentleman driving a donkey hooked to a stone boat loaded with milk cans. Question: "Neighbor, what is the best way to go to Cincinnati from here?" Answer: "Go right up to the forks in the road. Now, it don't make much difference which fork you take but, to tell you the truth, Stranger, if I was going to Cincinnati I wouldn't start from here." Sometimes people get to thinking like the old Kentuckian; but they must first recognize that, whatever their destination, they must start from where they are.

The leaders of the automobile industry have never been able to predict what the industry will offer the public two years in advance. If they knew what the car of ten or twenty-five years in the future was going to be like they would be making it now. To afford some idea of what the future might bring without taking all the responsibility upon his own shoulders the writer asked a number of men, holding responsible positions in the automobile and allied industries. On some questions they were almost unanimous in agreement; on others, they had entirely opposite ideas. All the various prophecies and predictions made in the past have been studied and, with these, the writer has tried to present, in some kind of complete picture, the requirements that will be demanded of those who will supply roads for the automobile of 1960.

All one can do now is to determine conditions as they are to-day, what they were yesterday, and, by projecting a line through the two known points, reach some idea of what the future may hold. It is an excellent idea for the road builders and automobile makers to get together and exchange ideas concerning their mutual problems. After all, the vehicle and the road must form a complete transportation system. Each one is dependent upon the other for the success of the whole. Civil engineers have the most difficult job because of the time factor. They must build a road to last twenty or twenty-five years for vehicles whose average life is only seven or eight years. Before a road is

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one-third used up, there is an entire new generation of motor vehicles with their new requirements; and before the road is finally ready for replacement the requirements have changed several times over. The aim of this paper is to predict what changes this time factor will demand so that civil engineers can build accordingly.

INTRODUCTION

The greatest problem involved in all thinking is to determine the time rate of action. The time rate of gravitation is known. If a man jumps from a window the time it will take for him to hit the sidewalk can be computed. There are other natural laws which are just as fundamental as the law of gravitation. Each of them has an entirely different time rate of action, and these are not understood at all.

An interesting incident occurred in connection with the salmon fishing industry in one of the large rivers of the United States. The industry was persuaded that it would be a good thing to have a salmon fish hatchery and, therefore, two or three of them were built. Fish hatching was begun, and the product sent out, to return again after the proper cycle. One year, one of the hatcheries burned down, and there was an immediate demand for a new one. People went to the authorities to get an emergency appropriation for building a new fish hatchery; but the time rate of political action is not that fast. The subject had to be discussed and the hatchery was not built that year. Some of the opponents argued, "they had just as big a run of salmon the year after the fire as they had the year before." The result of the delay in building was that, when the proper year in the cycle came around, the salmon catch was small because of the absence of the product of the hatchery. This has happened in some other problems, such as road building. Eight or ten years ago engineers missed something; so "the run is not so good" this year. This time factor must be studied so that similar troubles will not recur. Maybe the place to start is with the politicians.

The writer belongs to a group of men who believe the world is not finished. Nothing is constant but change. Men work day after day, not to finish things, but to make the future better. They want the future just as nice a place as they can make it because they will spend the remainder of their lives in the future.

Many things need to be torn down and thrown away. The country needs to be rebuilt and made better than ever before. The economists and bankers say, "Where are you going to get the money?" The money will come from the same source as when the country was built this far. There was no money with which to start. All that the people did was to dig each building out of the ground. It has always been here in the country, only in a different form.

Man-hours can be used to convert material into anything that Man needs. An intelligent people will supply the work. When materials are converted into human utilities money is only used as conveyor to carry it from one point to another. Sometimes, people calculate entirely too much in dollars. The wealth of the nation is not in dollars; it is in useful products. The positive side of economics is the movement of useful material through the channels of trade. That must always precede the return flow of money through the counting houses.

In preparing this paper the writer sent a list of eighteen questions involving factors that might affect road requirements in 1950 or 1970 to many people who it was thought might have valuable information on at least some of them. Their opinions were requested on any particular one subject upon which they had information; or, if they wished, they could comment on all eighteen. The subjects upon which the questions were based were:

(1) Growth in registration of passenger cars.
(2) Growth in registration, use, etc., of trucks.
    (*a*) Will railroads pick up and deliver less than car-load lots?
(3) Growth in registration and use of buses.
    (*a*) Will buses supersede street cars in cities?
(4) Effect of the trailer on automobile design, roads, laws, traveling habits, trailer camps, home building, population movement, etc.
(5) Road speeds of passenger cars, trucks, and buses.
(6) Size and weight of road vehicles.
    (*a*) Will this make segregation of different types of traffic necessary?
(7) The future of other types of motive power, such as the Diesel engine and the problem of suitable fuels.
(8) Design changes in the automobile which will affect road-building requirements and make safer vehicles.
(9) Lighting on the vehicle and highway.
(10) The future of tires—changes in coefficient of friction, life, etc.
(11) Possibility of high-speed trunk highways and separate truck highways between cities.
(12) Will travel be overhead or underground in congested areas?
(13) Will the population live in larger cities or will the trend be back to smaller communities?
(14) What mechanical devices are practical to control traffic conditions and actions of the driver?
(15) How will parking in cities be solved?
(16) How will laws regulating traffic, vehicles, and the driver affect road requirements?
(17) Type, kind, and number of highways that should be provided for the future.
(18) Possibility of chemistry contributing better road materials or treatment.

INCREASE IN REGISTRATIONS

The probable growth in population in the United States for the next 25 yr is quite easy to predict and such predictions have proved reliable in the past.

In Fig. 1, Curve (1) shows the population growth from 1900 to 1930, as taken from reports of the United States Census Bureau. From 1930 to 1960 the probable future population is shown as estimated by the American Petroleum Institute. The data represent the averages of five estimates from life insurance companies and others. Passenger-car registrations are plotted as Curve (2) with the estimated increase per 5-yr period to 1960. The estimated number of persons per car is plotted as Curve (3). Registrations in trucks are estimated to increase to 6 000 000 by 1960 (see Curve (4)). Curve (5), Fig. 1, shows the total estimated motor-vehicle registration by 1960 as more than 37 000 000 vehicles. The registration values were estimated by the American Petroleum Institute.

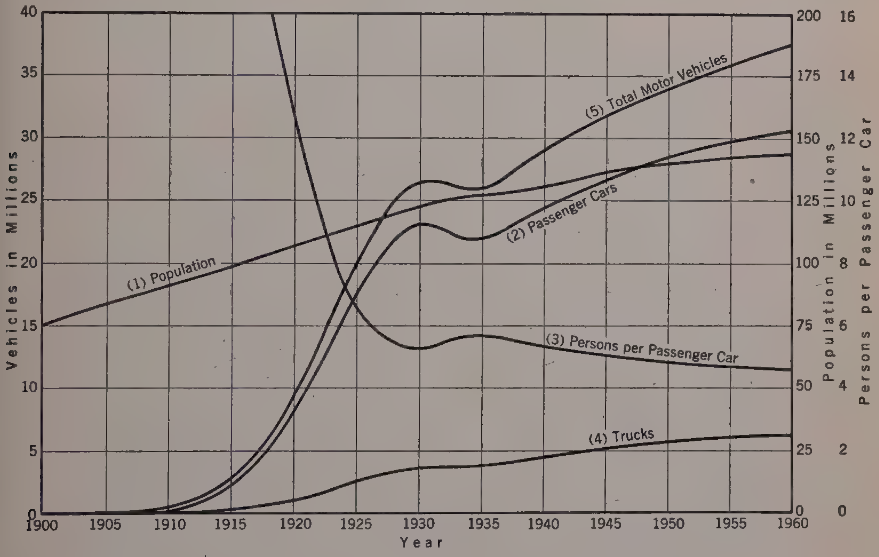

FIG. 1.—ESTIMATED MOTOR VEHICLE REGISTRATION TO 1960

From the foregoing it is apparent that engineers will have to provide roads for 50% more motor vehicles in the next 23 yr. Since many main roads are already inadequate to carry the traffic safely, it would seem that a tremendous task lies ahead. The area of the various countries per mile of road, and the number of cars per mile of highway, are as follows:

Nation	Area, in square miles per mile of road	Cars per mile of road
United States	1.0	8.2
Germany	0.8	4.4
France	0.5	4.9
Great Britain	0.5	10.3
Italy	1.1	3.6
Australia	6.3	1.3
Brazil	35.2	1.3

In comparison with most of the remainder of the world, highways in the United States are now congested. An increase in registration on the present system would almost prohibit much of the usual driving. What the optimum ratios should be is not known, but it is certain that more improved highways are badly needed.

Almost all those who answered the questionnaire believe that the population trend is away from congested areas. Railroads, unsupplemented by efficient highway transport, caused concentration of population at station points. Automobile transportation counters the railroad influence, and provides economic and social communication facilities, at greater distances from, and independent of, rails. Man has an inborn feeling for open spaces and a plot of ground, which he cannot lose entirely. There are a number of indications that the trend is away from cities and back to smaller communities. Several large manufacturers have definitely committed themselves to a policy of erecting new plants in small towns. One of the largest retail companies, with stores all over the country, is opening or building new stores only where large parking spaces can be obtained for the use of its customers. Traffic congestion and the problem of parking will have the greatest influence on this movement. A number of men think cities will be necessary for conducting many forms of business and government, but that a large percentage of the population can live more comfortably and find most of their needs cared for in small communities.

There are those who predict the "string" or "ribbon" city. This is an almost continuous population existing on both sides of a trunk highway. The ribbon city may be pictured as an elongated community like a long string with beads of different sizes strung upon it more or less erratically. The biggest beads are the large centers of population. They are certain to develop because of the need of centralizing large financial institutions, railway centers, ports, costly public facilities, such as water-works, gas-works, museums, libraries, and other agencies which have grown with large cities. Small "beads" will develop at strategic points and will consist of all gradations between the large city and the equivalent of small country cross-roads.

Others believe in the increase in importance and number of small communities which would be supported by a local industry. Each town would have at least one factory. Because of the greater amount of leisure which every one has, residents of small cities would be able to have plots of ground for cultivation and would raise part of their food requirements. This would make them less dependent upon manufacturing for their living. This type of rural life would make highways doubly important, both for pleasure and commercial use.

There are others who believe that faster means of transportation from outside into the center of large cities is coming. Workmen could live distances as great as fifty miles from their work. Homes would be situated on large plots in the open country. The city would be for work, business, and commerce. To accomplish this would require high-speed, limited-access trunk highways to the city limits. The city would have to have many elevated or depressed highways crossing it in every direction. Present level streets would be used for local travel. Local conditions will govern whether the thoroughfares are built underground or overhead.

PARKING

City parking is one of the most serious problems. It is inconceivable that motor-car owners will long continue to park cars almost bumper to bumper along busy city streets and suffer the damage and inconvenience which result. The traffic capacity of the present city street system could be almost doubled by eliminating parking and using the streets for travel only, as they should be used.

Experience seems to indicate that drivers and store owners have unconsciously, but effectively, accepted the idea that cities must provide the parking space required for the ordinary transactions of business and pleasurable pursuits of their citizens. The only regret in the mind of the average person seems to be that he has not been able to find any method of placing more than two curbs along a given street. The people in Detroit, Mich., have even found a way to do that on a few streets.

Accepting the idea that cities must provide space, it would seem far cheaper for them to provide parking lots than to supply expensively improved streets for parking purposes. If it is right for a city to go so far as to condemn and acquire strips from the front of existing lots, and even buildings, to convert these strips into additional highway widths, to pave them expensively, and then to have large parts occupied permanently for parking, why is it not legitimate and much more logical to condemn cheap but well-located property for conversion into city-operated parking lots?

In residential areas there is a grass plot between the sidewalk and the street. Why should not this space be used for parking instead of the traffic area of a street built for heavy duty? There are even city ordinances prohibiting such a practice.

Parking garages, either separate or located within the building they are meant to serve, are growing in number. Every large building should provide adequate parking space for its tenants and customers. Several well developed means of doing this are in use. Double-decked or triple-decked exposed parking lots may be necessary in some areas. However, the problem of parking is not one for either the automotive engineer or the civil engineer to solve directly.

PASSENGER CARS

What possible changes in the passenger automobile might affect road requirements? The car has, in no way, reached a state of standardization and there is no indication that it will. The car of ten or twenty-five years from now (1938) will be just as different and will have just as many improvements as they have had in the past ten or twenty-five years. The 1937 car will certainly seem as antiquated to those who see it in 1960 as the 1912 car does now. It is the policy of the automobile industry to improve its product constantly.

Sometimes an idea of the future may be gained by looking at the past. Suppose a buyer had picked out a brand new 1912 car just as it came from the production line. At that time every one agreed that it was the best car that could be obtained. Suppose it was sealed in an air-tight glass case and the price ($2 000) marked in gold letters where every one could see. As the years passed the buyer went back to look over his perfect car. The second year, its value had decreased to $1 800; and, as each year passed, its value decreased until in a short time (much less than the twenty-five years since it was sealed up) only the junk man would value it. Why? Remember it was sealed so that nothing could happen to it—no rust or corrosion, and no wear. The car remained exactly as on the day it was purchased; but what changes there have been since 1912 in the idea as to what an automobile should be! Cars now have self-starters, four-wheel brakes, better finishes, higher performance, improved appearance, fine riding qualities, and many more attributes than any one could mention. If a buyer were to do the same with a 1937 car now, he would surely have the same experience in 1960.

It is not possible to foretell exactly what the car of the future will be like. There are so many factors that might enter at any time to throw such prophecies out of line. It is feasible, however, to discuss some of the problems automobile engineers are talking about and experimenting with; questions they think important enough on which to put time and effort. The requirements for maneuvering and controlling the car, for example, are closely related to road conditions in many instances. These items are affected by such factors as steering, "hill climb," top speed, vision, lighting, riding quality, and ease of control, all of which are directly concerned with the safe use of the roads.

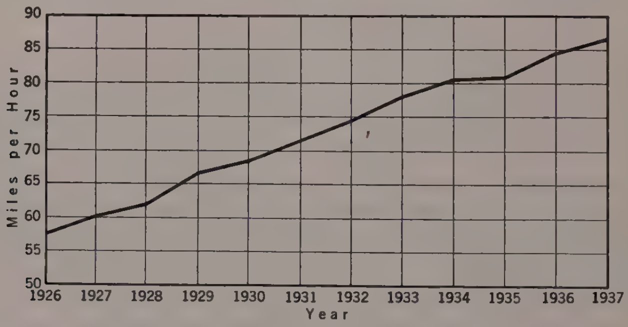

FIG. 2—CURVE OF MAXIMUM SPEED

Engineers do not always think of engine design as having anything to do directly with road building and safety and yet the power source may profoundly affect the type of surface upon which the car rides. It influences performance factors: Speed, acceleration, economy, flexibility, and "hill climb." From several sources of information one learns that there are indications that the practical usable maximum speed has been reached and that engineering emphasis will be placed on other factors in the future. Customer research surveys have shown that people, on the average, do not want to travel much faster than 60 miles per hr; but they want to go that fast without effort. Certainly, roads should not be designed for any less than 60 miles per hr. Other authoritative sources of information reveal that speed is an inherent reason for the widespread use of the automobile and that designers should do everything possible to maintain that advantage. As roads and cars become better, speeds may increase still further and with more safety than at present. Fig. 2 shows the trend in maximum speed of a representative group of cars for the eleven years, 1926–1937.

Although the question of maximum top speed may be controversial and may depend upon how safe the highways are made, every one insists that a higher average speed should be maintained between cities. This indicates the need for high-speed highways built to carry traffic safely. Elimination of grade crossings, limited access, separate lanes, and locations to pass around cities, are some of the requirements. These highways are necessary between main centers now, and their requirements will not depend upon changes in the vehicle that may be anticipated at the present time.

Acceleration and "hill climb" are closely akin. Accelerations will probably increase. This ability is important in "getting away" from a stop and increasing to traffic speed, to pass and return into line in a short distance, and to cross intersections without delay. It will affect the timing of traffic lights and regulating devices. It now appears that any passenger car will be able to negotiate a 10% grade without losing its speed. Undoubtedly, the trend is for the performance of trucks and buses to equal more nearly that of passenger cars. This will undoubtedly be true in the future.

The ability to stop is of major importance. Here is a problem that civil, as well as automotive, engineers can do much to solve. For its part the automobile industry will offer better and better brake equipment as it learns more and more how to do the job. However, the coefficient of friction between road and tire will always be the limiting factor in stopping distances. Although something may be done with the tire to increase the coefficient, the road surface itself seems to offer the greatest possibility for improvement. Everything possible should be done to increase this factor, and it might be well to include a minimum limit in road specifications. To show how much this may vary on different surfaces, the writer has computed stopping distances from 50 miles per hr for a number of coefficients, as follows:

Coefficients of friction	Distance, in feet, required to stop from a speed of 50 miles per hr
1.0	83
0.9	91
0.6	140
0.4	208
0.2	416

The large variations are clearly shown. Surfaces and materials vary the friction characteristics greatly and warrant important consideration in building new roads. The change with weather conditions also is of great importance. There is little that can be done with the tire to give higher values compared to the large variations encountered on present-day roads. At present, a coefficient of about 0.6 is all that can be relied upon in designing brakes.

The size of passenger cars will be determined largely by the space required to house the passengers. Probably this will not change very much. Road clearances of a little less than 8 in. have been common for a number of years and it would seem that nothing now being developed would reduce this value. Clearances are important in building alley intersections, intersection street grades, and driveways. Lower door clearances, apparently, are here to stay and must be taken into account in constructing curbs. The height of the center of gravity has a bearing on the capability of the car to "hold the road." For a number of years it has been about 24 in. above the road. Any lowering would only slightly affect roads. Turning radius is important only in parking. This value has averaged about 26 ft for some years.

What will be the effect of the engine in the rear? This subject seems to be much before the public at present. There are some good reasons why this design would be desirable and some, equally good, why it might not prove so feasible in production. A rear-engined car would provide better visibility with clear vision directly ahead. It would probably eliminate much engine noise, heat, and odors, and allow lower floor-boards. However, maneuvering, particularly around a corner, requires an almost equal distribution of weight on the front and rear wheels. To accomplish this in a rear-engined car of reasonable wheel-base will require an engine of about one-half the weight of the present units. The rear-engined car with acceptable handling characteristics must await power plant and drive developments. Even with a rear-engined car, road requirements would not be changed materially unless accompanied by radical changes in performance.

The rubber tire is one of the most important factors that made the present automobile possible. It is interesting to note that Mr. Dunlop was not thinking of automobiles at all when he developed the pneumatic tire. He undertook to make a better tire for his son's bicycle. He took a wheel and put a canvas over it, and inside inserted a rubber tube blown up by a football pump. That was the beginning of the pneumatic tire. Finally, an unknown bicycle racer beat a champion because he used pneumatic tires and the bicycle trade took them up. When the automobile came along the tire was ready.

Tire mileages have increased from less than 1 000 in the early days until now (1938) a tire will last for more than 20 000 or 30 000 miles and the owner objects if he does not get that many. One place where tires may be greatly improved is in blow-out resistance. At one proving ground tires have often been blown out with dynamite caps at speeds of 70 miles per hr or more without hurting any one. In fact, motion pictures taken by a following car and aimed at the tire show only a slight sidewise movement when the tire burst. Such procedure is not recommended, of course, as serious accidents can be caused by blow-outs, especially in front tires.

The trend in tire development is toward improvements in compounding technique that will result in more heat-resisting rubber compounds. A new tire is being produced which has a strong rayon cord that maintains its strength at a high temperature. More than 20 000 tires have been produced and 500 000 000 miles of wear imposed on them to be sure that the development is a sound one. The outstanding characteristic of this tire is its freedom from carcass failure and blow-outs, particularly where trucks and buses are operated at high speeds and under heavy loads. One company cited tests by which these new tires outlasted the specially constructed test cars themselves in runs at 120 miles per hr over the salt beds of Utah. It seems that the tire chemist has scored again and motorists will not be limited in road speed by tire weaknesses.

Mechanical Devices to Control Cars

It is not difficult to conceive of mechanical devices on roads and highways that might act to reduce the accident rate. Photo-electric cells, relays, and mechanical devices have been proposed for stopping cars automatically. This could be done in a practical manner just as is now done on railway trains. It is doubtful, however, whether engineers could justify the expense and difficulties of enforcement in the use of such devices.

For several years there has been a presistent clamor for governors on all motor vehicles. Speed is claimed to be the cause of most of the accidents. However, the public would not allow the designer to govern the speed to less than 50 miles per hr. Only a very small percentage of accidents happen at greater speeds. Limiting the speed to 50 miles per hr would not affect accidents in the city, where maximum speeds seldom reach this value; and it would only prevent a few accidents on the open highway. Many surveys have shown that the average speed is less than 45 miles per hr on good roads and only a small proportion of cars travel as fast as 60 miles per hr.

Automatic devices that limit speed or take the control away from the driver may be the cause of accidents. It is often necessary to accelerate rapidly to cross an intersection, pass around a car and back into line, or enter traffic and increase the speed to avoid trouble.

Lighting—Road or Vehicle?

Authorities agree on one fact: Night-driving conditions are far behind daytime conditions, and for one reason—lack of adequate lighting and glaring headlights. Many studies have been made on the relative accident rates between day and night driving. Most authorities agree that night driving is three or four times more hazardous than day-time driving. The future must avoid this condition. Here is a problem which both groups—the automobile industry and road builders—will have to solve individually. It seems logical that many miles of congested highways, especially between large centers of population, could be lighted economically. The first cost and the upkeep are high; and, with any electrical equipment now in prospect, these items make it impractical to light all highways. It has been estimated that the upkeep alone of the lighting for roads in the United States would cost more than the total annual expenditure for roads in the entire nation.

However, the writer would like to outline what has been done in the past generation in lights for other purposes. The present tungsten incandescent lamp is about eight times more efficient than the original Edison carbon lamps. The new sodium vapor lights are said to give nearly three times as much illumination as the best tungsten lamps. It does not take a very good prophet to guess that this progress will not stop. The future will certainly see vast improvements in methods of producing light with small expenditures of power. Entirely new methods will surely be found for producing light without the great heat loss which now must be accepted. The firefly, deep sea fish, and other of Nature's means of producing light can teach engineers an entirely new technique. Already, a few unrelated facts are known which may prove to be the starting point for large lighting industries. In a demonstration which the writer has often given, he pours two liquid chemical compounds together. Each is almost water-white and inert. When they are mixed, they create a pale blue light, strong enough to permit reading a newspaper in the dark, which persists for several hours. What this means for the future is difficult to say, but it is well known that many of the greatest industries are founded upon scientific facts as simple as this one.

For some time to come it seems likely that illumination on most roads will be supplied by the automobile itself. The problem is to obtain good vision without glare; and this the automobile engineer is working on all the time. From the crude oil lamps of twenty-five years ago the modern car has now progressed to fairly effective electric lights with several beams to suit varied conditions. This lighting can be improved still further, but how to do it is the problem. Since 1918, experimenters have tested thousands of means of eliminating glare, which would still provide adequate road illumination for the fast-driving pace most owners demand.

Only one solution has received widespread attention; that is, the use of polarized light. At first, this may seem to be the answer to the question, but there are still many problems to be solved before it can be utilized on a car. In this system a beam of polarized light is projected upon the road by a headlight which has a polarizing screen in front of the light bulb. The oncoming driver views the road through a screen in the windshield, or special glasses which have a polarizing screen set at right angles to that of the headlight of the oncoming car. Therefore, he sees only a small percentage of the light from the oncoming car, but the light from his own lights is little affected by the viewing screen. Glare is practically eliminated without much reduction in road illumination. The objection to the use of polarized light is that it requires about four to eight times the candle-power light of present systems to give the same visibility. To be effective, it would have to be fitted to all cars. The expense of the screens themselves is high, although this would undoubtedly be reduced with more experience in manufacturing.

Other systems have been proposed for providing adequate road illumination and, at the same time, eliminating glare from oncoming cars, such as a system of colored lights. Every car would be equipped with headlights capable of producing a beam of either one of two colors, together with viewing screens of these colors. Suppose the two colors were blue and orange-red. Cars traveling north and east, for instance, might be required to illuminate the road with the blue light and would use their blue viewing screen. Cars traveling south and west would use the orange-red light and screens. The colors would be selected so that the blue screen would transmit none of the orange-red and the orange-red screen would transmit none of the blue light. This would eliminate glare from oncoming cars although it appears that the efficiency of the system would be low.

A problem in road research closely akin to illumination is road color, and brightness, and their effect upon visibility, both day and night. This is probably one of the minor considerations in selecting road materials and yet it should be given some consideration. The related problem of visibility, front and rear, from the driver's seat is an important subject which has commanded increased interest, and should lead to much better visibility in the future. Narrower front pillars, better seat positions, glass angles, and locations to eliminate reflections, sun-glare shields, front-end shape, and many other considerations are involved. From the standpoint of the road, good highways signs, elimination of blind corners, location of signal lights, center lines and markers, and road color are problems of importance to the driver, especially when he drives over a new route.

Buses and Trucks

There will be a great expansion in the use of trucks and buses in the future. From the road-builders' standpoint the size and weight of the vehicle will be important. Almost every one agrees that this factor will be settled by law and that maximum sizes and weights will not exceed present limits. The trend seems to be toward many more smaller, faster, more mobile units.

There will be a tremendous growth in the use of buses, especially to replace street cars. Although buses cannot supplant street cars entirely, in all cities, it appears that there is a certain population below which buses can take their place. It has been estimated that this size is between 100 000 and 200 000. For large cities, and particularly in cities in which there are arterial highways, the street car will remain the backbone of the transportation system.

The records show how rapidly the public has accepted the motor bus in cities and, in the final analysis, their wants will always control. In 1922, only 18 cities depended exclusively on motor coaches for transportation. Within a period of 10 yr, 300 cities were using buses exclusively. To-day (1938) the number is well over 420. During the past five years 75% of equipment bought by city operators has been gasoline buses.

For inter-city use the bus has shown similar strides forward in the twenty-five years, 1913–1938. Passenger service is supplied by buses in 48 500 communities in America which are not served by rail lines and are totally dependent on highway transportation. These are the modern "lost Provinces" which have again been discovered by highway transportation. In several mid-western States, from 20% to 40% of the communities come under the heading of "Lost Provinces."

The public has "sold" itself on the value of buses, and their speed, economy, and convenient schedules. Comfort, appearance, speed, acceleration, "hill climb," and safety have shown marked improvement in the past few years and the limit has not been reached in these factors. The trend has been toward large units, seating more than thirty passengers, but of considerably lighter weight than those of the past. Aluminum and pressed steel reduced the weight of one large bus by 6 000 lb and with an increase in structural strength. The motor bus has definitely found a place in the transportation system of the United States and will have to be provided for in any future highway program.

Statements that the truck will eventually drive the railroads out of business can only come from those not acquainted with the problems of trucking. A "long haul" in truck language means a "short haul" in railroad language. At present, the truck is usually not economical for distances greater than 250 miles, or for other than the higher grades of freight. There is plenty of business for both the truck and the railroad.

Railroads, themselves, are using trucks in ever-increasing numbers. Door-to-door delivery has been tried sufficiently to show its value and this service will expand in the future. For short hauls there will be more trucks of every kind, their maximum size being limited by laws and regulations.

The possibility of separate highways for commercial vehicles presents many problems, most important of which, perhaps, is the cost. In most instances, the commercial development that has come in the wake of motor transportation has been along the present highways. Even if separate truck highways were constructed, it would still be necessary for trucks and buses to use present highways to serve the industries and business places along them. If separate highways are necessary, in some places, perhaps they should be constructed for passenger cars and leave trucks where they are. Trucks could be kept off the special highway for the passenger cars, but they cannot be kept off the present highways regardless of the construction of other truck highways.

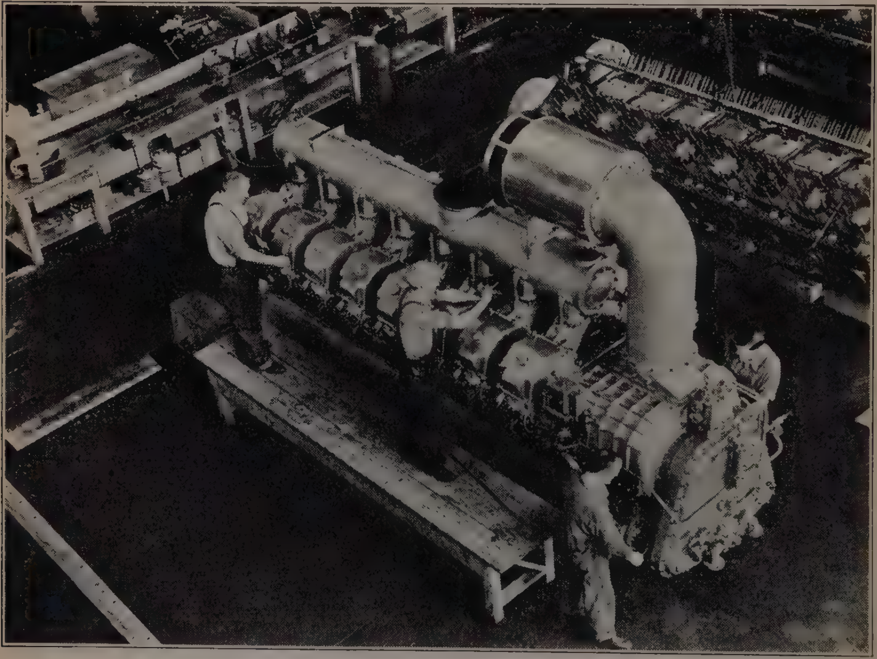

FIG. 3.—A MODERN TWO-CYCLE DIESEL ENGINE

DIESEL ENGINES FOR MOTOR VEHICLES

The Diesel engine is not new; it has been in use for more than forty years. Until given wide publicity by its use on the streamlined trains, it was little known by the public. Now, many car owners are talking Diesel engines as if they would displace the gasoline engine entirely in motor-vehicle transportation.

The Diesel is a remarkable engine for some purposes. It is the answer to many of the problems the railroad companies have been trying to solve for years. Passenger trains have been able to maintain faster schedules than they ever thought possible at a great saving in operation and fuel cost. Full-length trains have reached speeds exceeding 120 miles per hr. Time for upkeep in the roundhouse has been reduced one-half. Water is not a problem in dry, alkali areas because the loss of cooling water is negligible.

The Diesel engine is here to stay and will soon be common in large motor vehicles. Cross-country trucks and buses can now be obtained with Diesel engines and the future should show an increasing use of this economical form of power plant. The statement that they save one-half the fuel cost needs some explaining. One of the largest existing fleets of motor vehicles has a yearly gasoline bill of more than $2 000 000. To save one-half of that sum would be well worth while; but to do so, it would be necessary to spend more on the original equipment. It costs considerably more to make the Diesel engine, principally because of the expensive fuel injection system. By driving enough miles in a year, it may be possible to pay the extra first cost of the engine by savings in fuel, and still show a profit. This makes the Diesel engine possible in commercial vehicles, but not necessarily for passenger cars.

The main reason, however, why Diesel engines are not built in passenger cars is that no one knows how to build them. The writer has never seen a successful Diesel engine in automobile cylinder sizes. The larger the engine size (see Fig. 3) the easier it is to make a Diesel engine; and the smaller the size, the easier it is to make a gasoline engine. Truck and bus Diesels are still large engines as compared to passenger car engines. The use of Diesel engines should not materially affect road requirements in any event.

Trailers and Mobile Homes

A new factor that has entered the "picture" in the last few years is the house-trailer. Estimates vary as to the number now on the road, but it seems that there are several hundred thousand trailers, housing several million people. A conservative estimate is that 100 000 new ones were produced in 1937. These values are insignificant when compared to the 26 000 000 motor-vehicle registrations, but it must be remembered that trailer construction as an industry is just beginning. What the future holds, no one is sure. Opinions and prophecies vary from those by people who think that trailers are just a fad to those who predict that one-half the population of the country will be living in trailers within twenty years. There may be too much optimism in the trailer industry itself; and yet it has almost doubled itself every year for the past several years.

More than ten years ago (1926) the writer had some experience with living a trailer life. A stock bus was fitted out for living purposes and a party of seven took an 8 000-mile trip through the West. Traveling was surprisingly comfortable, and the accommodations, in many cases, were better than could be obtained in the country through which the party traveled. The present trailer is much better equipped and can find better parking places than it could ten years ago.

There are those who predict that the trailer will grow into a mobile modern home of several rooms, a home that can be moved when the head of the family changes locations of work. There are now several designs of expansible trailers which make three-room or four-room bungalows complete with all modern conveniences. They visualize a working population housed in these mobile homes, which can be moved from place to place as working conditions change. The owners would rent small plots of ground on the outskirts of industrial areas, where they could raise their own food, be out in the open country, and use the car for transportation to and from work. They would have all the benefits of owning their own home in the country, but would not be tied down because of high mortages or ownership of stationary property. Land could be rented from farmers and a new type of community life developed. After one studies some of the plans of these mobile homes, it becomes apparent that there are possibilities in this field, especially when it is realized that 80% to 85% of the present trailers are bought by working men for permanent homes.

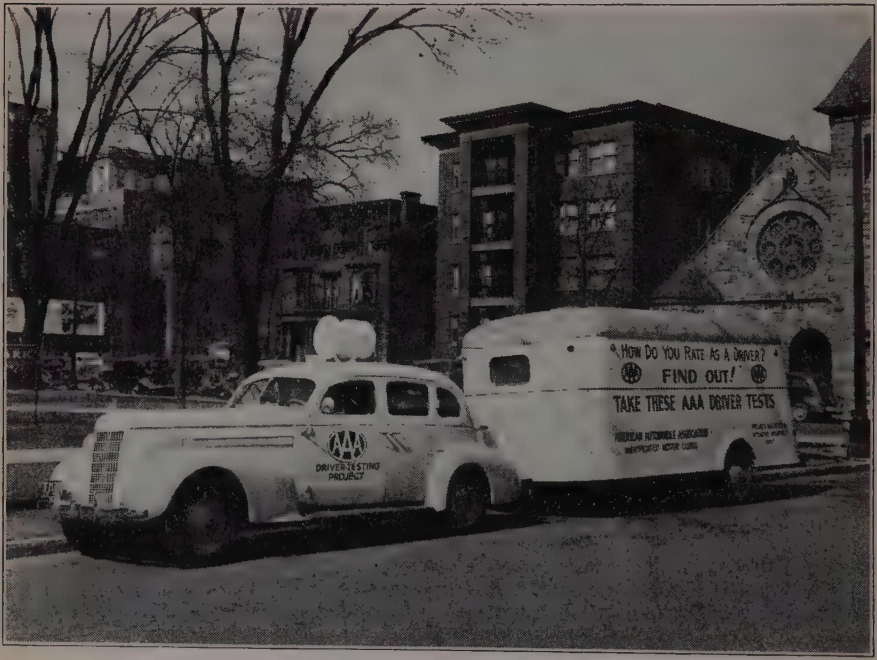

Fig. 4.—Display Trailer Used to Promote Highway Safety

The small present type of trailer may serve two purposes. It makes a satisfactory temporary dwelling for vacations and it can be used as a sample display room for commercial products. With the trend definitely toward more leisure and earlier retirement of men in business, it is certain that some effect will be felt in the modern way of living. The trailer is one answer to those who retire on a modest income or who depend upon the Social Security Act for a living.

Already a number of businesses use trailers for commercial purposes. Almost every one can remember the old days when commercial salesmen, or "drummers," traveled their territories by horse and buggy or by train. Then, along came the automobile, and they could cover as much territory in a day as they could before in a week. Now another change is appearing in methods of selling—the display trailer. It is doubtful whether any one conversant with the problems of commercial selling will gainsay the advantages of display trailers as an effective tool in the hands of the salesman. To bring an alluring show window to the prospective buyer has been the ideal of many a sales manager. This is exactly what the display trailer is accomplishing. The adoption of the house-trailer to display use is certain to have some far-reaching effects on sales methods.

At present, the varied products being displayed in trailers range from medicines for pre-natal care, and obstetrical instruments, to caskets and religious instructions. Such vehicles have the advantage of full-line displays, privacy, mobility, and spot delivery of small articles. They may become mobile retail stores, selling a wide variety of products (see Fig. 4).

Rubber-tired transportation is an integral part of modern life and has completely changed many of the current ways of living. The future will hold as many changes in the use of this new means of transportation as the past. It has been the experience of the past that the second twenty-five years of an industry always shows the greatest progress. The motor-car industry is only well under way on its second twenty-five years. It will make more use of highway transportation as better, and more adequate, roads are provided.

Assuming that the trailer is here to stay and that its production is only in its infancy as an industry, road requirements will be changed to supply these new needs. Highways will have to be wider, with wider bridges. Many more turn-outs for parking, repairing, preparing roadside meals, and feeding the baby, will be necessary. The responsibility will have to be fixed for over-night parking facilities. Will there be private ventures promoted by gasoline stations and others, or will communities provide the space as they did for tourists using tents? Combined with these questions are problems of water supply, power outlets, and sanitation as well as social problems of education for children, taxation, effect upon the birth rate, police and fire protection, and a host of others.

It appears that trailer camps will fall into five general classes, according to the use to which they will be put. These uses can be listed as follows: (1) Stop-over camps; (2) vacation or recreational parks; (3) semi-permanent and permanent parks; (4) de luxe parks; and (5) display trailer marts.

CONCLUSION

The highway of the future must be as different from the present highway as the auotomobile of the future will be from the present automobile. One thing is certain; the future will demand change. As an automotive engineer, the writer does not know how the civil engineer will accomplish his results. There will be (in fact, there are now) certain fundamental requirements that have not always been incorporated in roads: The best road for the automobile will have the least obstructions to the steady flow of traffic. Whether this means limited-access "free ways," such as Germany is building in its Reichsautobahnen, is difficult to state. Neither can one state definitely whether the engineer should build elevated highways, longer curves, wider roads, underground highways, more grade separations, better surfaces, or create new road materials, to solve the problems. Civil engineers are in a much better position to make surveys and to design roads than automotive engineers could ever be, if they are given the opportunity.

Neither group is suffering from a lack of information on how to handle the problem. Any expert could design and build a suitable safe road to connect two cities, or to cross a large urban area, if he were given the job to do it as he really thought it should be done; but there are too many other than straight engineering problems with which to contend. Probably one of the greatest problems is to prevent diversion of tax money collected for roads into other channels. The money collected for roads should be used for that purpose and not for building city halls, jails, and other projects. To concentrate on the worst of problems: First, solve the means of handling traffic on the most congested routes; and then build the main streets to handle their traffic faster and with greater safety. Instead of trying to solve all problems at once, solve the most pressing ones first. If this is done in a systematic manner, the highway system in the United States will be the best in the world within a few years.

Men in the automobile industry believe that a highway system, should be based on traffic surveys to determine requirements for the types of roads designed to carry traffic safely. Such a plan would require a primary system of high-speed highways crossing the country in all directions. It is estimated that about 50 000 to 60 000 miles of such super-highways would be sufficient. Leading from these would be a secondary system of good highways serving small cities and towns. The third system would consist of land service highways serving rural areas. In addition, a similar system is needed, in miniature, to carry the traffic in large cities.

The best type of road should embody all the knowledge the highway engineer has at his command for building a safe roadway to carry motor vehicles at a reasonably high speed. Where it is necessary, it should be above or below the ground. In any event, it should be a limited-access "free way" designed for the one purpose of carrying traffic smoothly. Where roads do lead into it, they should be provided with accelerating lanes so that traffic will not be interrupted by new vehicles entering from the side. Everything should be done to make for a free, uninterrupted flow of traffic. Crossings should be eliminated, curves should be banked, long-sight distances should be provided, bridges should be wide, and all "bottle necks" eliminated. Traffic lanes should be separated to prevent "side swipes" and head-on collisions. Surfaces should be non-skid. Where it is necessary, adequate signals and signs should be installed. The entire system should be governed by a uniform set of traffic regulations made to fit the new conditions.

This entire highway system should be paid for by the motorist. However, all the money collected from him in taxes should be used for this purpose. The automobile owner pays more than $1 000 000 000 per yr in taxes without much complaint as long as he knows his money is going for good roads. In some years almost one-fourth of his money has been diverted, directly or indirectly, to other uses. This "rifling" of the road funds must stop if the United States is ever to have an adequate highway system.

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THE TREND OF MODERN HIGHWAYS

By Thomas H. MacDonald,[2] Esq.

Synopsis

Highway construction and use are undergoing changes that are relatively slow but none the less sure and continuous. Changes in motor vehicles, in the sources and amounts of highway revenue, and technical advances in road-building methods, are some of the more important factors that will influence future highway trends.

Scientific highway planning and administration are necessary to provide highways to serve urban and rural needs with the road-user taxes that can be made available. Factual data are being provided that will permit the scientific planning of State-wide and nation-wide systems. This is the outstanding trend in the highway field.

Stabilization of soils to produce foundations of predictable behavior, elimination of intersections at grade with railroads and other highways, and provision of facilities for free flow of traffic from congested to suburban areas, will be important parts of future work.

An important beginning has been made in construction of super-highways and such work will be continued. Present policies indicate that their location will be integrated with population centers, and that the layout will not be on the transcontinental basis.

Introduction

Any attempt to reach far into the future of highway development invades a speculative field if limited to isolated examples of the unusual. However, if taken as a whole, each of the major public undertakings changes slowly in character through definite causes. These changes, if relatively slow, are none the less sure and continuous, depending upon the rate of progress in science and invention and upon movements in the social structure.

The most important causes of change in highway utilization and improvement, viewed nationally and in the mass, have in themselves a variety of checks and balances which determine their actual course and influence. For example, the national market for large numbers of motor vehicles has resulted in the investment of so much capital, the growth of industrial plants of such large dimensions, and the establishment of such complicated routine of production, that year-to-year changes in the product are limited by the necessity of conserving the investment in plant, by the requirements of mass production, and by considerations of time. Thus, the change in the motor vehicle becomes definitely pronounced over a fairly long period, rather than from year to year.

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The most serious loss in the highway investment over the decade 1928–1938 is the obsolescence resulting from changes in the number and speed of motor vehicles operating on the highways.

The revenues accruing to the public treasuries from the special taxes levied upon the motor vehicle and its users in all the various forms have been increasing in their total over a long period except for the temporary recession during the worst of the depression years

First intended and levied for highway improvement, the growth of income from these special taxes should permit highway officials to build more adequate and, consequently, more costly roads. However, because there have been large diversions of this income to other purposes, and because these funds have been spread over an ever-increasing mileage, many States are faced with a constantly growing financial dilemma.

Highway research is moving forward steadily on many "fronts," with definite objectives. The responsibility for carrying on intensive studies, in the physical and economic fields, has been accepted as a continuing obligation by highway engineers and scientists in allied fields. Concurrently, the necessity for greater street and highway safety has become a national emergency.

These are only a few of the factors that will determine the trend of modern highway development, but they are the most important.

Scientific Planning and Administration

Faced with the constantly increasing demand for the construction of new highways and the spreading of the income over a rapidly growing mileage, highway officials, State and Federal, realized the impossible situation which was developing and which, in a number of States, was actually current. Out of this situation was born the State-wide highway planning surveys which are now (1938) being conducted co-operatively in forty-four States and in the Territory of Hawaii. These surveys and supplemental studies will present a factual basis upon which to plan the complete administration of all the highways based upon sound principles and factual data. One of their important phases is the determination of the relation of the highways to other types of transportation and communication and to population distribution.

Beginning with water transportation and continued by railroad development, the influence of transportation in the past has been exerted to concentrate large populations in small areas. It is only necessary to examine maps of the United States, or any other country, to visualize clearly the concentrations of population in cities which have resulted inevitably from the characteristics of waterway and railway transportation.

Highway transportation by motor vehicle is the first great decentralizing transportation agency. The first notable effect of this dispersing influence is the formation of the wide bands of suburban development around the cities. Even cities of moderate size have developed, within a decade, a suburban band 5 to 10 miles in width (and depending upon topography) partly or completely enveloping the old city. The automobile, in conjunction with rail suburban service, has attracted city workers to make their homes in rural districts for distances easily as great as 50 miles from the city.

The next logical step is the break-down of over-sized industrial units into smaller units that will be free of many of the undesirable characteristics of over-concentration, yet will be of sufficient size to retain the economies of mass production. Sufficient progress in this direction has already occurred to indicate how inevitable must be this process even if slow. It is reasonably well established that economy of production can be secured in units of moderate size, and the national thought along social lines is becoming a potent force toward decentralization because of the opportunities presented for a more healthful and more desirable mode of life for the workers and their families.

It will be noted that no conjecture or uncertainty is involved in these statements, but simply a recognition of existing facts. The implications are clear that the scientific planning of highways and highway systems of State-wide and nation-wide dimensions will be the most characteristic trend in highway development.

Without such scientific administration there will be no possibility of providing adequate highways to serve both urban and rural needs, or to keep the cost of highway improvement, including maintenance, within the limits of revenues that can be raised by reasonable road-user taxes. This means a reversal of the present trend in many States where the maintenance cost of the rapidly growing mileage of local, poorly built roads is mortgaging far too large a portion of the highway budget. This situation has been produced largely by legal mandate and other public policies adopted and pushed into effect without any consideration for sound highway administration. The nation-wide movement, through the State and Federal highway departments, to place highway administration on a sound economic engineering basis, is not only the most important trend, but is also the cause of other trends which, on this account, may be predicted with some certainty. Out of the planning surveys will come definite specifications for the division of highways into groups classified by the service they are called upon to perform. By research and experience the details are being rapidly defined, that will determine the general type of highway design for each highway service group. In this field the principal items will be the alignment with limiting curves, sight distances, the number of traffic lanes, their widths, the shoulder widths, divided roadways for multiple-lane highways, and many other details.

DESIGN PROBLEMS

The trend here is, first, to classify highways, based upon service, and then to design closely in accord with the classification—contrasting with the all too prevalent practice, in the past, of applying single standards to long mileages, and without change, to roads of widely varying traffic service requirements.

In the field of highway design, the most important single development is found in the possibilities of soil stabilization. The intensive research work of more than a decade has borne fruit in the understanding that now exists of the physical and chemical properties of soils, and methods of utilizing the knowledge no longer stop at mere superficial applications to the immediate sub-grade, but go further, to affect the entire graded road-bed in both cut and fill sections. Among the most important additions ever made to the highway organization are the soil technician and his specialized soils laboratory. The application, on a broad front, of this new knowledge will come as rapidly as engineers may be given the specialized training, and as a result there will be, for the first time, the possibility of building really scientifically designed roadway sections since these will be placed upon foundations of predictable behavior.

With a large program of grade separation under way at railway and highway intersections, and with the changed public policy in paying construction costs largely from public funds, grade-crossing elimination has become a fixed policy that will continue until all important railway-highway grade crossings have been eliminated, and minor ones protected by adequate devices.

This same policy is being rapidly extended to the separation of important highways at intersections, and one of the important trends of highway design in the immediate future will be to rule out intersections at grade as a possibility in efficient highway design.

In this connection, incidentally, the widespread use of "stop" and "go" lights is not a solution for traffic movement, but has been a development of necessity imposed upon a system of highways designed and built before the present dimensions or speeds of highway traffic were considered possible. Obviously, the trend of highway improvement in the future must be to provide flow arteries in the congested-area traffic that will permit continuous flow of traffic from down-town areas well into the suburban areas. Although the cost will be high, it is only through such arteries that capital, invested now in land and buildings in the hearts of the business districts, can be even reasonably preserved.

The pioneer roadway, even on main traffic routes, was conceived as the single important objective. Now, with the recognition of values inherent in highway transportation beyond the bare utility, the roadway design has come to embrace the entire right of way. The trend of modern design is to provide landscaping of the roadsides, sidewalks, foot paths, bridle paths, and to stop, and protect against, soil erosion. The required additional attention and expense are paying large dividends through greater durability and through the recreational values inherent in attractive waysides.

Super-Highways

There is more or less discussion in which the term "super-highways" is used without any adequate definition of what is intended by this term. Perhaps, it is more frequently used in connection with a very limited number of transcontinental highways designed for high speed and with multiple-lane roadways to carry traffic from coast to coast.

The German system of super-highways embodies this idea. In that country a system of about 4 500 miles of highways (which gives approximately three lines across the nation in each direction) is constructed on entirely new, wide rights of way without access from abutting lands, except at infrequent intervals. This design is for high-speed, motor-vehicle, through traffic. The travel section is composed of two roadways about 30 ft wide, separated by a parking. Both the horizontal and the vertical alignments are exceptionally good. All cross-traffic is directed over or under these highways. No detail that comes within the purview of highway engineering that will make a safer or more efficient highway has been omitted. The most advanced highway design technique has been embodied in this development. The economic utilization is not so clear.

In the United States there is need for a considerable mileage of highways having similar characteristics, but the disposition of this mileage, to be most efficient, must be planned on the basis of the careful studies now going forward. The system of German roads is being built in advance of, and to promote the development of, highway transport. In the United States the situation is just the reverse. Highway builders are proceeding on the principle that the utilization of the highways must produce directly the revenues with which to finance their construction. As long as the United States adheres to this method of financing, the building of super-highways must be limited to areas where the present and prospective traffic will justify it. As a trend of highway development, it is apparent, from the important beginnings already made, that a considerable mileage of motor super-highways will be developed, that their location will be carefully integrated with the population centers, and that the layout will not be on the transcontinental basis.

In France, where a system of national roads has been developed over a long period, the present construction is to take care of the traffic around the metropolitan districts, particularly the vicinity of Paris, by a system of circumferential and radial roads in combination. The detail of outstanding importance in this design is the separation of cross-traffic.

From the developments abroad and in the United States, one can conclude that super-highways will be created, but only in the vicinity of metropolitan areas, for relieving traffic congestion within these areas and for connecting those that are separated by relatively short distances. The first function has already been served to a considerable extent by parkways. It is logical that there will be further developments of the type of the Blue Ridge Parkway designed to connect the Shenandoah and the Great Smoky Mountain National Parks. The development of such parkways recognizes the large use of motor vehicles for recreational purposes.

The Influence on Population Changes

Finally, the power of highway improvement to accelerate the shift of population from areas of low productive potentials to those more favorably conditioned will be consciously used in the national policies developed for the long-term attack upon land-use problems. A definite start is already being made in this direction and will become more apparent in the layout of the system of secondary or feeder roads. This thought definitely emphasizes that the country has completed the pioneer stage of road development and every trend of highway development of the future must be an intelligent meeting of the particular service to be rendered.

TRUNK-LINE HIGHWAYS IN METROPOLITAN AREAS

By Leroy C. Smith,[3] M. Am. Soc. C. E.

Synopsis

Elements essential to a master plan for trunk-line highways are discussed in this paper. The application of such a plan to the Metropolitan Area of Detroit, Mich., is offered as an illustration.

Introduction

Metropolitan areas in the United States are still in a condition of flux—the lines that might confine such an area to-day will, in general, be outgrown to-morrow; the uses made of any part of the area to-day may not be the uses of to-morrow; the pleasure route of to-day may become the industrial highway of to-morrow; the traffic and population of to-day will be greater to-morrow; the cross-roads at major highways of to-day may be a secondary center of to-morrow; and, unless the hub of the area of to-day is adequately served by arterial routes, it will not be, to the same extent, the hub of to-morrow.

Certainly, no one influence has been more responsible in giving impetus to the changes that are occurring in metropolitan areas, and to the uncertainties of the ultimate crystallization that may occur, than the automobile; and, certainly, there is no saturation point in evidence in the usage of automobiles or in their numbers. In the first place, the ratio of the number of automobiles to the number of people is still on the increase and, at the same time, the population is increasing. In the second place, it is becoming increasingly apparent that the sphere of use and influence of the truck and bus is enlarging, and it is dangerous to attempt to define their probable limitations.

This condition of flux and uncertainty as to the ultimate demands that may be made on trunk-line highways in metropolitan areas would present a much more complicated problem to the highway engineer in planning for the future, if it were not for the axiomatic fact that the essence of the solution lies in providing adequate right of way, and that such right of way is essential to any plan and to any use which may ultimately be made of any such highway.

Essential Elements of a Plan

The first step must be a plan providing for a system of trunk-line highways, so located and of such widths that the future highway requirements of the entire area can be met. Although, perhaps, it may be necessary to reinforce such a plan with layouts showing the ultimate developments requiring those widths, the engineer should remember that radically different details may be required to meet the results of changing conditions, but that with adequate right of way available the future can be viewed with confidence.

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Such a plan will provide for certain distinct locations and frequency of trunk-line highways to serve the various primary needs of the area, and will include:

(1) Arterial highways piercing to the hub of the area from the hinterland outside the area.

(2) Traffic headers or trunk-line loops which may serve several purposes, such as:

- (*a*) Permitting through traffic to by-pass congested areas and to avoid the hub of the metropolitan area.
- (*b*) Distributing traffic of the several arterial highways to the local street system and *vice versa.*
- (*c*) Meeting the demands of periods of peak loadings, such as Sundays and holidays in the case of pleasure traffic, or daily commercial peaks and seasonal conditions affecting industrial traffic.
- (*d*) Cross-town arteries for local traffic. Ultimately, in many cases, this may become the major function of certain such headers or loops.

(3) Boulevards and parkways for the exclusive use of pleasure traffic and to serve the recreational needs of the area.

(4) Secondary highways which are not quite trunk line in character but which are dispersed at frequent intervals as major local streets within the trunk-line network.

(5) Some of these trunk-line highways must meet the present and ultimate needs of mass transportation, whether on rails or rubber.

(6) Express roadways with separated opposite-direction free-traffic lanes may be provided in certain highways; or, the entire highway may become an express-way from which local traffic is excluded, except at long intervals and definite points; and, in still other cases, the entire highway may become a free-way without vehicular access from abutting property.

(7) Such trunk-line highways must almost invariably serve the needs of the area as trunk lines for public utilities, and inadequacy of right of way, in addition to cramping the highway possibilities and increasing the cost of the ultimate highway development, will affect the utilities in the same way. The public pays the cost of both. The minimum provision for such utilities may become a controlling factor in determining the width of the right of way in certain special cases, as, for example, a trunk line that becomes a subway route.

(8) Super-highways having sufficient right of way to permit all highway functions to be combined in one highway at grade, including provisions for mass transportation and local service to abutting property and complete grade separation and interchange at major intersections. In certain conditions such a super-highway may carry several highway functions on an elevated structure in a right of way sufficient to contain such elevated structure adequately, and the elevated structure may become a free-way without depriving abutting property and local traffic from the use of the highway at grade.

(9) Double-decked streets in which the entire right of way is used on two levels. This is a special case applicable generally only in extremely congested

areas where a built-up condition and right-of-way costs preclude other solutions. This subject will not be dealt with in this paper.

(10) Parking facilities. It is a moot point whether such facilities should be provided in highways.

Except in the case of a few of the major metropolitan areas of the United States, and except for the present need of widenings or other provisions to relieve "bottle-necks" and lack of capacity existing on arterial streets of most cities, there is usually no immediate necessity for making or deciding the ultimate structural development of the system of trunk-line highways. The immediate need is for a system of rights of way of adequate width and location and if that need is recognized, the future is reasonably insured. Structural development of such rights of way can be progressive as the need arises.

Right-of-Way Width

The Master Plan for the Metropolitan Area of Detroit fixes a minimum width of 120 ft for right of way of trunk-line highways. For secondary highways, as mentioned in Item (5), the plan calls for a minimum width of 86 ft and, for super-highways, a width of 204 ft.

Boulevards serve as something more than traffic arteries and the right of way must be sufficient not only for traffic needs but also to permit some beautification and park characteristics. A right of way 150 ft wide is a practical minimum and will permit two roadways each 36 ft wide, a center parkway strip, 38 ft wide, and 20 ft on each side for pedestrians, plantings, etc.

Parkways are essentially long narrow parks, served by an included roadway which is not directly accessible to local abutting property, and are generally planned to have grades separated at major intersections. The right of way is usually variable to conform to local topographic features; at minimum sections, however, it should not be less than 200 ft. Topographically, parkways often follow streams; they utilize low-lying stream valleys and, in such locations, can often be carried into and through comparatively well developed areas at relatively small cost, thereby replacing unsightly stream-valley dumps with a value that will enhance parkway facility.

Master Plan for the Metropolitan Area of Detroit, Michigan

In 1925, acting in conjunction with the Rapid Transit Commission of Detroit, the authorities controlling streets and roads in the metropolitan area adopted a Master Plan for the area. Although the plan dealt primarily with conditions within a 15-mile circle from the City Hall, the essential elements extend over a much larger area.

The plan provided that on each section line a right of way 120 ft wide would be available and that at every third section line the right of way would be 204 ft wide. The right-of-way width for roads on quarter-section lines is 86 ft. In addition, it provided that the right of way for the six major radial arterial highways (Fort Street, Michigan Avenue, Grand River Avenue, Northwestern Highway, Woodward Avenue, and Gratiot Avenue) would be 204 ft wide beyond the Six-Mile Circle (approximately) and 120 ft wide within that circle. Actually, these radial arteries have already been made 204 ft wide, and the system of 120-ft highways has been adhered to for many miles beyond the 15-mile limit of the original plan.

It may be well to explain that these radial arterial highways are of more than local significance and are, in fact, the major routes to the various sections of the State and to the neighboring States.

Five radial highways were established in 1825 under the direction of Governor Lewis Cass as the five great military highways for this section of the country. With a right-of-way width of 100 ft (which in many cases has since been encroached upon), they radiated in all directions and comprise the River Road, from Detroit to Perrysburg (Toledo), Ohio; Michigan Avenue, from Detroit to Fort Dearborn, in Chicago, Ill.; Grand River Road, from Detroit to the mouth of the Grand River, at Lake Michigan, on the west coast of the State; Woodward Avenue, from Detroit to Fort Saginaw, Mich., the major artery running north through the State; and Gratiot Avenue, from Detroit to Fort Gratiot north to Port Huron, Mich. (serving the "thumb" district of Michigan).

These five arteries are now super-highways under the plan, except River Road, the major portion of the traffic from which is locally taken by the Fort Superhighway. The sixth radial arterial highway, Northwestern Superhighway, is of recent origin and, although at present it terminates in the resort lake region of the adjacent county, it is ultimately projected to be extended northwesterly through the State to Ludington, on Lake Michigan.

Fig. 5 shows the Master Plan. It will be noted that more than 80% of the 204-ft rights of way for super-highways has already been secured as well as most of the land for all lesser widths. Furthermore, on the super-highway sections of the aforementioned six radial arteries, double pavement sufficient for present needs has been built, and the same is also true on seven other super-highways (Base Line, Mound, Kelly, Stephenson, Southfield, Schoolcraft, and, partly, on Telegraph).

In addition, 38 miles of the 42 miles of the Outer Drive have been constructed as a boulevard on a 150-ft right of way and many of the 120-ft trunk-line highways have been developed. Several parks have been acquired and developed, a plan for parks and parkways has been adopted, and a parkway more than 8 miles long has been completed on right of way varying from 250 ft to approximately ½ mile in width.

This progress since 1925 (when practically no right of way wider than 100 ft existed for any of these highways and only an inconsiderable part of that width) has been attained on a pay-as-you-go basis, without bond issues, and has been largely due to the following factors: (1) The unanimous adoption of a plan; (2) legislation enforcing the plan on sub-dividers of property; (3) adequate condemnation legislation; and (4) conserving automobile taxes to highway uses.

In respect to radial arterial highways, the Master Plan provided that they would be super-highways and the reduction from 204-ft width to 120-ft width at the Six Mile Circle (approximately) was predicated in layouts showing that, by carrying provisions for mass transportation in a 4-lane or 4-track subway

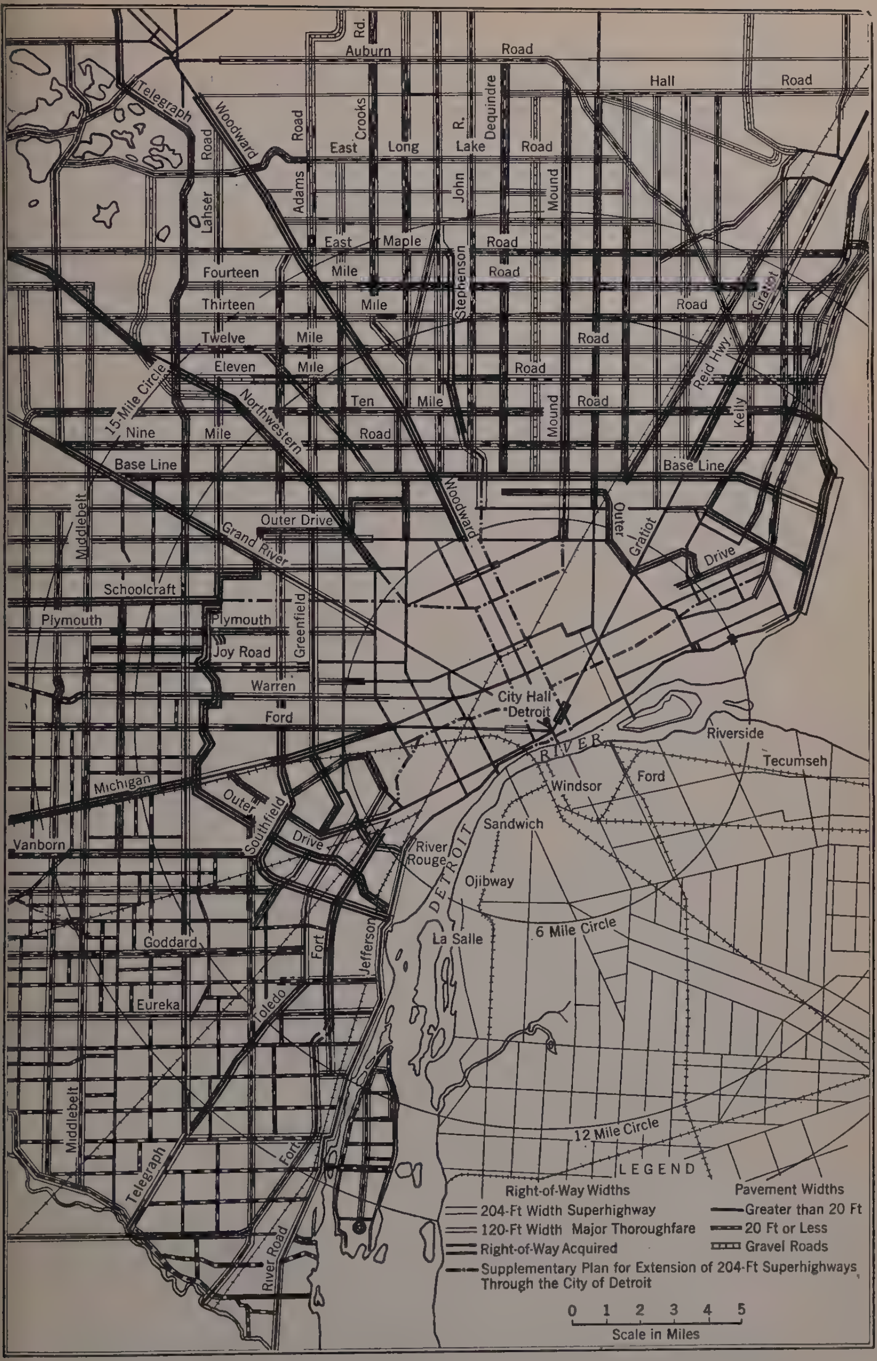
Auburn Road
Rd.
Hall Road
Telegraph
Woodward
Road
Road
Crooks
East Long Lake Road
R.
Dequindre
Adams
Lahser
John
Mound
East Maple Road
Fourteen Mile Road
Stephenson
Thirteen Mile Road
Gratiot
Twelve Mile Road
Eleven Mile Road
Reid Hwy.
15 Mile Circle
Northwestern
Ten Mile Road
Kelly
Mound
Nine Mile Road
Base Line
Base Line
Middlebelt
Outer Drive
Grand River
Woodward
Outer
Gratiot
Drive
Schoolcraft
Plymouth
Plymouth
Greenfield
Joy Road
Warren
City Hall
Detroit
Ford
Riverside
Tecumseh
River
Michigan
Windsor
Ford
Outer
Southfield
Drive
Sandwich
Vanborn
River Rouge
Detroit
Ojibway
6 Mile Circle
Goddard
La Salle
Fort
Jefferson
Eureka
Toledo
Middlebelt
Telegraph
Fort
12 Mile Circle
River Road
LEGEND
Right-of-Way Widths
204-Ft Width Superhighway
120-Ft Width Major Thoroughfare
Right-of-Way Acquired
Supplementary Plan for Extension of 204-Ft Superhighways Through the City of Detroit
Pavement Widths
Greater than 20 Ft
20 Ft or Less
Gravel Roads
0 1 2 3 4 5
Scale in Miles

FIG. 5

below the 120-ft street, that street would have approximately the capacity of the 204-ft super-highway.

The congestion of surface traffic on these radial arterial streets within the Six Mile Circle had already reached a stage which demanded immediate widening, and the width adopted was 120 ft, with the aforesaid layout as a background. Since that time these widenings for Woodward, Gratiot, and Michigan Avenues have been almost completed, a start has been made on the others, and steady progress is under way toward ultimate completion of this element of the plan. Thus, the entire original Master Plan is almost a reality to-day (1938) in so far as ultimate widths of right of way with sufficient development for the needs of to-day are concerned.

Supplementary Plan

With these several great super-highways converging toward down-town Detroit, the Wayne County Road Commission believed that they should be extended into, and connected across, the city proper on similar widths of right of way. Accordingly, a plan to accomplish this end was presented in a report to the County Board of Supervisors and adopted in principle. This plan calls for the widening of unimportant streets through the unprosperous area of old Detroit to provide new 204-ft thoroughfares. By means of 61 miles of such highways, all the outlying super-highways will be connected to a down-town loop approximately 1 mile square. This supplementary plan is shown in Fig. 5.

In presenting this plan the Road Commission was influenced by the following considerations:

(1) It is improbable that the few arterial streets that are being widened to 120 ft will prove adequate if the central section of the city is to be properly served in the long future and if the high values and concentrated uses of the down-town area are to be preserved.

(2) The cost of utilizing these 120-ft streets ultimately to accommodate the future capacity of the 204-ft super-highways feeding into them, will be so great that the excess cost in sub-surface works would finance a better and less expensive plan and additional and more adequate arteries in other locations.

(3) The supplementary plan would open blighted areas to renewed prosperity and would traverse sections of the city now inadequately served. The street widening in stagnant sections, where lots and buildings are of little value, will increase values along these new thoroughfares, the highest type of development will be attracted to these broad thoroughfares, and the central area of Detroit will be enabled to expand to meet future demands.

(4) The width of 204 ft can often be secured at a cost not appreciably greater than a much lesser width.

Having in mind the general street layout it is found that, in condemnation processes to widen a street to 120 ft, the taking line, whether the widening is on one or both sides of the street, passes through all buildings. The result is that the award, besides covering the land taken and damages to the remainder of the land parcel, also includes a value for the part of the building taken and damages to the remainder of such building as well as an allowance for business loss during alteration. These latter items approximate the reconstruction cost of the building.

On the other hand, if the widening is all on one side of the street and if the entire parcel is taken, only the present value of the property is involved, and that value may be less than the composite damages awarded. If the widening is so laid out, it will generally produce a street width of at least 186 ft, including the present street (say, 66 ft), the lot depth (say, 100 ft), and the alley width (say, 20 ft). In many cases it will produce even more, but in any event the condemnation of an additional strip, 18 ft or less in width, off the rear of the next and much cheaper series of lots, will produce the desired 204 ft.

Rapid Transit by Rail and Express Highways

Rapid transit, whether on rail or rubber, demands freedom from traffic lights and from local traffic interference. Although express highways and free-ways built at grade with grades separated at major intersections may be possible for outlying districts, the permanence and adequacy of such a solution, as the future develops, is open to question. Exclusion of local traffic and of abutting property from use of the facility, if at grade, makes of it a Chinese Wall as the territory develops. To be effective rapid transit by rail and express highways must be above or below grade and local needs must be met at grade.

Layouts and perspectives prove the possibilities of the 204-ft right of way to permit facilities for mass and rapid transit on rail and on express highways, together with adequate surface facilities for local traffic. Furthermore, at intersections, such right of way will permit complete grade separation and, at the same time, complete local interchange. In the case where these rapid transit facilities are on an elevated structure, it will be noted that the area below the structure provides valuable parking space in addition to location for surface street cars. It will be noted, furthermore, that if in the future mass transportation on rubber rather than by rail becomes the rule, a much simpler structure can be adopted.

The usual objections to elevated structures are minimized or overcome if the street width is such that adequate roadways at grade flank the structure, and under such conditions the structure can become one of beauty rather than ugliness and, through proper design, noisiness can be reduced to a point where it becomes non-objectionable. In general, where such an elevated structure accommodates both rail and rubber express-ways the part of the width utilized at one point for express loading platforms will be used at another for local loading platforms of the rail facility, and at still other points will be utilized by the ramps to and from the motor express-way.

Another plan for express-ways or free-ways which is worthy of consideration to meet the needs of arterial highways in congested districts where property values are high, would entail the use of the alleys at the rear for single-direction elevated express-ways. This is particularly applicable if there is a legal method provided for condemning air rights.

Assuming a 20-ft alley, the construction of a 30-ft express-way would entail the use of approximately 8 ft of air rights on each side of the alley; a 40-ft express-way would require air rights over approximately 13 ft on each side.

In addition, unless it proved feasible to cantilever the part of the structure occupying the strip of air rights, it would be necessary to condemn occasional small areas (probably at the lot lines) to accommodate columns and footings.

At long intervals such express-ways would be served by ramps to and from the surface and, at those locations, additional condemnation will be unavoidable. The space under such a structure, on private property, could serve all present uses; or it could be enclosed to enhance those uses, or to provide new uses. The height of the structure would be governed by present uses and by minimum clearances at cross-streets or at crossings of similar elevated structures. The cost of increasing the height to meet local considerations is relatively small.

Given legal condemnation of air rights still another plan is applicable in certain instances. Predicated on the fact that railroad rights of way pierce most cities in a strategic manner, the use of air rights for elevated highways over such available routes would add express-ways or elevated highways for industrial uses where they would be most effective. However, consideration of such a plan must include weighing carefully certain opposing factors:

(*a*) It is important that the elasticity of the railroad facility to expand or to be altered to meet future needs be conserved (this is as important as any highway function).

(*b*) Such a plan may entail partial or complete electrification of the complicated terminals of the railroads at a cost that would preclude its further consideration.

Another general observation on the problem of rapid transit by rail or express-highways is that sub-surface structures are almost invariably much more costly than elevated structures. In many cases the excess cost of a facility of ample proportions below grade is so great that new and ample right of way in parallel locations can be secured and served by an elevated structure at less cost.

Furthermore, the use of sub-surface structures is much more inelastic; if designed for rails, it cannot (except with further additions at great expense) be used for highway traffic. On the other hand, the change from a rail facility to a highway facility, or *vice versa*, is easily accomplished in an elevated structure.

PARKING

It is difficult to foresee what the future may demand in the way of parking facilities as an adjunct of the highway.

In considering long-range highway and street plans the engineer should not pass the parking problem with a gesture and the assertion that parking is not a highway function. Perhaps the following comments may point to a future trend:

(1) The automobile owner now pays the cost of parking and at a high rate. The money comes from the same pockets as the highway money, and the expense is an unavoidable part of the cost of the highway transportation system.

(2) The present investments in covered parking facilities run into millions and, in addition, the public pays an enormous price for parking in open lots.

(3) Perhaps the cost of this element of highway transportation would be less if it were planned to be included in the highway system and, with adequate rights of way, it is conceivable that sub-surface regions, or elevated layouts, could be utilized for the purpose. In cases where ultimate development of the trunk-line highway in congested districts is through the use of elevated structures flanked by streets at grade, the space at grade below the structures may become a valuable parking area and its value in that regard may be a partial justification for the adoption of any elevated plan.

(4) If often occurs that all of a parcel of property or of contiguous parcels may be secured quite apparently at no greater cost than the pay needed for the highway width. Perhaps, if parking were recognized legally as a highway function, there would be justification for, and necessity could be proved for, taking such excess property for such use.

(5) The value of such facilities is recognized by business, particularly those businesses which are responsible for the growing trend toward decentralization through the establishment of smaller concentrated business centers in outlying districts. In many such cases the store or group of stores involved are providing free parking facilities (sometimes on a large scale) for patrons. This is perhaps a major factor in the growing success of such outlying business centers.

To a large extent such decentralization is at the expense of existing businesses and property values in the heart of the metropolitan area. To the extent that a plan aims to conserve such central values it should, perhaps, take cognizance of the parking problem as being a considerable factor.

Legislation to Make Plans Effective

A plan of a system of trunk-line routes with adequate right of way, adopted by all highway authorities of the metropolitan area, is essential to real and steady progress; but it is impotent and unproductive in the absence of legislation to enforce it, to preserve it from encroachments or abandonments, and to provide adequate methods for obtaining right of way by condemnation and otherwise.

Perhaps a review of the legislation of Michigan which has been found effective in creating, rapidly, a network of adequate rights of way in accordance with a definite plan for the Metropolitan Area of Detroit, may not be amiss even if the underlying constitutional provisions, highway laws, etc., may not be quite duplicated in other States.

The fundamental highway laws of Michigan set up jurisdictions, etc., as follows:

(1) Except as affected by home-rule constitutional provisions in the case of incorporated cities and villages, the State Highway Commissioner has jurisdiction over State trunk-line highways and is required to meet one-half the cost of maintenance, widening, and betterment thereof in metropolitan communities of the population of Detroit.[4]

(2) All other highways and sub-division streets and alleys, except in incorporated communities, are under the jurisdiction of each County Road Com-

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[4] PA 131 of 1931 amending Section 1 of PA 19 of 1919.

mission. With the consent of the incorporated community such commission may take jurisdiction over any street or part thereof within the community. Such commission is also empowered to take over, in whole or in part, the obligation of any such community arising out of a contract with the State Highway Commissioner in conection with the widening or improvement of a street which is a State trunk-line; and, in connection therewith, or as may be entailed on streets taken over by the commission, it may condemn property within the community.

Super-Highway Law.—Shortly after the Master Plan for the Metropolitan Area of Detroit was adopted by all municipalities and highway authorities, an effective legal instrument was made available in the super-highway law[5] which included the following:

(*a*) A contract may be entered into by boards of supervisors, and a super-highway commission can be formed by any two or more adjacent counties; and such commission includes the three members of the respective County Road Commissions and the State Highway Commissioner.

(*b*) Super-highways are defined as including highways of widths varying from 106 to 204 ft, or more.

(*c*) It is the duty of the super-highway commission to prepare and record a plan of the proposed highways and their widths, and where there are existing highways the plan must show on which side of the road private property is to be used for the widening. After recording of the plan no plat of land shall be accepted which does not conform thereto.

(*d*) The law sets up a limitation of 0.5 mill property tax in each county for the use of the commission. Except to cover the initial office expenses involved, this section of the law was never utilized and, in practice, each road commission has acquired right of way at its own expense from other funds (automobile tax revenues).

(*e*) The commission can accept donations of land and property and dedications of land, and may purchase, option, or condemn land.

The major effect of this legislation was to enforce the plan on sub-dividers of property and through that provision alone dedication of many miles of frontage in accordance with the plan was secured without cost, thereby saving millions of dollars.

Approval of Plats.—If a plat of property includes county roads or State trunk-line or Federal Aid roads, it must be submitted to the County Road Commission or the State Highway Commissioner for approval before being eligible for recording.[6]

Abandonments.—Although a Court may vacate a plat, it cannot abandon any part of a county road or State trunk-line highway. Such highway can be abandoned in whole or in part only by the County Road Commission or the State Highway Commissioner as the case may be. Abandonment by the County Road Commission must be recorded. Thus, the right of way, once secured, cannot be abandoned except by these authorities.[7]

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[5] PA 381 of 1925.

[6] Sections 31 to 37, PA 172 of 1929.

[7] Section 66 of PA 172 of 1929; Section 18 of Ch. 4, PA 283 of 1909 being Section 3993 of Compiled Laws (1929) as amended by PA 135 of 1935.

Encroachments.—Title to any part of the right of way of a highway cannot be secured by adverse possession.[8]

Condemnations.—County road commissions and the State Highway Commissioner have several methods of condemnation available: (1) By jury under two Acts;[9] (2) by a commission of three appointed by the Circuit or Probate Court;[10] and, (3) immediate possession after hearing of necessity and after tender of damages estimated by the County Road Commission. If tender is not accepted, condemnation must be done by processes provided to determine damages.[11]

Set-Backs Without Excess Condemnation.—If lots at the rear of the lots desired can be secured by agreement and if owners of front lots will accept a set-back by agreement, this process can be adopted. In outlying districts this provision has enabled the purchase of much frontage at the price of interior lots.[12]

Conveying Title.—By statute, County Road Commissions are incorporated bodies and are empowered to convey title to lands held by them and which are not a part of, or required to be used for, a road or park.[13]

May Condemn All of a Lot.—County Road Commissions can take all of a lot (if such part is required for the public improvement) as will destroy the value of the remainder as a single parcel or lot.[12]

Needed Legislation.—To meet the future, as the need presents itself, legislation may become necessary for condemnation of air rights, and for the exclusion of abutting property from access to free-ways. Free-way legislation has been enacted in the States of New York and Rhode Island.[14] Quoting from *Information Bulletin No. 36*, issued by the Regional Plan Association of New York:

> "This act [the Rhode Island act] is to be recommended for its directness and simplicity of treatment.
>
> "The adoption of freeway legislation by these two States may be hailed as the beginning of the most significant movement in several years in the progress of highway development. A look into the future would envision the idea extended to the other states of the Nation, expanded to include county and local highway departments and perhaps influencing the program of Federal highway aid."

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[8] PA 46 of 1907 and PA 314 of 1915.

[9] PA 149 of 1911 and PA 124 of 1883.

[10] PA 352 of 1925; PA 283 of 1909, as amended, being Sections 3986 to 3991 of Compiled Laws of 1929.

[11] PA 352 of 1925.

[12] Section 1(*a*) of PA 124 of 1883, being Section 3785 of Compiled Laws of 1929.

[13] PA 283 of 1909, being Section 3984 of Compiled Laws of 1929.

[14] Chapter 2537 of the laws of 1937.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

RELATION OF RAINFALL AND RUN-OFF TO COST OF SEWERS

By John A. Rousculp,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The preliminary design of storm sewer systems often requires a number of laborious studies involving various plans and perhaps various combinations of the two design factors—rainfall frequency rates and run-off percentage. It is the purpose of this paper to describe the development of a method, using time-saving charts, based on a study of relative costs of a system, which can be a valuable aid in preliminary design work.

The relative costs of storm sewers, as studied herein for various combinations of design factors, are based on the application of such combinations to an assumed district. The selection of this assumed district was based upon a study of the concentration of area of a number of actual districts.

The rational method of estimating run-off was used in computing the various cases. Explanation is made of the method of determining the assumed district, and a sample computation is given to illustrate the method of determining the run-off, Q, for one of the cases. The results of the run-off Q-computations for all cases are plotted on charts showing the run-off, Q, for corresponding area. These charts are used as the basis for computing relative sewer-capacity requirements in connection with the study of cost relations. A discussion of results is given, together with examples suggesting some practical uses of a study of this nature.

The writer does not intend that the charts, as developed, should cover the full range of design stipulations that may be encountered, or be of widespread applicability. Rather, it is hoped that the development, as a method, may be helpful to other engineers in making similar studies to satisfy various special requirements.

Introduction

In the design of storm-sewer systems, the run-off at various points and the cost are dependent on the values adopted for the rainfall frequency rates and

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Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **October 15, 1938.**

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the run-off percentage. The use of various values for either or both of these design factors will result in corresponding values of computed run-off and cost. In order to study the effect of such design factors it is necessary to re-compute the entire system for each set of values. For example, suppose a design has been made on the basis of 10-yr rainfall and 40% run-off, and the cost estimated. Then, if for some reason, the estimated cost on a design basis of a 5-yr rainfall and 30% run-off is required, the system must be re-computed entirely. The relations of run-off and cost resulting from the use of various combinations of design factors as determined by this study can be used to determine such relative cost quickly, and to lessen the computing work greatly. Hence, the results of this or similar studies should be especially useful to engineers when confronted with preliminary design problems of this kind. Clearly, with such a quick means of estimating the relative costs, more thorough and comprehensive preliminary studies would be possible. Since it was necessary to vary slope and area to determine the effect of the rainfall and run-off factors, it was convenient to analyze also the relative effect of the slope and area factors on run-off and cost.

SELECTION OF TYPICAL DISTRICT

As a guide in the selection of the typical district, several representative sewer districts were used (see Fig. 1). The area concentrating and time of concentration, in minutes, were computed for a number of collection points along the main trunk sewer in each district. The areas concentrating for each district were then plotted in Fig. 2, and a curve for each district was drawn.

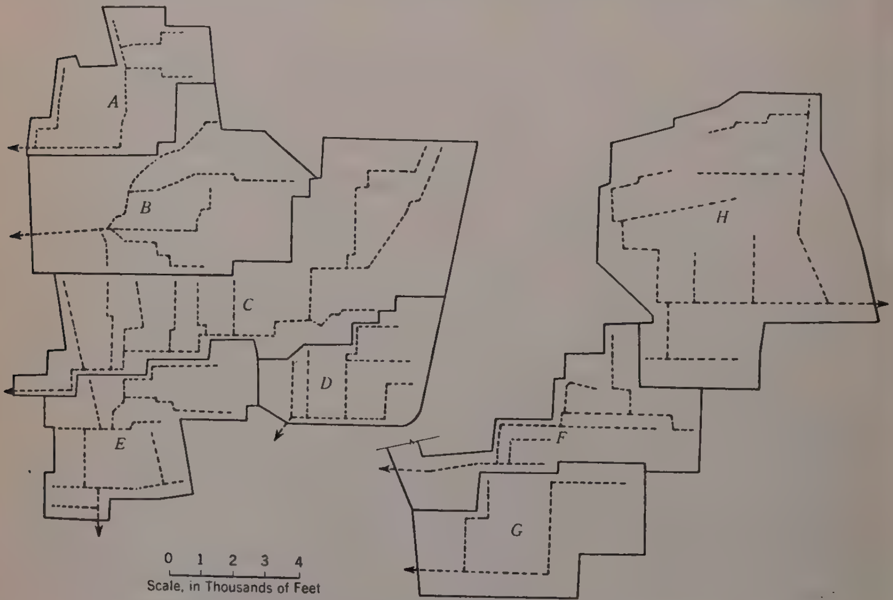


FIG. 1.—OUTLINE OF VARIOUS DISTRICTS USED IN STUDY OF CONCENTRATION OF AREA, SHOWING LOCATION OF PRINCIPAL SEWERS

A smooth curve, which was assumed to represent approximately an average of the curves of the various districts, was then drawn. This average curve represents the typical district, and areas concentrating with respect to time, as given by this curve, are as follows:

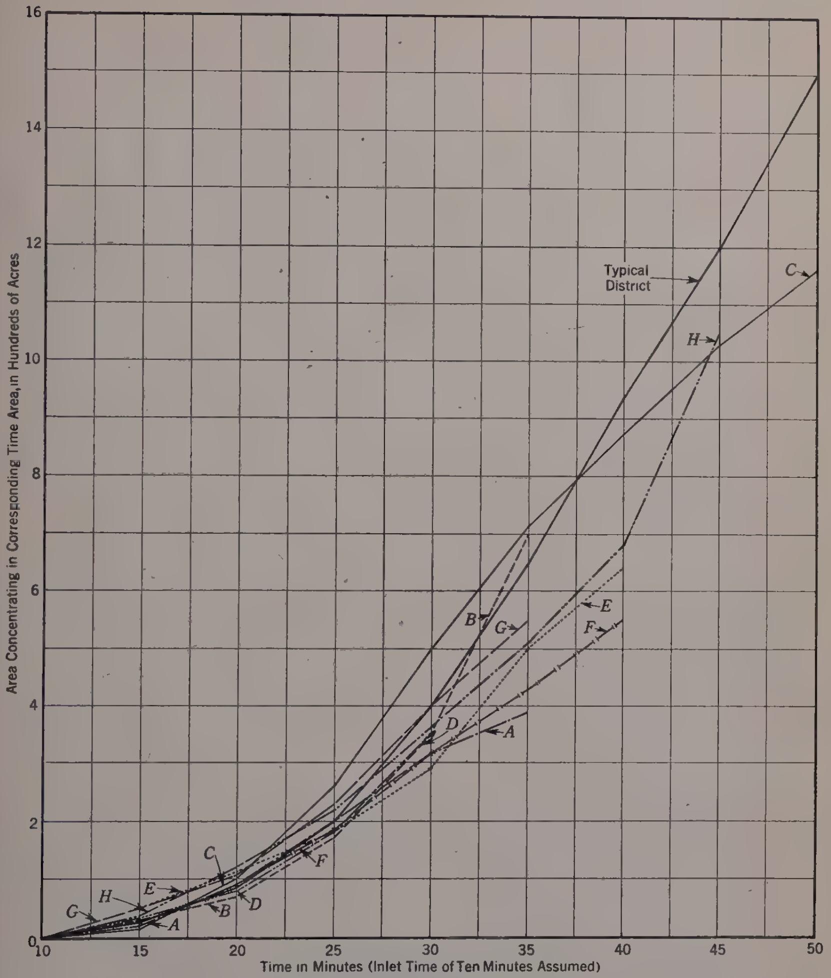


FIG. 2.—CONCENTRATION-OF-AREA CURVES OF VARIOUS DISTRICTS USED AS GUIDES IN SELECTING TYPICAL DISTRICT

Time, in minutes (based on $S = 0.003$)	Area, in acres, used for typical district	Time, in minutes (based on $S = 0.003$)	Area, in acres, used for typical district
15	25	35	650
20	80	40	940
25	200	45	1 200
30	400	50	1 500

The next step was to convert time in the typical district to distance, since the element of distance between points of concentration is necessary in a design problem. In computing the time element for the representative districts, velocity was based on sewer sizes required when using a design basis of 10-yr rainfall intensities, $C = 30\%$, and $S = 0.003$. Hence, these same conditions were used as a basis in computing distance from the foregoing list of time and area concentrating for the typical district. The results of the calculations to determine distance are given in Table 1. Note that the acres accumulating appear in a different order than in the foregoing list. This change makes the typical district more comparable to an actual design problem, and the distances shorter between the up-stream points of concentration where the velocities are smaller and changing rapidly. It was found that much larger distances between points of concentration could be used at the lower end of the system (where flow is large and where velocities do not change to any extent even for considerable increase in tributary area) than at the upper end of the system, for the same relative accuracy in computing time of concentration. It was desired to use as many points of inlet as were required for reasonable accuracy, and no more, for the reason that each additional point used would add materially to the computations.

TABLE 1.—AREAS CONCENTRATING AND DISTANCES BETWEEN POINTS OF CONCENTRATION

Area, in acres	5	5	10	20	30	40	60	80	100	150	200	200	300	300
Accumulated area, in acres	5	10	20	40	70	110	170	250	350	500	700	900	1 200	1 500
Distances between successive points of concentration, in feet	440	530	780	830	850	1 100	1 200	1 450	1 950	2 350	2 200	3 800	3 600	....

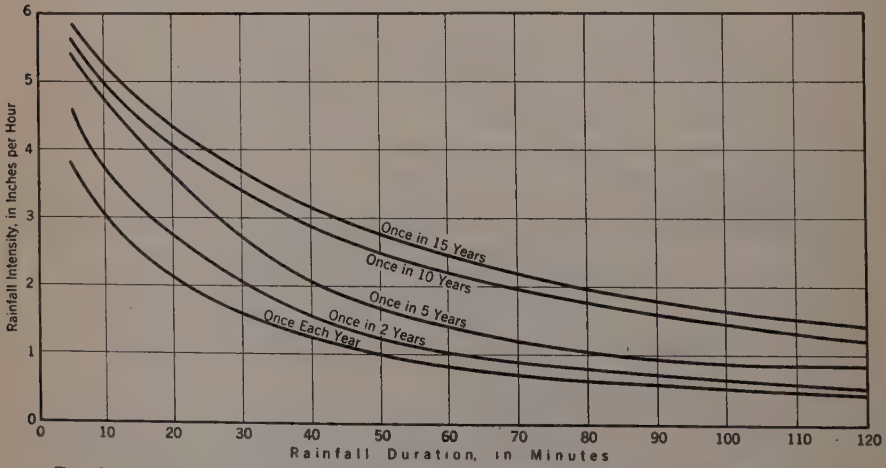


FIG. 3.—INTENSITY, DURATION, AND FREQUENCY RAINFALL CURVES FOR COLUMBUS, OHIO

SCOPE OF STUDY

In the usual storm-sewer design a basis is selected as, for example, 10-yr rainfall intensities and 30% run-off. The sewer slopes usually vary somewhat in each separate problem, and, hence, slope is not a part of the design basis. In this study, however, four different slopes were used, principally to determine whether the relative results due to the rainfall and run-off factors were affected by the slope factor. The range of factors covered in this study are presented in Table 2.

TABLE 2.—RANGE OF FACTORS COVERED BY STUDY

Rainfall intensities, yearly bases	10	5	2
Run-off percentages	30	40	50
Slopes	0.001	0.003	0.01 and 0.02

The total number of possible combinations of these factors is 36, and, hereinafter, they are referred to as cases. Hence, 36 sets of computations were made, using the typical district areas and distances, as given in Table 1. The rainfall intensities used are given in Fig. 3. Table 3 is an example of the computation to determine the run-off, Q, for one of the 36 cases. The values of Q thus determined for all 36 cases were summarized and plotted on Fig. 4.

TABLE 3.—SAMPLE COMPUTATION; 10-YEAR RAINFALL; 30% RUN-OFF; AND 0.003 SLOPE

Accumulated area,* in acres (1)	Distances* between successive points of concentration, in feet (2)	Time of concentration of area based on Column (2) and Column (6), in minutes (3)	Rate of run-off ($I \times C$), in cubic feet per second per acre (4)	Run-off, Q, Column (1) × Column (4), in cubic feet per second (5)	Velocity of flow in sewer required for Q in Column (5), in feet per second‡ (6)
5	440	10†	1.47	7.3	3.5
10	530	+2.1 = 12.1	1.42	14.2	4.3
20	780	+2.2 = 14.3	1.35	27.0	5.0
40	830	+2.6 = 16.9	1.28	51.0	5.8
70	850	+2.4 = 19.3	1.23	86.0	6.5
110	1 100	+2.2 = 21.5	1.18	130	7.5
170	1 200	+2.4 = 23.9	1.13	192	8.3
250	1 450	+2.3 = 26.2	1.09	272	9.0
350	1 950	+2.7 = 28.9	1.05	368	9.7
500	2 350	+3.3 = 32.2	0.99	495	10.3
700	2 200	+3.8 = 36.0	0.93	650	11.0
900	3 800	+3.3 = 39.3	0.88	792	11.5
1 200	3 600	+5.6 = 44.9	0.81	970	12.0
1 500	....	+5.1 = 50.0	0.75	1 125	....

* See Table 1. † Sewer size not given since velocity was taken directly from flow curves. ‡ 10-min inlet time assumed; velocities based on Kutter's $n = 0.013$.

TABLE 4.—SEWER COST PER LINEAR FOOT

Sewer diameter, in inches (1)	Depth to sewer invert, in feet (2)	Sewer cost, in dollars per linear foot (3)
12	10	2.00
15	10	2.60
18	10	3.30
21	11	4.10
24	11	4.80
30	11	6.40
36	12	8.10
48	13	11.70
60	13	15.40
72	14	19.60
84	15	23.90
96	16	28.20
108	17	32.70
120	18	37.00

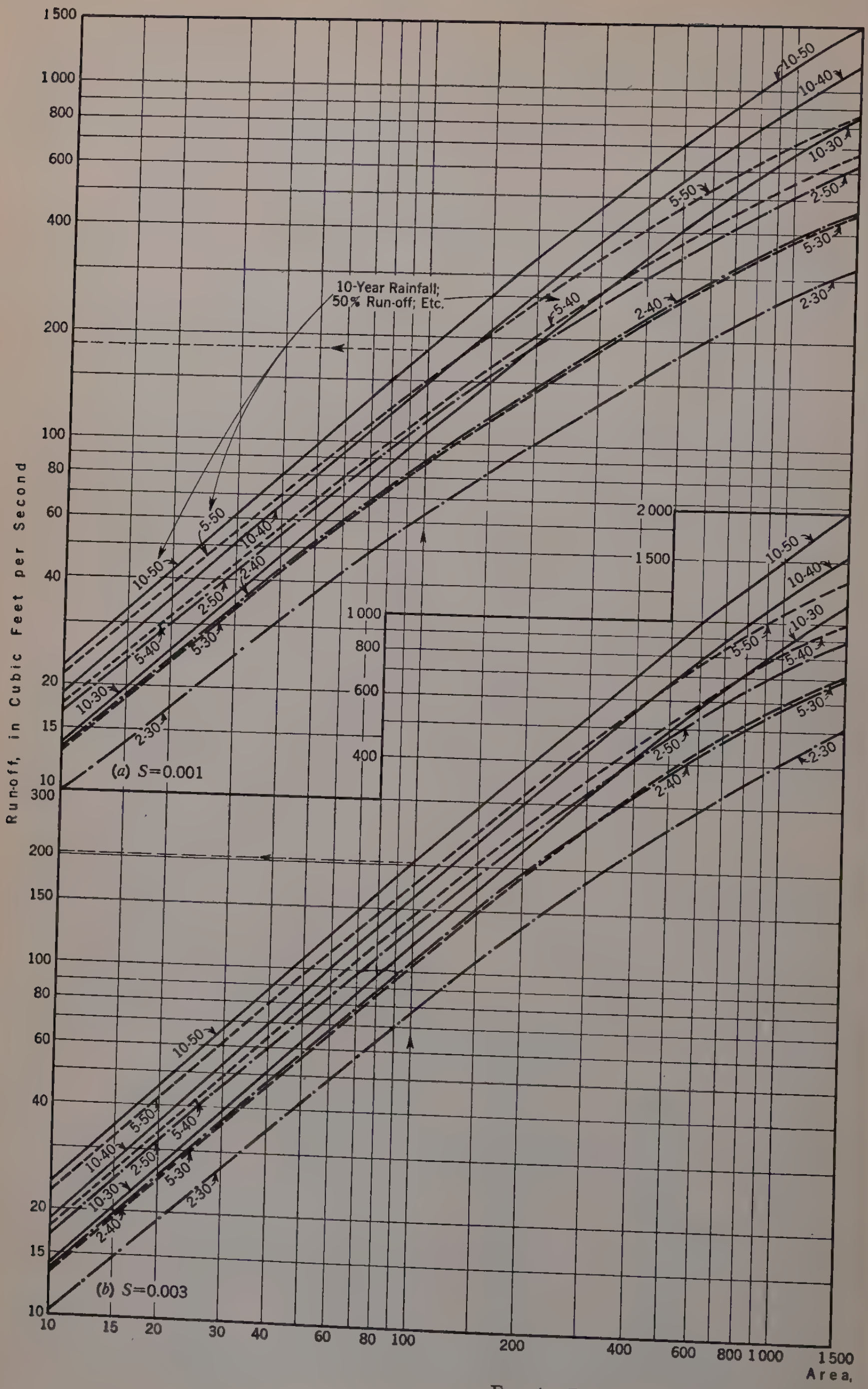
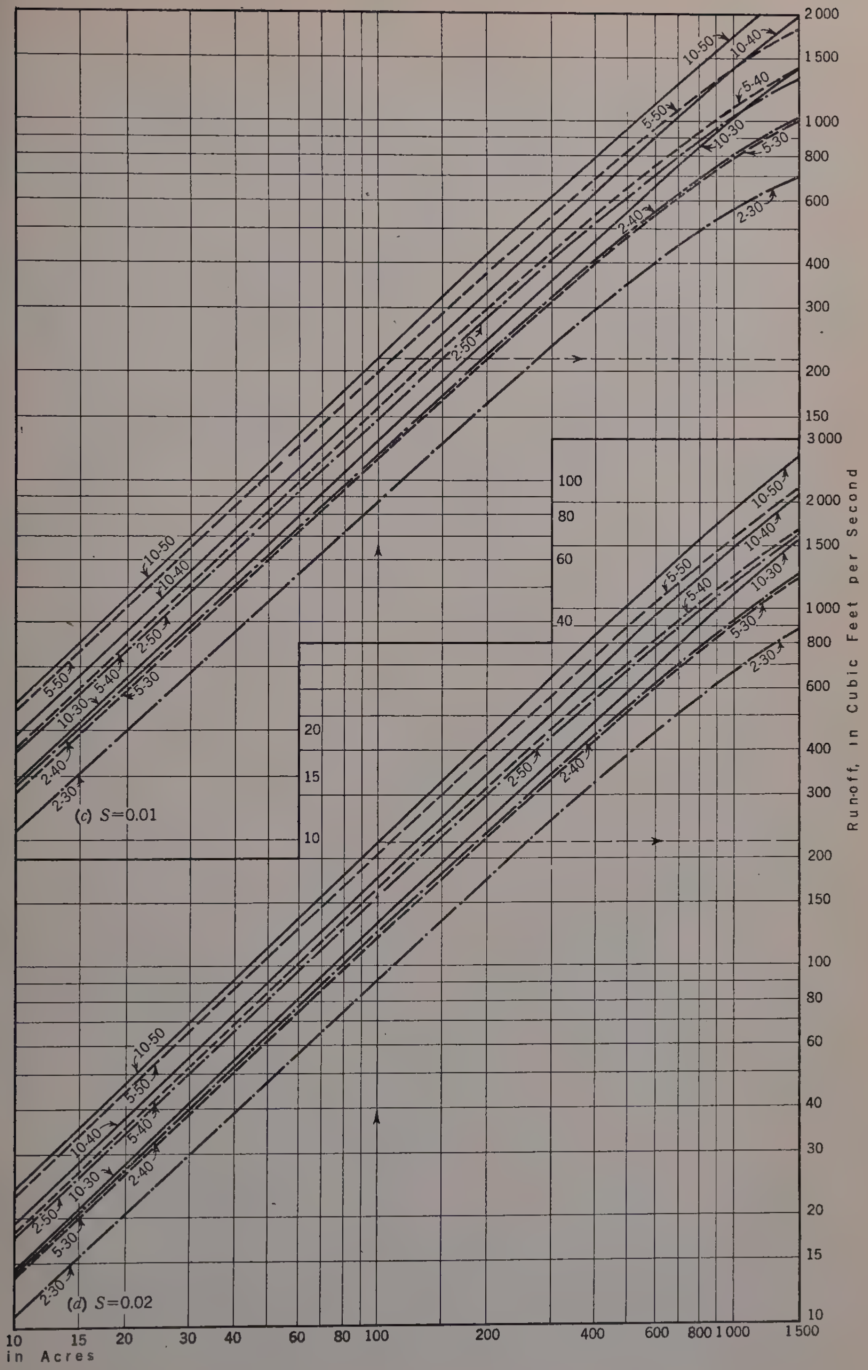


Fig. 4.—Run-Off for Various Rainfall



Curves and Run-Off Coefficients

Cost Computations

The costs per linear foot of sewer given in Table 4 were used as the basis of the cost studies. The depths in Column (2) were considered as average and the costs per linear foot (Column (3)), as representative of the average 1936 prices. Although prices varying considerably from those selected could be justified as being representative of various localities, it is believed that any such variation would have but slight effect on the relative costs. The next step was a tabulation setting acreages opposite the capacity, in cubic feet per second, of the various sewer sizes for each of the 36 cases. The areas were read from the curves representing the various cases in Fig. 4. In reading these values, the chart being used was entered on the run-off side with the value of the sewer capacity in cubic feet per second, the corresponding area being read from the proper curve.

Table 5 is a sample tabulation on the basis of a 10-yr rainfall and $S = 0.001$, giving the values for 3 of the 36 cases. Values of area in the tabulations (of which Table 5 is an example), where the acreage values were less than 10 acres, could not be read from Fig. 4, but were computed by using the 10-min rainfall intensity with the proper run-off coefficient. Such computations give results approximately correct, since the concentration time of all areas smaller than 10 acres may be assumed to be 10 min.

TABLE 5.—Areas Corresponding to Sewer Capacities—10-Year Rainfall; and $S = 0.001$

Sewer diameter, in inches (1)	Sewer capacity* ($S = 0.001$), in cubic feet per second (2)	Area, in Acres, for the Following Run-Off Percentages			Sewer diameter, in inches (1)	Sewer capacity* ($S = 0.001$), in cubic feet per second (2)	Area, in Acres, for the Following Run-Off Percentages		
		30% (3)	40% (4)	50% (5)			30% (3)	40% (4)	50% (5)
12	1.1	0.75	0.55	0.43	48	45.6	37.0	27.0	21.0
15	1.9	1.3	0.95	0.75	60	83.0	75.0	53.0	42.0
18	3.2	2.2	1.6	1.3	72	135	130	90.0	72.0
21	5.0	3.4	2.5	2.1	84	202	210	143	112
24	7.0	4.7	3.6	2.9	96	287	325	215	165
30	12.9	9.1	6.6	5.3	108	392	480	310	240
36	21.1	16.0	11.5	9.0	120	518	710	455	340

* Based on Kutter's $n = 0.013$.

From the tabulations, of which Table 5 is an example, the curves of Fig. 5 were developed, these being the basis of the cost studies.

Computing Costs per Acre

Preliminary to computing costs per acre, it was necessary to select a typical sewer layout, from which to secure sewer lengths per acre serving various size areas, against which to apply costs. The layout adopted was one of 100-acre units made of blocks of approximately 4 acres each, which were 350 ft wide and 500 ft long (see Fig. 6). These blocks were arranged in rows, 4 blocks wide and 6 blocks high, to make up the 100-acre units. This would actually be about 96 acres, but was taken as 100 acres to simplify computa-

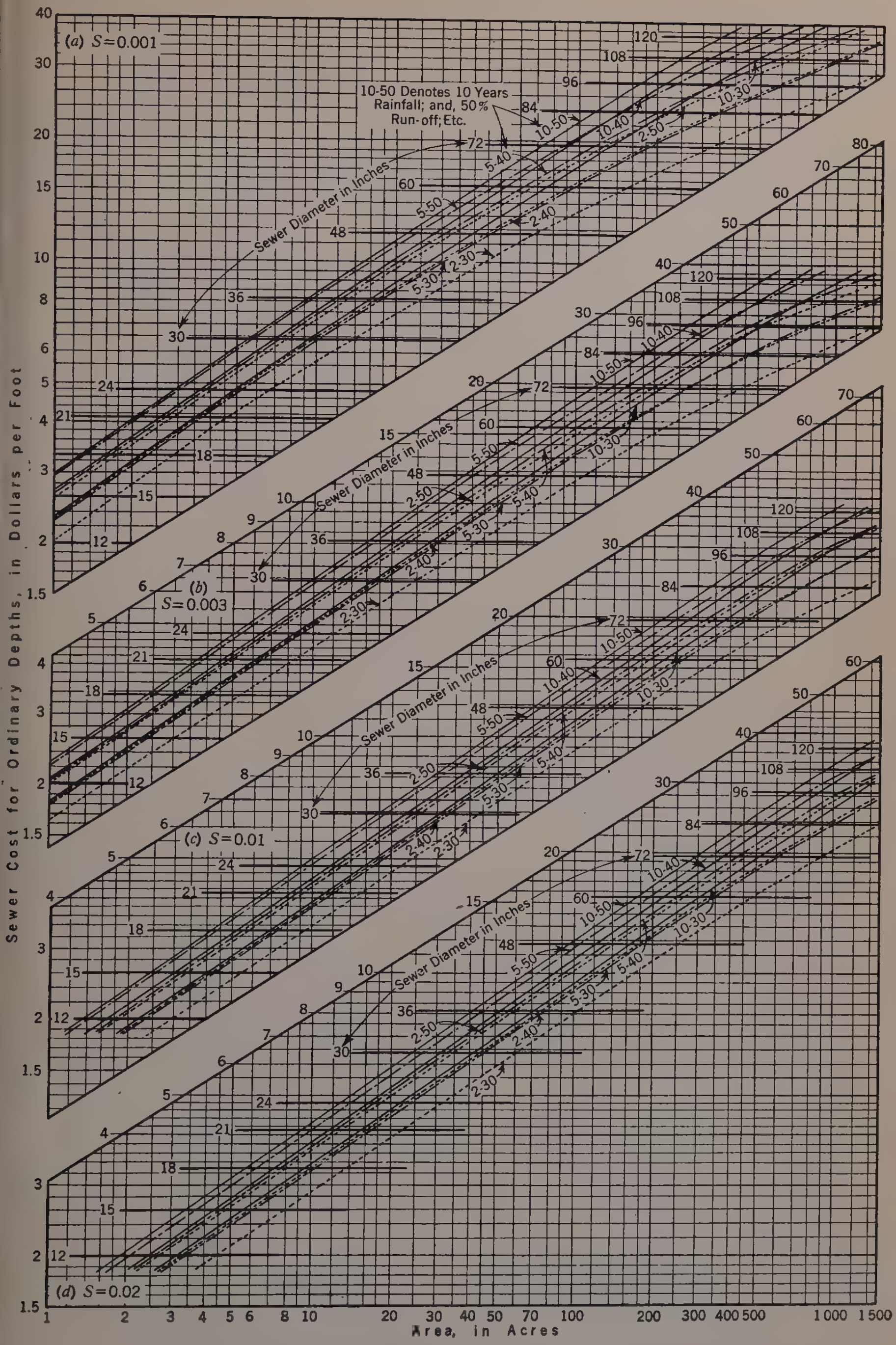


FIG. 5.—COST AND SIZE CURVES FOR TWO-YEAR, FIVE-YEAR, AND TEN-YEAR RAINFALL FREQUENCIES AND RUN-OFF COEFFICIENTS OF 0.30, 0.40, AND 0.50

tions. In this arrangement, the 4-block width of the unit was 4 × 530 ft = 1 400 ft, and the 5-block length was 6 × 500 = 3 000 ft. The collecting lines were arranged with one line to each row, extending the width of the blocks; these lines connected with a line which ran along the 2 500-ft side of the 100-acre unit. This line and the corresponding ones for each 100-acre unit connected with the main line serving all the units, by extending along the 1 400-ft width of the units. This layout, with the main trunk running along the lower edge of the 100-acre units, was estimated to represent, closely enough, the typical district used in computing the run-off, Q, as shown in Table 3. The arrangement in blocks and units made it possible to compute costs per acre for the various cases with a minimum of labor. Table 6(*a*) gives the computed lengths of sewer per acre for various areas served, and accumulated sewer lengths per acre. Using the sewer lengths per acre given in Table 6(*a*) and costs per foot taken from Fig. 5, the costs per acre were computed. Table 6(*b*) is an example of the computations for one of the 36 cases.

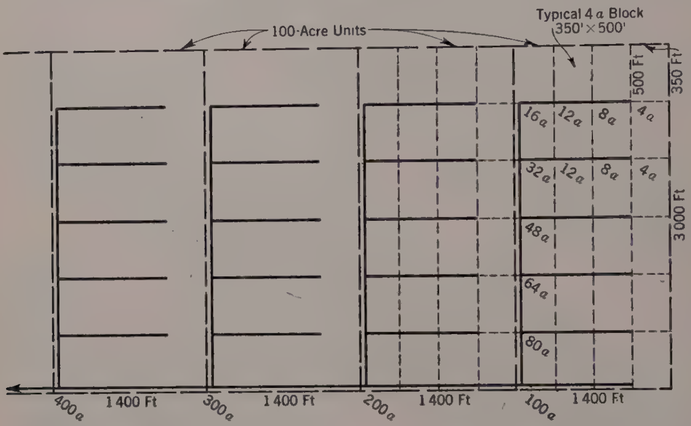


Fig. 6.—Assumed Layout Used as a Basis of Sewer Lengths per Acre

Using the results of the cost computations, curves representing costs per acre for 32 of the 36 cases were plotted on Fig. 7. The studies of cost per acre do not give values for areas smaller than 100 acres, as the purposes of the study are amply served without dealing with smaller areas.

The principal purpose of computing costs per acre, as given by Fig. 7, was to relate or compare costs of the various cases. Although the lengths per acre used may vary considerably from those for an ordinary problem, they are so proportioned for the various areas that the relative costs are fairly representative of the ordinary problem. To compare results in any ordinary problem with the results of this study, merely determine the accumulated sewer lengths per acre as in Table 6(*a*), and proportion the results to a comparative basis.

Note that in connection with computing costs per acre, the use of the interpolated costs per foot as taken from Fig. 5 was necessary to eliminate the 5% to 10% variation or error that would result from using the cost per foot of the sewer size which came nearest to serving the area in question.

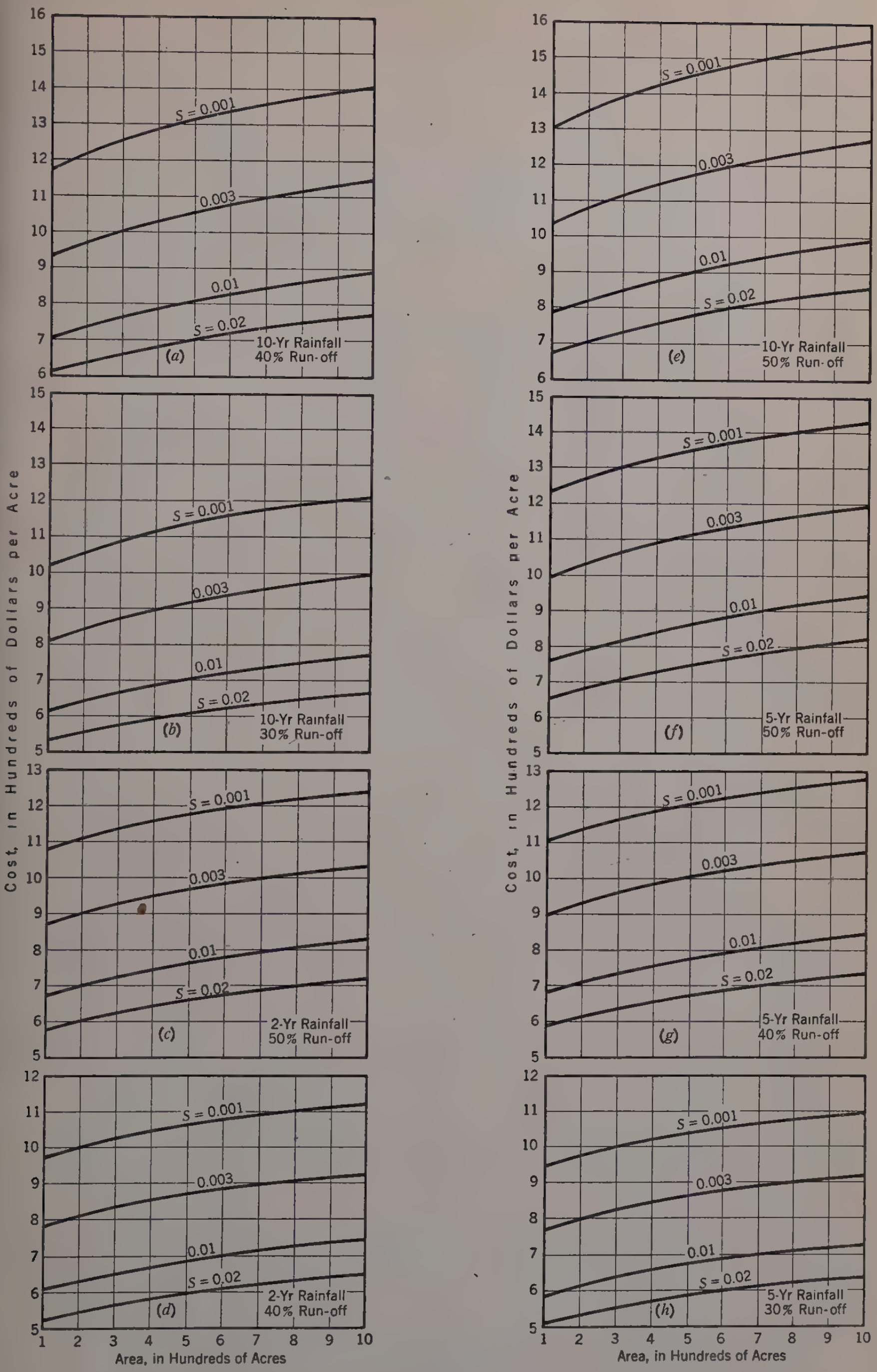


Fig. 7.—Costs per Acre

TABLE 6.—Costs and Lengths of Sewers

Area served, in acres	(*a*) Lengths		(*b*) Costs‡				
	In feet per acre	Accumulated lengths, in feet per acre	In dollars per foot	Total, in hundreds of dollars (Column (2) × Column (4))	Accumulated Costs		
					In hundreds of dollars	In Dollars per Acre	
						Column (6) ÷ Column (1)	$610 + Column (7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
4	350 ft per 16 *a* = 21.9	21.9	3.40	....	....	74*	....
8	350 ft per 16 *a* = 21.9	43.8	4.70	....	....	103*	....
12	350 ft per 16 *a* = 21.9	65.7	5.60	....	....	123*	....
16	500 ft per 80 *a* = 6.25	71.9	6.40	....	....	40*	....
32	500 ft per 80 *a* = 6.25	78.2	8.70	....	....	54*	....
48	500 ft per 80 *a* = 6.25	84.4	10.30	....	....	64*	....
64	500 ft per 80 *a* = 6.25	90.7	11.60	....	....	72*	....
80	500 ft per 80 *a* = 6.25	96.9	12.80	....	....	80*	610
100	1 400 ft per 100 *a* = 14.00	110.9†	14.10	197	197	197	807
100 to 200	2 800 ft per 200 *a* = 14.00	110.9	19.10	267	464	232	842
100 to 300	4 200 ft per 300 *a* = 14.00	110.9	22.80	319	793	261	871
100 to 400	5 600 ft per 400 *a* = 14.00	110.9	25.80	361	1 144	286	896
100 to 500	7 000 ft per 500 *a* = 14.00	110.9	28.20	394	1 538	307	917
100 to 600	8 400 ft per 600 *a* = 14.00	110.9	30.00	420	1 958	326	936
100 to 700	9 800 ft per 700 *a* = 14.00	110.9	31.80	445	2 403	343	953
100 to 800	11 200 ft per 800 *a* = 14.00	110.9	33.20	465	2 868	358	968
100 to 900	12 600 ft per 900 *a* = 14.00	110.9	34.80	487	3 355	372	982
100 to 1 000	14 000 ft per 1 000 *a* = 14.00	110.9	36.10	505	3 860	386	996

* Actual cost, Column (2) × Column (4). † No increase in accumulated sewer lengths per acre for areas greater than 100 acres, for a layout on which sewer lengths are based (see Fig. 6). ‡ For 10-yr rainfall, 30% run-off, and 0.003 slope.

Effect of Various Factors on Cost

Using the costs per acre from curves in Fig. 7, an analysis of relative costs was made and plotted in the form of curves in Fig. 8. The relative costs for each factor were determined by using various combinations of the other three factors and then running through the range of values of the factor under consideration with each combination. In the cases of slope and run-off coefficient, it was found that relative costs for the factor under consideration did not vary materially with the various combinations of the three other factors. Hence, the cost relations for each of these factors are represented by one curve.

In the cases of the rainfall and area factors, it was found that there was some variation of cost relation due to different combinations of the other three factors, and limiting curves were necessary to represent the cost relations for these factors. In the case of the rainfall frequency relations, the upper limiting curve is shown for slopes of 0.01 and 0.02 (steep slopes) and an area of 100 acres (small area), whereas the lower curve is for slopes of 0.001 and 0.003 (flat slopes) and an area of 1 000 acres (large area). The analysis revealed that only slight difference was due to variation of run-off coefficients. The difference due to variation of area from 100 acres to 1 000 acres was found to be about the same as that due to variation of slopes from 0.02 to 0.001. Hence, these factors should be taken into consideration in using the chart to estimate decrease or increase in cost due to the use of a lower or higher rainfall frequency. For example, for a 2-yr frequency, a slope of 0.002, and a 500-

acre area, the value would be found slightly above the lower curve; or, the cost for a 2-yr frequency would be about 80% of the cost for a 10-yr frequency.

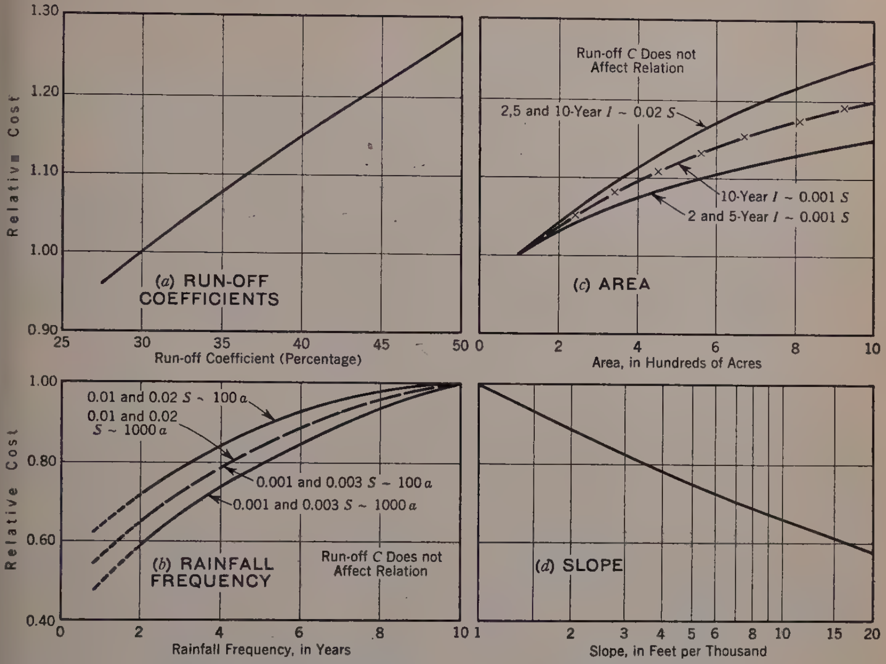


Fig. 8.—Relative Effect of Various Factors on Cost

In the case of area and slope factors, the cost relation curves are of no practical use other than as a matter of general interest because there would be a variation of these factors only in separate problems; for example, in the case of slope, if in a certain problem the slopes were 0.01 instead of 0.001, the cost relation would be 64%, or 36% less. In the case of area on the basis of a 10-yr rainfall and a 0.001 slope, the cost relation for 1 000 acres would be 120% of that for 100 acres, or 20% more.

Examples

To illustrate the practical use of the results of this or similar studies, a number of examples are given.

Example 1.—Assume a 10-yr rainfall, a 30% run-off, and a slope of 0.003. The design of a main trunk in accordance with these data is indicated in the following tabulation:

Areas to be served, in acres	Read run-off, in cubic feet per second, from Fig. 4(*b*)	Read diameter, in inches, from Fig. 5(*b*)
20	27	33
50	62	42
100	118	54
200	220	72
400	405	90

Example 2.—Assume the case of an area, for which it is desired to determine the capacity required, or the size of a main trunk outlet, without computing the time of concentration for the system. The area is 200 acres; the slope, –0.01; the frequency, a 5-yr rainfall; and a 30% run-off. Fig. 4(*c*) gives 215 cu ft per sec as the capacity required; or, Fig. 5(*c*) gives 54 in. as the required diameter.

Example 3.—Suppose the cost of a system of 1 000 acres has been computed on the basis of a 10-yr rainfall, a 30% run-off, and a slope equal to 0.001; and, the cost is found to be $1 000 per acre. If the relative cost on the basis of a 5-yr rainfall should be desired, Fig. 8 gives the 5-yr cost relation as 90% of that for 10 yr; hence, the cost would be 90% × $1 000 = $900 per acre.

Example 4.—Suppose the cost per acre of a system computed on the basis of 40% run-off has been found to be $800 per acre. To find the cost on the basis of 30% run-off, Fig. 8 gives the cost relation as 115 for 40% and 100 for 30 per cent. Hence, the cost would be $\$800 \times \frac{100}{115} = \$700$ per acre.

Example 5.—By the use of Fig. 7, the cost relations for any problem with any combination of factors can be determined. Assume a problem with 500 acres, and S = 0.01. Then, suppose it is desired to know the comparative cost computed on the basis of a 10-yr rainfall and a 40% run-off as against a 2-yr rainfall and a 30% run-off. From the 0.01-curve in Fig. 7, for 500 acres (10-yr rainfall, and 40% run-off), the cost per acre is $810. From the 0.01-curve in Fig. 7 for 500 acres (2-yr rainfall, and 30% run-off), the cost per acre

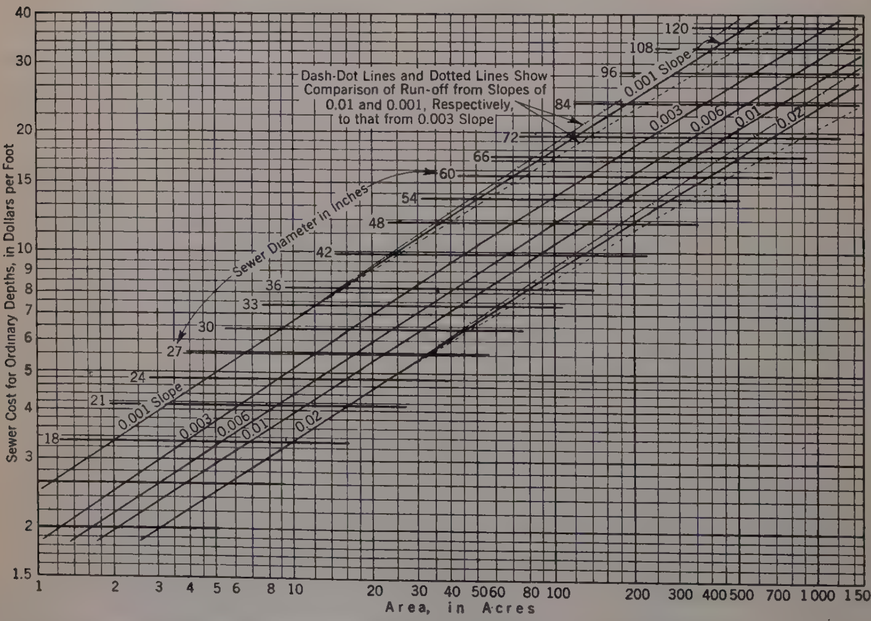


FIG. 9.—DESIGN CHART FOR STORM SEWERS; BASED ON TEN-YEAR RAINFALL; 30 PER CENT. RUN-OFF; S = 0.003; AND KUTTER'S n = 0.013

is \$600. Hence, if a problem had been solved on the basis of a 10-yr rainfall, and a 40% run-off and if the cost had been found to be \$800 per acre, the cost on the basis of a 2-yr rainfall and a 30% run-off would be $\$800 \times \frac{600}{810} = 800 \times 74\% = \$595$ per acre. This example should make clear that the use of the costs per acre in Fig. 7 is as an index of cost for comparative purposes and not for direct answers, in cost per acre, for a particular problem.

DESIGN CHART

As a result of this study, the writer has developed a chart (Fig. 9) for use in storm-sewer design which has proved to be a valuable aid in preliminary design and in checking designs. The sample-design problem in Table 7 is solved by the customary method and the diameters required are found to be identical with those determined by Fig. 9.

TABLE 7.—DESIGN BY THE USUAL METHOD

(Design basis: 10-year rainfall; 30% run-off; $S = 0.003$)

Accumulated area, in acres (1)	Distances between inlet points, in feet (2)	Time, in minutes (3)	Rate of run-off* $I \times 30\%$, in cubic feet per second per acre (4)	Run-off, in cubic feet per second, Column (1) × Column (4) (5)	Diameter† of sewer required, in inches (6)
30	....	17	1.29	39	36
60	700	17 + 2.5 = 19.5	1.21	73	48
140	900	19.5 + 2.5 = 22	1.18	165	63
180	700	22 + 2 = 24	1.13	203	69
200	600	24 + 2 = 26	1.10	220	72
250	600	26 + 1 = 27	1.08	270	78
350	800	27 + 2 = 29	1.04	364	84
600	2 500	29 + 5 = 34	0.96	576	102

* See Fig. 3 for rainfall intensities. † Sewer sizes based on 0.013 value of Kutter's *n*.

ACKNOWLEDGMENT

This paper is based on a thesis, "Storm Sewer Cost Studies," submitted to Ohio State University by the writer, in 1933, in partial fulfillment of the requirements for the professional degree of Civil Engineer.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

A NEW THEORY OF RAIL EXPANSION

Discussion

BY ALFRED AFRICANO, ASSOC. M. AM. SOC. C. E.

ALFRED AFRICANO,[29] ASSOC. M. AM. SOC. C. E. (by letter).[29a]—Those who have been kind enough to discuss this paper have added immeasurably to its value by clarifying some points perhaps too briefly treated, by contributing their experience with continuous welded rail in support of the theoretical analysis, and, finally (and of equal importance), by emphasizing certain objections to the fundamental assumptions made.

Although most of the discussion indicates an acceptance of the simple mathematical derivation of the formulas as valid for practical purposes (leaving to the judgment of the engineer who uses them the determination of the best average resistance, T, per tie—as the writer intended), the objections offered by Professor Talbot seem sufficiently important to be given first attention.

Of the questions which Professor Talbot raises the principal one seems to be the uncertainty as to whether or not the longitudinal resistance set up at each tie is a constant. It is not a constant. It varies under the influence of so many factors that the writer purposely avoided attempting to assign any quantitative value to those factors at all. If a simplifying average or effective value of the tie resistance under all conditions can be used in such a manner that the theoretical analysis will show a practical agreement with observed end movements, its use is obviously justified.

The experimental data given for the two half-mile rails of the Delaware and Hudson Railroad Company at Mechanicville, N. Y., were the average of forty reported observations of end movements made over a period of 1 yr, for various temperature changes. It is extremely interesting and instructive to plot these individual results against Δt, the temperature change in degrees Fahrenheit from the rail-laying temperature. This has been done in Fig. 11(a). For comparison, the curves of theoretical end movements obtained by the use of Equations (9) and (18) are also shown.

NOTE.—The paper by Alfred Africano, Jun. Am. Soc. C. E., was published in February, 1937, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: April, 1937, by Messrs. Chester F. Gailor, and E. F. Kenney; and June, 1937, by Messrs. C. W. Baldridge, George W. Hunt, Frank B. Walker, Arthur N. Talbot, Randon Ferguson, A. N. Reece, G. M. Magee, and H. D. Hussey.

[29] Asst. Engr., Interborough Rapid Transit Co., New York, N. Y.

[29a] Received by the Secretary January 15, 1938.

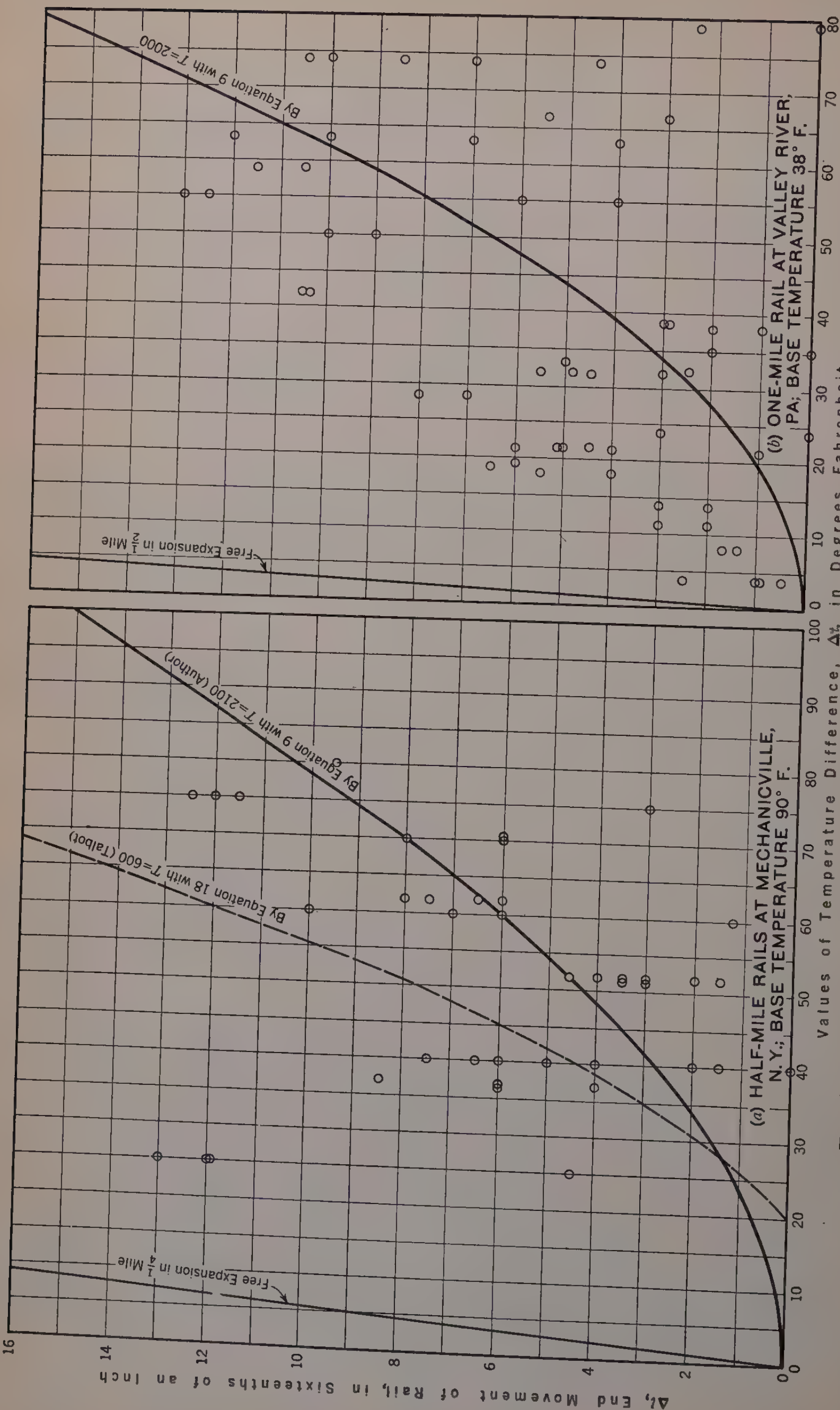


FIG. 11.—COMPARISON OF THEORETICAL WITH OBSERVED END MOVEMENTS OF LONG RAILS

The scattering of the observed points indicates the difficulty of attempting to formulate an exact equation accounting for every variation and, consequently, the desirability of using the simplest possible analysis that will give practical results. Equation (9), based on a constant value for T, apparently gives such results. The important consideration here is that the theoretical or predicted expansion or contraction is approximately 1 in., as observed for the ordinary range of temperature change, and not the large value heretofore calculated by assuming free expansion. That the fear of dangerous free expansion actually existed at the time these rails were laid, is proved by the fact that an elaborate expansion joint was provided by the engineers of the Delaware and Hudson Railroad Company to take up the maximum of 22-in. end movement computed in this way if one end remained fixed and the other received the total expansion.

Equation (9) is plotted with $T = 2\,100$, whereas Equation (18) is plotted with $T = 600$ and a joint restraint of 35 000 lb, or about 3 000 lb per sq in. transmitted by the joint (averaged from Fig. 10 given in Professor Talbot's discussion).

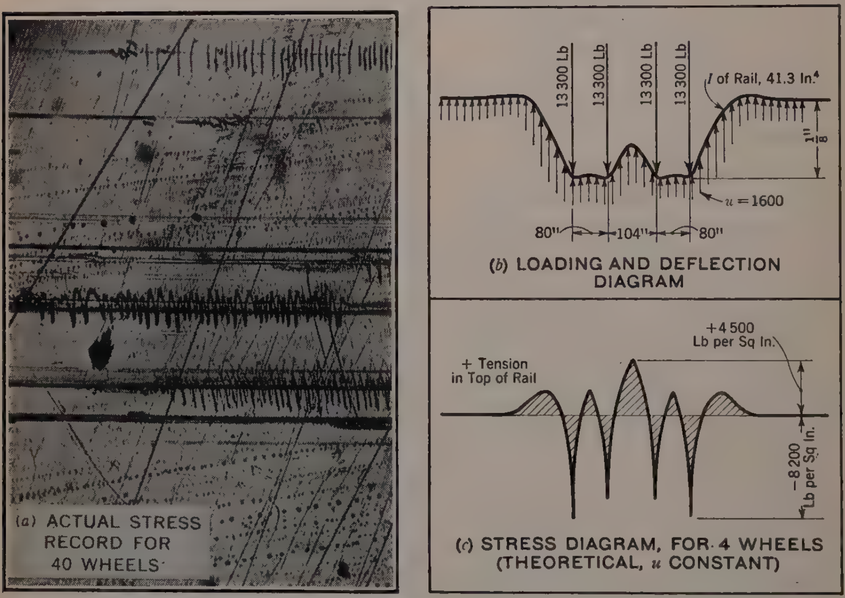
Although joint restraint no doubt exists, it is too erratic to be depended upon and, in the writer's opinion, should not be used at all in designing fastenings and track for new installations, since the really serious situation occurs when a rail breaks, giving the condition of two free-rail ends. By controlling the value of the tie resistance in future installations the magnitude of the resulting gap can also be controlled. In Fig. 11(*a*), a value of 2 100 in Equation (9) shows a better agreement with the observed data than the value of 600 in Equation (18) with any magnitude of joint restraint. The high value also gives a better basis for explaining why the gaps were only 1 in. or 2 in. when rails broke instead of the 3 in. to 6 in. that should be expected if only the lower value was available.

It should be noted that the writer did not arbitrarily assign or "place" the high value of 2 100 lb for T for the reason Mr. Hunt suggests, but solved for the average value that must have been effective at various temperatures to produce the evident correlation with the small observed end movements.

From additional experimental data made available since 1935, when the paper was written, the writer plotted Fig. 11(*b*) showing a comparison of theoretical with observed end movements for the one-mile continuous welded rail installed by the Bessemer and Lake Erie Railroad Company, at Valley River, Pa. The agreement is as good as can be expected for so variable a structure as a railroad track, and the scattering of the points again precludes the use of any assumption other than simple constant tie resistance. It should be noted that very little joint resistance must have existed since Equation (9) passing through the origin obviously fits the data better than Equation (18). The latter would cut the horizontal axis at some particular value of temperature change determined by the degree of joint resistance which acts to prevent any movement until that temperature change is exceeded. No distinction has been made in this graph (Fig. 11) between movements (expansions) corresponding to positive temperature changes and movements (contractions) corresponding to negative temperature changes. The writer agrees with

Professor Talbot qualitatively in believing that the direction of the temperature change, up or down, as well as the magnitude, exerts some influence on the available tie resistance. Quantitatively, however, to judge from Fig. 11(*b*), it would seem scarcely worth evaluating.

Use of a simplifying assumption for practical purposes under such circumstances is by no means without precedent. As Chairman of the Special Committee on Stresses in Railroad Track, already referred to, Professor Talbot used an analogous simplification in the case of the exceedingly variable vertical modulus of foundation, or rail support, u, in the development of the equation for rail stress due to static wheel loads.[5] Those familiar with this equation know how variable this modulus may be in different tracks and even from point to point in the same track. When the writer began investigating rail stresses in the tracks of the Interborough Rapid Transit Company, in 1931, he found by studying the reports of the Committee on Stresses in Railroad Track and by correspondence with authors of papers on rail stresses, that although the extreme variation in individual values of this assumed constant was generally acknowledged, the results obtained from the assumption were sufficiently close to actual observed stresses to warrant the method.



(*a*) ACTUAL STRESS RECORD FOR 40 WHEELS

(*b*) LOADING AND DEFLECTION DIAGRAM

(*c*) STRESS DIAGRAM, FOR 4 WHEELS (THEORETICAL, *u* CONSTANT)

FIG. 12.—ILLUSTRATION OF ANALOGOUS CASE IN WHICH MODULUS OF FOUNDATION, u, AS A CONSTANT GIVES PRACTICAL RESULTS FOR RAIL STRESSES DUE TO WHEEL LOADS

To illustrate this point, the similarity is shown in Fig. 12 between actual stresses measured by the writer in 1934 with a scratch extensometer and the theoretical static stresses from Equation (24) by using a constant value of $u = 1\,600$ and superimposing stresses due to the four wheels. The scratch record was made while the 10-car train passed slowly over the instrument, recording actual strains in the steel, and later was magnified 100 times and photographed. Since the recorded stress represents the effect of different values of the vertical elasticity of the track, the agreement based on a constant value is surprisingly good.

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[5] Introduced in the First Progress Report of the Special Committee on Stresses in Railroad Track, *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1191.

Referring again to Fig. 10, an independent conclusion may be reached as to whether or not the tie resistance varied appreciably for the same, and for different, temperature changes. If it is constant, the accumulating longitudinal force should increase uniformly along a straight line, and this the graph shows.

By noting that the slopes of the lines showing the accumulating stress in Fig. 10 are approximately 30° from the vertical, for three of the four temperature changes shown, the conclusion may be drawn that, as far as practical results are concerned, this assumption is valid for all temperature changes. The rounding off near the top of the sloped line is to be expected, since, as the writer was careful to point out, there is no doubt a slight effect, more pronounced at this point, of the longitudinal resistance being proportional to the longitudinal movement. As the accumulating resisting force approaches that required to fix the rail completely, the movement of the rail diminishes to so small an amount that the tie resistance does not have a chance to come into action, especially if there is any looseness in the ties. In other words, there may be a limiting degree of movement above which the tie resistance is fairly constant and below which the longitudinal restraint is more nearly proportional to the rail movement. In the writer's opinion, the latter effect is a negligible factor in the computation of the end movement.

A point of practical importance to many engineers—the stress in the fixed part of the rail—seems satisfactorily taken care of by Equation (2), but the writer's choice of value for the coefficient of linear expansion has been generally criticized. In view of the difficulty of predicting end movements within more than, say, 75% accuracy, it seems poor judgment to bother about extreme accuracy in this one factor just because more or less precise values happen to be available. The writer chose the value for hard steel, 7.3×10^{-6}, given in the steel handbooks, not to recommend its use particularly, but because it better fits the case of rail steel ordinarily having about 0.75% of carbon with a higher Brinell hardness number than the structural steel which has about 0.33% carbon and the lower value, 6.5×10^{-6}. In addition, where there is a choice of values, the conservative one would be that giving the worst condition, in this case, the maximum expansion. Thus, the highest coefficient in the range for rail steel, 5.5×10^{-6} to 7.2×10^{-6}, which Professor Talbot kindly furnished from the International Critical Tables, should unquestionably be used.

The handbook value for hard steel checks this maximum coefficient very closely. From the temperature stresses for the fixed part of the rail shown in Fig. 10, it is possible to compute an average value for actual track conditions. In Fig. 13, the writer has plotted the observed temperature stress against the corresponding average temperature differences from the base of 53° F for each case, choosing the straight line that passed nearest the data as the best stress-temperature relationship. The slope of this line is the same as that for the straight-line Equation (2), or $E\,n$. Substituting a constant value of 30 000 000 for E, and solving for n results in the value, $n = 7 \times 10^{-6}$, for the best actual track value of the coefficient of linear expansion in these cases, as against the laboratory test value of 6.3×10^{-6} which Professor Talbot favors. This calculation corroborates the higher value used by the writer, and, in his opinion, shows that the inaccuracy involved is not only negligible, but that the round value of 7×10^{-6} is entirely adequate for the degree of precision warranted and should be used in all computations of end movements for long rails.

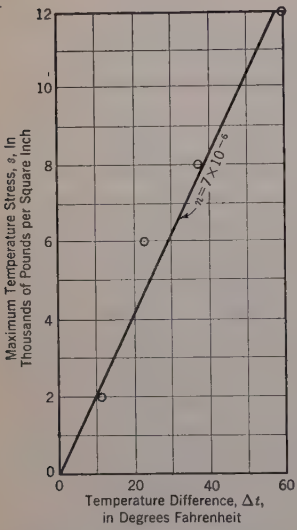

FIG. 13.—COMPARISON OF OBSERVED MAXIMUM TEMPERATURE STRESSES (FROM FIG. 10), AND WRITER'S THEORETICAL VALUES BASED ON RECOMMENDED COEFFICIENT OF LINEAR EXPANSION FOR RAILS.

The remaining points raised by Professor Talbot can be considered more briefly. It is to be expected that the special problems involved in the maintenance of long welded rails will have to be solved satisfactorily before they come into general use. Nevertheless, since it is known that they do not expand or contract as free rails, and have been in actual use for several years, as pointed out in Mr. Baldridge's valuable contribution of historical background, they are no longer experimental except for these problems. If this paper serves only to show to the Engineering Profession that the existing statistical evidence in favor of the practicability of long welded rails is supported by known engineering principles even if the theoretical analysis may not be completely satisfactory, the writer's intention has been fulfilled.

Regarding the use of calculus for deriving what proves to be a simple second degree equation, this is the usual method for determining the integral or total effect of a variable factor.[30] The writer took special care to show every step of the precise reasoning upon which the resulting formulas are developed. It may seem unnecessary to those who have devoted years of study to a specialized subject.

Figs. 9 and 10 are important in showing conclusively the existence of the fixed intermediate part of the rail as predicted by the photographs of the writer's elastic model shown in Fig. 2. The use of "equivalent stress per square inch" for the measured unit change in length may be confusing to some readers. Since a temperature change requires the length to change to avoid becoming stressed, it might have been more informative to plot the actual data to some convenient scale, especially since its use is desired for developing a general principle. Such data, if obtained near the ends of an intentionally broken rail, along with the corresponding end movements would yield valuable information on the actual maximum holding power of the rail fastenings.

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[30] "Mechanics of Materials," by L. A. Martin, p. 16. Here a similar use of calculus is made to determine the extension of a cylindrical rod due to its own weight. The interesting and analogous result is that the extension is one-half that which would occur if the total weight were concentrated at the lower end.

In cautioning readers in the use of Equations (18), (20), or (21) (or for that matter any equation or its application), the writer heartily concurs with Professor Talbot. Both the derivation of the equations and their applications, the writer agrees, are too new to be plucked blindly from a handbook. It is of equal importance to point out that inaccuracies may be conceded to exist, and yet be so negligible as not to affect the usefulness of the theory for all practical purposes.

Mr. Gailor appears to be satisfied with the calculations and formulas as a basis for future studies but states that the possible out-of-alignment of a long rail on a hot day is a different condition from the straight line assumed in the paper. It is true that for the purpose of analysis the writer used the case of straight tangent track. However, even if the rail is laid in the winter, which is not recommended, the buckling that may be caused vertically and laterally by excessive expansion the following summer is not likely to affect the end movement appreciably, and remains as a problem to be solved by providing proper lateral resistance.

Statement (6) by Mr. Gailor seems to show a misinterpretation of one of the fundamental points of the analysis, namely, that the temperature stress set up in a rail when fixed is independent of the length and would be the same whether for one tie-spacing or for one mile. Since it may be of the order of 200 000 lb it is impossible for any single tie-fastening to resist. The cumulative resistance is the only remaining explanation for the fact that such large forces must have been developed to restrain the long rails in present use. This cumulative feature of the tie restraint has been well established by the experience presented in several of the discussions.

The writer regrets having given the impression that he would have considered using rigid fastenings at the end portions only; yet according to the data submitted by Mr. Walker in Table 5, even ordinary spike fastenings prevented the ends of broken rails from separating "in excess of 2 in." so that the fear of "disastrous results" expressed by Mr. Gailor and Professor Talbot in such a case seems to be unfounded. However, the writer would recommend using some relatively economical type of rigid fastening for the mid-section and the best available rigid fastenings at the ends. In this way the definite end movements occurring with temperature changes are reduced to a minimum, thus avoiding trouble with the insulated joints which, to a great extent, will determine the practical maximum lengths of welded rail. The number of rail breaks for standard rail is of the order of 1 per track-mile in 5 yr of service. The temperature stress may increase this, but it will depend on the experience and judgment of the engineer as to what degree of rigidity and extra expense he will consider absolutely necessary to offset the risk of this occasional failure.

Mr. Kenney believes the vital problem is not so much the control of linear expansion as the control of lateral expansion without, possibly, increasing the maintenance cost. Since the scope of the paper was limited to the discussion of temperature stresses and rail expansion the writer will not attempt to prove any economy in the use of welded rail. Such conclusions depend to a great extent on the additional life of the rail due to elimination of end batter, and this factor requires several years to determine statistically.

The case mentioned by Mr. Baldridge of the shearing of joint bolts following a sudden drop in temperature is significant as it indicates the need for investigating even existing standard track for the possibility that it may act as a continuous rail. The writer has observed frequent cases in which rails have "ganged up" or closed their gaps in warm weather and then have acted as a unit in ensuing cold weather, in one case permitting a gap to open almost an inch.

Applying the analysis to the data given by Mr. Baldridge of a 2.5-in. maximum gap opening for a temperature difference of 100° F it is possible to estimate the length of anchorage and the average tie-resistance that had to come into action in this case. A 2.5-in. gap means that 1.25 in. of contraction occurred in the critical length on each side of the joint. This is the free expansion for 160 ft of rail, as Mr. Baldridge calculates, by substituting $\Delta l = 1.25$ in., $n = 6.5 \times 10^{-6}$, and $\Delta t = 100°$ F, in Equation (1), and solving for l. However, this length, l, represents expansion or contraction without restraint, and since the actual degree of expansion varies from that for a free rail at the gap end, to zero at the opposite fixed end, the average rate of expansion is only one-half the free expansion as the writer demonstrates by Equation (9) and Theorem 1 in the paper. Consequently, the length of the anchorage must have been twice 160 ft, or 320 ft, on each side of the joint. Assuming 2-ft tie-spacing, the average longitudinal resistance of 160 ties will be available to equal the total temperature stress set up when the rail is finally fixed. The unit stress from Equation (2) is 19 500 lb per sq in., if n is taken as 6.5×10^{-6} and E, as 30×10^{6}. Multiplying by the area of the 112-lb rail (11.05 sq in.), the total temperature stress to be resisted by the ties is about 216 000 lb, which, divided by the 160 ties, gives as the probable effective resistance of each tie 1 300 lb. Equation (9) would have given this value immediately since all other quantities are known.

Mr. Hunt emphasizes the fact noted by the writer that Equation (9) is simply a special case of Equation (18) when the joint resistance, P, is equal to zero. He contributes a useful graphical solution in Fig. 5 for this general case. The writer developed the "special" case first for two good reasons: It is logical to start with the simplest case; and since this tells the maintenance-of-way engineer what maximum gap opening to expect, it is the most important case.

Table 3 is useful to show the effect of joint restraint on the calculated value of the tie resistance. Both Messrs. Hunt and Magee believe the influence of joint restraint was under-estimated, thus leading the writer to what may be considered too high a value for T in the illustrative examples. These examples show that two different values may be obtained by using a definite joint restraint and by considering the gap at a free end. Hence, it should follow that the correct application of Equation (18) at a joint to determine the tie restraint depends upon an accurate knowledge of the degree of joint restraint. It is much simpler to obtain accurate values for the maximum available tie resistances, by reproducing the condition of a free end. Thus, by loosening the joint bolts and noting the free end movement, the elementary Equation (9) can be applied. Purposely breaking the long rail at some point on the fixed length would be an accurate method of obtaining the data to guard against such an accidental contingency. The effect of the joint restraint on the magnitude of the temperature stress seems of little importance since this stress becomes a maximum in the fixed part of the rail regardless of whether or not joint restraint exists. Its only effect on the end movement is to decrease it.

In Table 4 Mr. Hunt demonstrates that an 8-in. gap might result if ordinary track were made continuous. The actual data submitted in one of the discussions for such track showed that the gaps were much smaller so that a higher restraining force must be considered to have been in effect. His calculations and diagrams showing the local stresses set up when a short length of a long rail under temperature stress is replaced, are interesting, but such stresses may not be serious since they would have a good chance of being "ironed" out by vibration and wave action in the rails due to the wheel loads. Creeping of rails produces a similar effect and is no doubt responsible for some of the scattering in the observed points in Fig. 11.

Mr. Walker's photograph of the "sun-kink" (Fig. 8) on a welded rail is extremely interesting, and his experience in eliminating this trouble in the 210 miles of unpaved welded track of the Eastern Massachusetts Street Railway, is a valuable addition to the paper. He reports that the actual expansion in the 660-ft lengths was about one-half the expected 6 in., thus checking the phenomena observed in the Victorian Railways in Australia and corroborating Theorem 1. The incident of the rail "popping out" of the remaining spikes after some had been removed is worth noting, both for the warning it gives of this possible source of danger, and as an illustration of how the total resisting force is an accumulation of many small forces.

Mr. Ferguson's objections to the use of a uniformly increasing anchorage due to constant tie resistance are similar to those of Professor Talbot which have already been considered. He is correct in stating "the force or stress diagram is not necessarily a straight line"; but the straight line, nevertheless, seems appropriate for the analysis.

Mr. Reece's numerical example showing how joint restraint may cause even short rails to expand and contract less than they would if free is of special interest to those who have noticed discrepancies with the expected gaps based on the standard tables for rail laying.

Mr. Magee agrees that the tie resistance is more in the nature of a frictional than an elastic resistance. To explain end movement where no surface evidence of movement is noticeable, he suggests that slippage may occur along the horizontal plane at the bottom of the ties—the rail, ties, and ballast moving as a unit. However, cases are on record in which sliding of the rail occurred through the fastenings, particularly when the temperature dropped suddenly and the rail broke.

A question frequently raised regarding the magnitude of the buckling forces in curved rail due to temperature stresses is answered by Mr. Hussey's Equation (29) which gives the required radial resisting force in each tie to keep the rail in place. It should be stated that this equation involves the assumption that the half-chord of the curve enclosing two tie spans is equal to the tie span, *c*. This is a close approximation. Radial forces of about the same order may exist when the speed of a train around a curve exceeds that corresponding to the superelevation. The same lateral rail anchors that are already in use for the one radial force may be sufficient to hold the additional radial force due to temperature stress, at least for curves of large radius. For sharper curves, this effect might become serious, and the added factor of a shorter life may make welded rail inadvisable in certain cases.

In the writer's opinion the entire track structure of the future will be altered radically from that called "standard" to-day in the effort to obtain the maximum economy and the smoothest riding from continuous welded rail. A continuous elastic support under the rail in addition to adequate cross-members and a concrete sub-grade should do much as an approach to the more rational structural unit required to parallel the advances made in modern stream-lined trains.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

EFFECT OF DOWEL-BAR MISALIGNMENT ACROSS CONCRETE PAVEMENT JOINTS

Discussion

BY ARTHUR R. SMITH, M. AM. SOC. C. E., AND
SANFORD W. BENHAM, ASSOC. M. AM. SOC. C. E.

ARTHUR R. SMITH,[16] M. AM. SOC. C. E., AND SANFORD W. BENHAM,[17] ASSOC. M. AM. SOC. C. E. (by letter).[17a]—In the discussions of this paper attention has been called to some very important considerations in dowel-bar installations and their functioning as a load-transfer device across joints in concrete pavement.

It was appropriately suggested that, in view of the results of the experimental work to determine permissible error, the recommended specification for accuracy was too liberal, particularly since the joints in the experimental slabs were opened and closed with no vertical loads, simulating wheel loads, at or near the joint. If the slabs had been loaded during the tests, it is quite likely that different results would have been obtained since the effect of wheel load and some directions of dowel-bar error are additive. In outlining the program of tests it was the original intention to conduct the tests in this manner on different types of sub-grade and, in addition, to construct experimental slabs for test at a time when pronounced curling was present at the joints. However, time and expense made such an elaborate program impossible. Because of the need for a specification covering accuracy of installation, to correct careless construction practice, the study was necessarily limited to a single phase of the subject; that is, to misalignment alone.

A dowel-bar stresses the concrete in which it is embedded because its axis is being bent, due to such causes as traffic load transfer, curling of the pavement at joints, unequal heaving due to frost action, or binding when the joint

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NOTE.—The paper by Arthur R. Smith, M. Am. Soc. C. E., and Sanford W. Benham, Assoc. M. Am. Soc. C. E., was published in June, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1937, by Messrs. L. W. Teller, David J. Peery, and L. J. Mensch; December, 1937, by Messrs. W. O. Fremont, and George A. Smith; and January, 1938, by L. E. Grinter, Assoc. M. Am. Soc. C. E.

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[17] Research Engr., State Highway Comm. of Indiana, Bureau of Materials and Tests, Indianapolis, Ind.

[17a] Received by the Secretary March 31, 1938.

opens or closes (due to misalignment). With the exception of misalignment all the other causes which tend to bend the bar, thereby damaging the concrete, are entirely independent of the accuracy with which the bar is installed; that is, a misaligned bar across a joint that has not changed in width will transfer wheel loads as effectively as a perfectly installed bar. The coating of bituminous paint (approximate thickness, 0.020 in.) to prevent bond provides a clearance which, although undesirable in load transfer, is of great advantage in permitting some movement of a joint with misaligned bars before the bars begin to bind in the concrete. Although clearance is undesirable as regards load transfer because the bar does not act until the loaded slab has deflected and because it also increases the funneling action of the bar, clearance must be had because, without it, it is not possible to install the bars and hold them in position during the placing and finishing of the concrete with sufficient precision to permit movement without cracking the concrete.

The joints in the experimental slabs were opened and closed at the rate of 0.1 in. per min. In practice, movement is very slow. The pavement has attained considerable age before 1-in. expansion joints have closed 0.75 in. (to a width of 0.25 in.). During this period funneling has probably occurred around the dowel-bar providing clearance which, with that created by the paint and oil, is sufficient to permit closing of the joint without distorting the bar enough to crack the concrete. This is a possible explanation for the apparently satisfactory condition of the installations shown in Table 1, in some of which the bars are in extremely faulty positions. These installations are now (1938) three years old and all the joints are at present in apparently satisfactory condition.

For these reasons it was believed that a specification for accuracy of installation more rigid than that recommended would add very little to the satisfactory functioning of the joints; certainly not enough to warrant additional cost. Because of the already high cost of joints it is undesirable to specify additional refinements unless it appears rather certain that improved structural efficiency will be obtained. The recommended specification for accuracy, if it had been in effect during the construction season of 1935, would have required the contractor to replace the joints identified in Table 1 as Tests Nos. 1, 10, 11, and 13. Field tests conducted during the following season, when the specification was in effect, showed that the bars were being installed with a degree of precision considerably better than that obtained during the previous year, indicating that the specification resulted in an improvement in construction practice rather than an encouragement of carelessness, as has been suggested.

The question has been raised concerning the tension produced in the pavement due to the resistance to movement of misaligned bars when the pavement is contracting. As was stated in the paper the greatest resistance to the opening of a joint was 3 000 lb per bar which creates 50 lb per sq in. tensile stress in 5-in. pavement in addition to the tensile stress caused by sub-grade friction. The observed loads to open and close joints included, of course, both dowel-bar and sub-grade resistance but the experimental slabs were so short in comparison with slab lengths in actual pavements that sub-grade friction can be ignored. In this instance the dowel-bar error was maximum (1.5 in.). To open the corresponding slab in which the bars were installed perfectly required 2 100 lb per bar, or 35 lb per sq in. tension in the slab. Thus, reducing the dowel-bar error from 1.5 in. to zero (perfect installation) reduced the tensile stress in the pavement 15 lb per sq in. It is believed that this reduction in tensile stress is not great enough to add appreciably to the structural efficiency of the joint; certainly not enough to justify an increase in cost of construction.

It is impossible to state definitely whether or not there was a progressive destructive action during the successive cycles of opening and closing the joints. After the appearance of the first crack or cracks, additional cracks occurred during the remaining cycles of movement in very few of the test slabs, due probably to the partial relief of stress in the slab. This observation is subject to a certain amount of modification since, in some instances, cracks were not detected until after the second or third cycle of movement. It is possible that they were caused by the first movement but did not open wide enough to be seen until after additional cycles. The loads required to open and close joints decreased slightly and rather uniformly with successive cycles of testing which leads to the belief that continuing the tests beyond ten cycles was not necessary.

It has been suggested that vertical errors are more serious than horizontal errors since it would seem that the pavement would offer less resistance to failure of the type shown to the right in Fig. 9 than to the formation of a vertical crack throughout the depth of the pavement as would be caused by a bar having horizontal misalignment. This seems to be a reasonable opinion although failures were about equally divided between the two directions of error.

It is hoped that the interest displayed in this subject is an indication of increased attention to the perfection of load-transfer devices across joints in concrete pavement.

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DISCUSSIONS

SOIL REACTIONS IN RELATION TO FOUNDATIONS ON PILES

Discussion

By R. M. Miller, M. Am. Soc. C. E.

R. M. Miller,[21] M. Am. Soc. C. E. (by letter).[21a]—One of the most interesting experiences that can come to the engineer is to have repeated contacts with engineers and architects of every professional status, who have piling foundation problems to solve. Perhaps no other work has been given as little actual study by the mass of engineers and architects and in no other field are mistakes repeated so persistently. The very general attitude seems to be that brains are needed to design and build a structure but that almost any one can drive piles into the ground, and that once driven the matter is settled for all time. The courage of ignorance has been so general that the writer was tempted and wrote "Soil Reactions in Relation to Foundations on Piles" in the hope that he might emphasize in simple terms, and in readable form, why some of the commonest mistakes occur. If he has made a few of the most dearly cherished fallacies seem questionable, something may have been accomplished. The discussion of this paper has been most considerate although by no means in entire agreement.

Mr. Hill feels that the writer gives more weight to pile-driving and loading tests than he should. Perhaps the thought was not well expressed. In substance, it should have been much as follows: In impervious soils or combinations of pervious and impervious soils, the commonly used "yardstick," the dynamic pile-driving formulas are of but small value whereas the test loading of single piles is not much better; nor is the test loading of single piles, a cluster or mat in such soils, if done too quickly after driving. If, however, a cluster is test loaded after a sufficient lapse of time to allow for some re-arrangement of the soil particles, even if the cluster is not of an area comparable in size to that of the foundations, the results will more nearly approximate those of the founda-

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Note.—The paper by R. M. Miller, M. Am. Soc. C. E., was published in June, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1937, by Messrs. Hibbert M. Hill, and George P. Stowitts; October, 1937, by Messrs. George A. McKay and Chandler Davis; and December, 1937, by Messrs. Harry E. Sawtell, and J. Stuart Crandall.

[21] Project Engr., PWA, Cincinnati, Ohio.

[21a] Received by the Secretary April 28, 1938.

tions than those indicated by the dynamic formulas, single-pile load tests, or load testing the cluster, or mat, immediately after driving. Test loading a group as suggested should offer an improvement over the usual methods, but not the complete answer.

The writer wishes to concur particularly in the closing sentence of Mr. Hill's discussion which states that "only the inexpert are courageous enough to proceed, where compressible materials are involved, on the basis of ordinary boring samples."

Mr. Stowitts advanced the thought that the order in which piles are driven is an important factor in the final result. Such matters appear to be lost sight of in favor of apparently more important considerations; but, if mishandled, they may become of as much importance as any other factor of the problem. The sub-soils underlying any two projects are never exactly alike: In one, it may be advisable to drive alternate runs of piles and return a second time over the same ground; in another, as suggested by Mr. Stowitts, it may be well to begin driving at the center and work outward toward the edges; and in another, it may be best to work uphill rather than down, or *vice versa*. In any case, careful consideration should be given to the control of heave and to lateral movement. Broken, cracked, or pinched piles have resulted from this cause in sufficient number to warrant thoughtful planning.

The writer agrees with Mr. Crandall that the dynamic pile-driving formulas should be used only where piles are driven in sands, gravels, and such incohesive soils, and also that his modification of Hiley's formula offers more dependable results than can be had from the generally used *Engineering-News* formula. On the other hand, he feels with Mr. McKay that the unknown effect of the use of the several kinds of driving cushions of different thicknesses and different materials should be more thoroughly examined. That this factor affects the results in considerable but unknown degree cannot be doubted, and, yet, how many engineers give the matter consideration?

The use of Equation (6) for the resistance to movement of a pile through clays, silts, etc., appears logical provided the friction value of the soil and the type of pile is known within reasonable limits. Mr. Crandall states that "friction tests should be made for each locality and for the lengths of piles used unless other tests are available for the same soil and locality."

Sub-soil formations may lie fairly uniformly under a project site, or they may change radically within a comparatively small area. If the former is found to be the case one friction test should be sufficient; but if the latter is true the number of friction tests should be governed by the conditions. As an example, refer to the writer's description of the sub-soils underlying one of the buildings of the Laurel Homes Housing Project, in Cincinnati (see heading "Average Practice"). In the space given as 35 by 150 the use of Equation (6) would insure equal settlement, or no settlement, only if at least six or perhaps more friction tests were made. However, as equipment installations represent a large part of the cost of such tests a few added tests would affect the cost very little.

Mr. Sawtell states that the writer has cited no examples of the effect of the remoulding of clay during pile-driving upon subsequent settlement. This is quite correct, due to his inability definitely to connect effect to this cause. In fact, he agrees with Mr. Sawtell that "driving piles into clay cannot be compared to remoulding a clay sample in the laboratory," and that "he (Mr. Sawtell) does not think that the results of actual construction or any published experiments can justify any assumption that driving piles into soft or medium clay will completely or largely remould it." In fact, the writer's observations incline him to the belief that, although some remoulding of the clay surrounding a pile undoubtedly occurs during driving, the soil particles re-adjust themselves after driving so that, in time, the effect of any remoulding disappears. Furthermore "the slow but sure compacting of underlying clay beds" is a characteristic of this type of soil and furnished a more logical reason for settlement than the remoulding theory.

Although insistence should come from engineers that more thorough soil investigations be made than is customary, it should go further than that. So long as contracts for borings or open-pit explorations are let through competitive bidding to the low bidder, with little regard for his knowledge and experience, so long will much of the foundation work be based on inexact information.

Occasionally, the contractor's superintendent or foreman in charge of such preliminary investigation is competent to secure representative samples and to classify soils; but many times this is apparently not the case, judging from results. It has been the writer's experience, so many times, to work with boring records that subsequent construction has proved to be inaccurate that he looks with suspicion upon any records of soil exploration unless he is sure that competent men have been in charge. Results indicate that too much care can scarcely be taken to secure sufficient and accurate preliminary information.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

MEASUREMENT OF DÉBRIS-LADEN STREAM FLOW WITH CRITICAL-DEPTH FLUMES

Discussion

BY MESSRS. R. L. STOKER, AND J. C. STEVENS

R. L. STOKER,[19] ESQ. (by letter).[19a]—Model tests of various modifications of the San Dimas Flume developed by the authors were begun at the University of California at Berkeley, Calif., in 1937, in co-operation with the California Forest and Range Experiment Station. As this type of flume presents an analytical problem that should be susceptible to solution by established methods,[20] it is interesting to make a comparison of the observed and computed water-surface profiles. The following comparison is for the case of clear-water flow through the flume.

The ordinary equations for non-uniform flow in open channels are derived on the assumptions that steady flow exists and that the flow lines possess negligible curvature and slope. These assumptions regarding the flow lines make it possible to express the pressure anywhere in the flow as being greater than atmospheric by an amount equal to the unit weight of the fluid, times the vertical distance beneath the free surface. However, in open channels where the section changes size or shape, or where the bed slope varies abruptly, it is evident that the flow lines are not in conformance with the foregoing assumptions. Nevertheless, in the following discussion, the computations are based on the simplifying assumptions of negligible curvature and slope of flow lines and have been made for a particular run of one of the models of the San Dimas Flume shown in Fig. 16, in spite of the observed conditions shown in Fig. 17.

The particular flume was coated with sand of uniform diameter of approximately 0.02 in. The value of n in Manning's equation was found to be 0.0148

NOTE.—The paper by Messrs. H. G. Wilm, John S. Cotton, and H. C. Storey, was published in September, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1938, by Messrs. R. L. Parshall, and Martin A. Mason; March, 1938, by Edwin S. Fuller, Esq.; and April, 1938, by Messrs. Harold K. Palmer and Fred D. Bowlus, and Harry F. Blaney.

[19] Instructor in Mech. Eng., Univ. of California, Berkeley, Calif.

[19a] Received by the Secretary March 14, 1938.

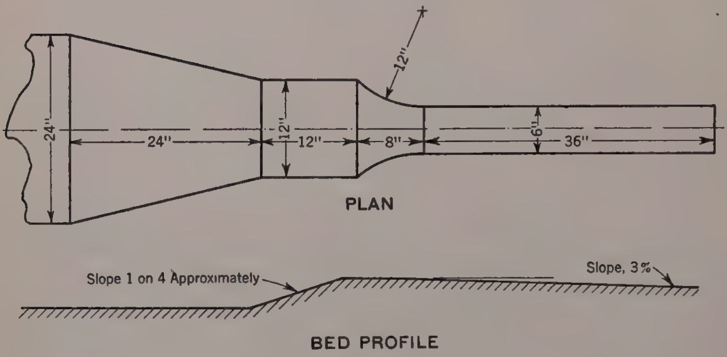
[20] "Hydraulics of Open Channels," by B. A. Bakhmeteff, M. Am. Soc. C. E.; and "Applied Fluid Mechanics," by M. P. O'Brien and G. H. Hickox, Associate Members, Am. Soc. C. E.

from the formula,[21]

$$n = 0.043\ D^{1/6} \qquad (12)$$

in which D is the mean sand grain diameter, in feet. Various experimenters have obtained approximately this value of n. For example:

Authority	Manning's n
Strickler[22]	$0.039\ D^{1/6}$
O'Brien[23]	$0.049\ D^{1/6}$
Chang[21]	$0.043\ D^{1/6}$



PLAN

BED PROFILE

FIG. 16

It is easily shown that the section at the beginning of the part of the flume that is 6 in. wide should act as the control section because the 3% bed slope is substantially greater than the critical slope for the given channel roughness and discharge. The discharge was 0.927 cu ft per sec for the particular run considered herein. Thus, in the computations the critical depth was assumed to occur at the beginning of the 6-in. width, and the back-water was computed up stream and down stream from this section by the specific energy method. The foregoing value of Manning's n was used in determining the energy loss. A comparison of the "observed," "computed," and "frictionless" water surface profiles is shown in Fig. 17.

It is interesting to note that although no contraction loss was assumed to occur in the flume transition, the frictionless and observed profiles are at very small variance in the up-stream section of the flume. In closed conduits the entrance loss due to a well-rounded transition is of the order of 2% of the increase in kinetic energy, which appears to be approximately the case in this open channel transition. According to much of the literature, a contraction loss of 10% of the kinetic energy increase is commonly assumed in designing transitions.

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[21] "Laboratory Investigation of Flume Traction and Transportation," by Y. L. Chang, *Proceedings*, Am. Soc. C. E., November, 1937, pp. 1701–1739.

[22] "Geschwindigkeitformeln und Raughigkeitzahlen," by A. Strickler, *Mitteilungen* des Amtes Wasserwirtschaft, Bern, Switzerland, 1923.

[23] "The Vertical Distribution of Velocity in Wide Rivers," by M. P. O'Brien, Assoc. M. Am. Soc. C. E., *Transactions,* Am. Geophysical Union, 1937.

The observed and computed profiles are in fairly good agreement except in the immediate vicinity of the control. This would be expected, however, as it is here that the flow lines have their greatest slope, and this influence was neglected by the original assumptions. Some divergence is seen to occur at the free discharge end of the flume, but this is explainable in part because no allowance was made in the computed profile for the draw-down at discharge which is measurable 5 or 6 in. back from the brink.

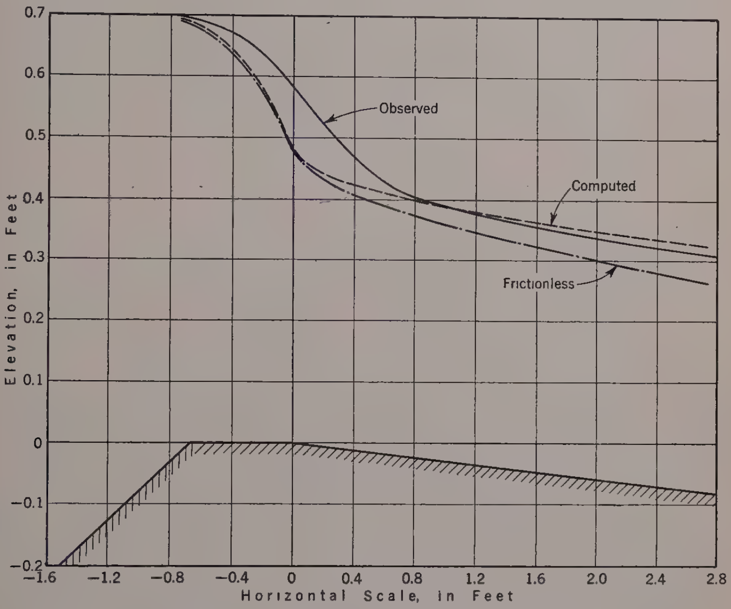

FIG. 17.—COMPARISON OF OBSERVED AND COMPUTED WATER SURFACE PROFILES; $Q = 0.927$ CUBIC FOOT PER SECOND

The good agreement between theory and experiment demonstrated by the example in this discussion indicates the value of applying the same method to all the data presented by the authors. Substantiation of the theoretical method in this manner makes interpolation possible in other designs, other locations of pressure taps, and variations in surface roughness.

J. C. STEVENS,[24] M. AM. SOC. C. E. (by letter).[24a]—The authors of this paper offer a solution to a perplexing problem wherever the conditions permit the installations indicated. A prime requisite is an abundant drop at the structure. The profiles of Figs. 10 and 13 could not be maintained if the tail-water should cause a hydraulic jump on the bed of the flume and thus deposit much of the débris. A definite drop of substantial proportions below the end of the flume seems to be essential.

It would be interesting to determine some of the characteristics of the hydraulic jump when heavily ladened with débris. With coarse material this may be impossible but with finer sands some data might be secured.

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[24] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[24a] Received by the Secretary April 1, 1938.

It must be remembered that the Parshall flume was not designed for any such service as it has been subjected to here and it must not therefore be condemned as a measuring device. Its chief field of usefulness is in irrigation systems where, at most, sand or silt and not cobbles are encountered, and also where it is necessary to sacrifice a minimum amount of head. For such service the Parshall flume gives remarkably accurate and consistent results.

The authors mention additional data to be submitted with their closing discussion. Since the paper was written, the San Dimas Experimental Forest has experienced some floods that have doubtless given these devices a real test. It is to be hoped the authors will take the Engineering Profession into their confidence in discussing the effect of these floods on their measuring equipment.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

WATER TRANSPORTATION VERSUS RAIL TRANSPORTATION

A SYMPOSIUM

Discussion

BY RUFUS W. PUTNAM, AND S. L. WONSON,
MEMBERS, AM. SOC. C. E.

RUFUS W. PUTNAM,[30] M. AM. SOC. C. E. (by letter).[30a]—In preparing one of the two basic papers for this Symposium, it was hoped that the main points at issue in this question of rail *versus* water transportation would be set up in such a manner that in the subsequent discussions there would be avoided rehearsals of the many minor subjects which in the past have been responsible for the apparent confusion of thought obscuring the few facts and principles involved. The almost complete absence of direct discussion or criticism of the paper itself leads the writer to the conclusion that it failed in its purpose and that he inadvertently so expressed himself as to stir up a "hornet's nest" of indirect comment.

It has been held that the public necessity requires a satisfactory transportation service—one which is adequate, efficient, and economical. A general statement, such as this, necessarily becomes relative only when applied to specific cases. It would not do to condemn river transportation because of the closure of northern waterways during the winter season, or on account of circuity, or because of rigidity, any more than it would be correct to condemn transportation on the Great Lakes for the same reasons. From the point of view of the user these considerations affect only his over-all transportation costs, which include the cost of storage of requirements during the closed winter season.

NOTE.—The Symposium on Water Transportation Versus Rail Transportation was presented at the meeting of the Waterways Division, Little Rock, Ark., April 25, 1936, and published in September, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: October, 1937, by George Hartley, Esq.; December, 1937, by W. D. Faucette and J. E. Willoughby, Members, Am. Soc. C. E.; January, 1938, by C. D. Bordelon, Esq.; February, 1938, by J. E. Goodrich, Esq.; and March, 1938, by H. R. Faison, M. Am. Soc. C. E.

[30] Pres., Maritime Eng. Corporation, Chicago, Ill.

[30a] Received by the Secretary May 11, 1938.

No general plea for the necessity of waterway improvement should be entertained these days on account of the inadequacy of rail service. Even in times of national emergency, although the waterways themselves might have excess capacity available for use, the floating and terminal equipment might not have; therefore the value of this means of transportation, as a reserve against inadequacy of other forms, is not great.

On the tangible side, the entire question is one of economics; but, unfortunately, this seems to be a very difficult question when it comes to obtaining any degree of agreement as to the factors involved. In the first place whatever method of comparison is used it would be best to apply similar, if not identical, tests to rail as well as to water transportation. It is believed that the use of the method of comparing freight rates is generally recognized as incorrect. Rates of water carriers do not carry the investment, maintenance, and operating costs of the projects; those of rail carriers in some cases are no more than, if as great as, out-of-pocket costs, whereas, average rail rates are known to be insufficient to carry fixed charges on the entire railroad investment of the country. Rail rates should not be compared with water costs, even if the latter include fixed charges on the waterway improvement plus maintenance costs (see heading "In Justification: Comparison of Costs," in the writer's paper). The only solution apparent to the writer seems to be to attempt to compare rail costs with water costs, and, at best, such a procedure can result only in an approximation.

As to out-of-pocket operating costs of the carriers there appear to be no particular difficulties except the reluctance of water carriers to reveal their confidential data. Maintenance-of-way costs for the railroads and maintenance costs for waterways are readily available. The principal obstacle seems to be in reaching even a reasonable approximation of what should be the capital cost in each case.

If it is assumed that capital cost is to be measured in terms of expenditures to produce these transportation agencies, difficulties are encountered in either case. For the waterways, it is possible to go back and sum up the expenditures for "New Work" for any project. Where flood control and bank protection are not involved this is comparatively simple, but where they do form a part of the improvement, segregation is not so easy. An approximation is necessary and this the writer made in his paper in arriving at a 1937 cost of $550 000 000 for the system under discussion (refer also to Mr. Hartley's discussion). This estimate did not take into consideration the invisible costs of bridges which, in a more accurate determination, should be included. Merely for purposes of illustration, the writer has roughly estimated that the extra bridge costs involved on the 850 miles of the Mississippi River above Cairo, on account of greater clearances, and the provision and operation of draw-spans (capitalized), would approximate $16 500 000. The item is substantial but not large in comparison with the totals involved.

The writer's estimate made no deductions for the cost of early river improvements, long since superseded, and which had very little value as parts of the more modern projects now under way or recently completed (see Mr.

Faison's discussion). A more accurate determination of costs would make allowance for these substantial items of expense.

To measure the cost of any part of the railroad system of the United States in a similar manner is next to impossible. Calculation of the actual sum of money expended on capital construction would not give the answer as there are costs of very substantial proportions which the investing public have paid during the lifetime of the railroads for which there are no visible, tangible returns. Several of the railroads in the Mississippi Valley have gone through the experience of alternate periods of prosperity, receivership, sale, and reorganization. Securities have been scaled down and in some instances "wiped out"; the cost of any such losses should be included as they represent expenditures by the public made for the purpose of providing transportation.

The use of the cost-of-reproduction method as a basis of comparison is not believed to be correctly applicable in this case. To do so would be to supplant fact with theory, and theoretical returns on theoretical investments cannot be used for the payment of taxes, interest, or dividends. As far as waterways are concerned, it would be necessary to calculate the cost of reproduction on the basis of an assumed rate of construction which is next to impossible to predict with irregular Congressional appropriations. Assumptions or theories would have to be applied to determine rights-of-way costs, which are substantial factors in railroad costs.

The writer made use of this method in his computation of railroad transportation costs merely because he was unable to determine the capital cost of the railroad system in the Mississippi Valley on the basis of past expenditures. His results, therefore, are only an approximation and should be so considered.

It is noted that Mr. Goodrich is of the opinion that the load factor of the floating equipment on the inland waterway system is an important factor, and he expresses the hope that the writer would present data as to actual existing load factors in the closing discussion. Such data are quite confidential so that only generalities may be given. One large operator has a towboat load factor of about 21% with a barge load factor of only 6.4%; this case represents about the minimum. Large coal operations on the Ohio River and the Monongahela River are conducted with towboat load factors of about 40% and barge load factors of 25 per cent. Only where an operator has a fairly well balanced two-way movement of bulk commodities do the load factors exceed these values.

Much of the confusion with which these questions are attempted to be solved, and most of the difficulties that exist without confusion, are borne of the effort to make a scientific comparison of two activities which cannot be reduced to terms with a common denominator. An approach to a satisfactory determination might be made by a committee of engineers skilled in this subject, but accord through published papers and discussions seems to be hopeless.

S. L. WONSON,[31] M. AM. SOC. C. E. (by letter).[31a]—The writer desires to express appreciation of the discussions of his paper and of the various considerations of interest and value that have been presented, both pro and con.

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[31] Asst. Chf. Engr., Mo. Pac. R.R., St. Louis, Mo.

[31a] Received by the Secretary May 19, 1938.

To the extent that the discussions present a viewpoint opposed to that of the paper, it would appear that a rebuttal would involve substantially a repetition of facts and conclusions already expressed.

There is one collateral consideration that seems worthy of mention, namely, that the natural and primary function of a river is that of a drainage channel, and that works for the improvement of navigation, or for other purposes, should not be allowed to impair the drainage function, or to impede any necessary improvement of it.

Of the force of this consideration, the disaster of 1937 along the Ohio River is a grim reminder. Along another major stream of the Mississippi Basin, upon which extensive works for the improvement of navigation are being constructed, there are communities in apprehension of the effects of a major flood following even the present stage of those improvements.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

PRACTICAL APPLICATION OF SOIL MECHANICS
A SYMPOSIUM

Discussion

BY A. STREIFF, M. AM. SOC. C. E.

A. STREIFF,[80] M. AM. SOC. C. E. (by letter).[80a]—There is an extraordinary difference in the flatness of slopes of the Mississippi River levees built by the U. S. Army Engineers when compared with dikes abroad. Mr. Buchanan's instructive description is interesting and reveals this startling difference very clearly. In China,[81] Germany,[82] The Netherlands,[83, 84] France, Hungary, Italy, or other countries, dike slopes are much steeper, and volumes are much smaller. The Mississippi dikes are not higher than foreign dikes in several places.

In China, the Yellow River dikes have slopes of 1 on 2. In Cochin-China,[85] the river side has a 1 on 2, and the land side, a 1 on 3, slope. Steeper slopes also prevail in Germany and Denmark.[82, 86] In The Netherlands the inside slope, even on the most exposed sea dikes, is 1 on 1.75. A. Caland[83] states that the slope is determined mostly by the capacity of the bank to grow a good sod. The outer slope is determined by the location. Where exposed to heavy gales, the outer slope may be as flat as 1 on 12, but along the rivers this consideration does not exist and the slopes are 1 on 2.

NOTE.—This Symposium was presented at the meeting of the Soils Mechanics and Foundations Division, at Boston, Mass., October 7, 1937, and published in September, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1937, by the members of the Committee of the Society on Earths and Foundations; November, 1937, by Messrs. S. C. Hollister, T. T. Knappen, and L. F. Harza; December, 1937, by Edward Adams Richardson, Esq.; January, 1938, by Messrs. Richards M. Strohl, William P. Creager, Jacob Feld, and Y. L. Chang; February, 1938, by Messrs. Charles Senour, Donald M. Burmister, and Donald W. Taylor; April, 1938, by Messrs. Lee H. Johnson, Jr., and Gregory P. Tschebotareff; and May, 1938, by William L. Wells, Jun. Am. Soc. C. E.

[80] Vice-Pres., Ambursen Eng. Corporation, New York, N. Y.

[80a] Received by the Secretary April 12, 1938.

[81] "Flood Problems in China," by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1405.

[82] "Wasserbaukunst," by Hagen, 1880.

[83] "Handleiding tot dyksbouw," by A. Caland, 1833.

[84] "Waterbouwkunde," by Storm Buysing, 1851.

[85] "Digues du Tonkin," by J. Gauthier, 1930.

[86] "Bau der Flussdeiche," by Paul Ehlers.

It is interesting to note that another branch of the U. S. Government uses slopes of 1 on 2 for the dikes along the All-American Canal at the Imperial Dam. These slopes compare with 1 on 6 and 1 on 8 along the Mississippi River! The difference in yardage is enormous. Clay is said to have been unsatisfactory along the Mississippi. This can only be due to the fact that there apparently is no moisture control in the process of building. In The Netherlands, Germany, and Denmark, clay has been used extensively since Roman times. There are also many sand dikes, but these are always covered with clay. The present dikes through the open sea for the closure of the Zuider Zee are made of sand and boulder clay. The treatment described of Mounds Landing is very ancient practice in Dutch dikes. The writer has built many miles of clay dikes along the Saramacca Canal in Surinam, having slopes of 1 on $1\frac{3}{4}$ which proved quite satisfactory. The unsatisfactory experiences with clay along the Mississippi River can only be due to lack of moisture control.

The Dutch system, however, is more elaborate than the U. S. Army system along the Mississippi River, as described by Mr. Buchanan. A preliminary dike called "spekdam" is thrown up on the seashore. Behind this dike a ditch, called "verskade sloot," is dug deep enough to drain the borrow-pits. The borrow-pits are separated by strips, left in place, to prevent erosive currents along the dike. The ditch is drained or pumped. Clay is placed and rolled in convex layers draining to both sides. In this manner no trouble is experienced in obtaining satisfactory clay dikes.

On the great rivers of Cochin-China[85] the outer slope (1 on 2) was provided with a clay cover, rolled with corrugated rollers. The inner slopes are 1 on 3.

In the course of time the ancient art of dike building has been greatly perfected in those countries that were in need of it. Higher profiles are needed in many places, however, as Mr. Buchanan shows for the Mississippi River. In Europe, thirty-nine major catastrophes have occurred since the Twelfth Century. The problem remains to be able to predict maximum flood heights. The art of building leaves little to be desired; but the means of protection are much more intricate than the single lines of dike shown by Mr. Buchanan. An entire network of dikes exists in The Netherlands, forming auxiliary lines of defense. There are "sleeper" dikes, "bosom" dikes, inner and outer, winter and summer, river, and sea dikes. In many places slopes are protected in an elaborate manner.

The expedient of flattening slopes, which Mr. Buchanan describes, to make the structure stable in soft ground, is an ancient one. Caland, an experienced dike builder, writing in 1833,[83] describes the building of dikes on soil into which a stick could be thrust 12 ft without effort. He states that the side upthrust can be counteracted by flattening the slopes, and describes a case that occurred in Middelburg Harbor, in 1816. The Dutch have another ancient and successful method of constructing dikes on very soft foundations. It consists of covering the dike seat in the center by a "zink stuk," or sink piece of fascines. This prevents the lateral escape of the dike material, and of the foundation.

Doubtless the great differences in design in Europe and the United States are due to differences in local conditions. In the low countries of Europe, river engineering is a special trade, just as coal mining or weaving. Fascine work has been developed to a specialty. Dikes 40 or 50 ft high have been constructed between two submerged dikes of fascines, made of sink pieces. There is a great variety of methods of slope protection. In that manner the Dutch are able to construct clay and sand dikes through a depth of 25 or 30 ft of open sea, subject to the force of storms. Such methods would neither be possible nor economical in the United States where, apparently, an inordinate flattening of slope has been found to be the best solution.

The method of computing these slopes, described by Mr. Buchanan, does not seem to the writer to offer any better guaranty for their stability than that obtainable by older methods. Unfortunately, the great advances in soil mechanics are confined to the laboratory. The writer disagrees with Mr. Buchanan and feels that, for the present at least, the application of soil mechanics has caused no visible progress in: (1) The art of foundation design; and (2) the methods for computing soil stability. The first has always been quite adequate, and computation methods are as approximate as they ever were.

The practical art of constructing earthwork has been entirely successful since ancient times. Feeder dams on the Grand Canal in China, built 500 yr ago on alluvial soil, are still in use.[81] In the Indian Province of Madras alone there are 43 000 native earth dams used for storage reservoirs, of a total length of 30 000 miles (more than nine times the length of the Mississippi dikes) containing 200 000 separate masonry outlet works. One of these, the Madduk Masur Dike, has stored 800 000 acre-ft every monsoon for 400 yr.[87] The entire Netherlands has been diked since the Middle Ages. Entire cities containing the heaviest medieval towers stand on ancient pile foundations in soft soil below sea level. Among the many dams constructed on soil foundations[88] in modern times may be mentioned the Alcona Dam, built on a fine quicksand foundation 100 ft deep under artesian pressure, and successfully holding 40-ft head since 1923. None of these works needed the modern soils laboratory.

That the computation methods are as approximate as they ever were is admitted by the foremost workers in this field. Thus, Prandtl[89] states that his theory on plastics is no longer applicable if stress changes with deformation (which condition generally prevails in soils). Professors Terzaghi and Froehlich[90] as late as 1936 admit that "the investigations perforce have the character of approximations for the purpose of obtaining a rough guess as to the settlement expected [writer's translation]."

Mr. Buchanan states that earth dikes on impervious strata can "now" be analyzed by methods such as the one developed by Glennon Gilboy, Assoc. M. Am. Soc. C. E.;[91] but it appears that in this, and in another, paper Professor Gilboy only applies the methods of Rebhahn[92] and Coulomb[93] and indeed specifically mentions that fact. In one of these two papers[94, 91] Professor Gilboy resorts to doing by trial and error what Coulomb did in 1821 by differentiation. The soils laboratory cannot be credited with developing ancient methods that have simply survived to the present time.

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[87] "Ways and Works in India," by G. W. MacGeorge, 1894.

[88] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), pp. 1303–1307.

[89] International Congress for Applied Mechanics, Delft, 1924.

[90] "Theorie des Setzung von Tonschichten," by Charles Terzaghi, M. Am. Soc. C. E., 1936.

[91] "Hydraulic Fill Dams," by Glennon Gilboy, Vol. 4, No. 46.

[92] "Theorie des Erddruckes," by Rebhahn, Vienna, 1871.

[93] "Theorie des Machines," by Coulomb, Paris, 1821.

The efforts to compute soil stability date from 1764[95] and before. Since Moeller's book on earth pressures was published[96] nothing has been added which is any more serviceable than was stated therein. Retaining walls everywhere have been built with success by these theories; the 90-ft reinforced concrete retaining wall at the Junction Dam on a silt foundation[97] ranks among the highest examples.

Mr. Buchanan applies the method of Fellenius to the dike slope. Fellenius, in his book, "Erdstatische Berechnungen," attempts to introduce shear or cohesion and friction separately. The latest edition of Emperger's "Handbook for Reinforced Concrete" states that this leads to contradictions. The method referred to by Mr. Buchanan concerns circular shearing planes. Such shearing planes are already to be found in Coulomb's original treatise (1821).[93] Coulomb preferred the plane shear surfaces, considering the entire wedge-shaped sliding earth mass as one piece for the simple reason that it lends itself more readily to differential calculus. Fellenius, by using circles, has to resort to a trial-and-error process in order to obtain the conditions for a minimum stability; but Fellenius' method suffers from many more approximations than the Coulomb method. He attempts to divide the slope segment in vertical slices which are supposed to be subject only to the weight and the friction at the base of the corresponding strip. This is definitely a gross approximation as each strip is also subject to lateral pressure on both sides, with opposite directions of friction, which are not equal and cannot be omitted in a rigorous solution. The bases of Strips 8, 9, 10, and 11 (Mr. Buchanan's Fig. 8) are subjected to more pressure than merely the weight of the strips. Coulomb, therefore, preferred to regard the sliding mass as one piece. A sub-division into strips without considering lateral pressures is a rough approximation that does not approach closer to the truth. If the vertical pressure is unknown, a division of resistances into shear and friction is also not possible. Engineers, therefore, are as far from achieving an exact solution with Fellenius' method as they were with Coulomb's solution. No one can say which method gives closer results. In addition, a laborious trial-and-error method must be used to obtain results as open to question as ever.

The Dutch method of building dikes on very soft foundations, by means of a sink-piece in the center of the dike, utilizes the upthrust in the center to hold the sink-piece while preventing the downward movement on both sides of the center by means of the strength of the sink-piece. It is self-evident that computing the strength of the foundation merely by multiplying the depth of the clay foundation with the vertical load per square foot and dividing this by the allowable shearing stress, in order to obtain the base width of the dike,

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[94] "Mechanics of Hydraulic Fill Dams," by Glennon Gilboy, *Journal*, Boston Soc. C. E., Vol. XXI, No. 3.

[95] "Recherches sur la construction des digues," by Bossut and Viallet, Paris, 1764.

[96] "Erddruck," by M. Moeller, 1902.

[97] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1321.

can be classified only as a rule-of-thumb. The actual conditions are far too complicated to be computed accurately by this means. The writer uses Rebhahn's method in such a case. The shear plane proposed by Mr. Haines as reported by Mr. Hough in Fig. 38 can conveniently be rectified into two straight lines joining in the perpendicular through the toe of the dike. The active pressures on one side must balance the passive pressures on the other and can easily be obtained graphically.

Soil mechanics, at least to the present, has not visibly enriched the "tool box" of the practicing engineer. Nevertheless, continued research remains of the greatest importance in spite of the paucity of practical results. A sober, critical examination finds recent enthusiastic reviews subject to considerable discount; but tests such as those described in 1923, in the very complete paper[98] by Jacob Feld, M. Am. Soc. C. E., will always be of great value.

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[98] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1448.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE DESIGN OF ROCK-FILL DAMS

Discussion

BY MESSRS. RALPH J. REED, F. J. SANGER, AND C. S. JARVIS

RALPH J. REED,[36] M. AM. SOC. C. E. (by letter).[36a]—The successful service of the typical rock-fill dam depends upon the maintenance of an impervious facing supported by the up-stream slope of the fill. Although small settlements can be provided for in the design, any considerable settlement of this facing may result in such failure of it as to affect seriously the structure of the dam, or at least to result in excessive leakage. It seems to the writer, therefore, that such provisions should be made in the design and such measures should be taken in the construction of a rock-fill dam as will result in the larger part of the dam's settlement taking place before the permanent facing is completed. In accomplishing this result it is obvious that the character of the rock used is of primary importance. The selection of rock that is hard, sound, and does not readily disintegrate, is emphasized by the author. The usual quarry operation results in the production of rock varying considerably in size, so that, without resorting to measures which increase the cost of production, it is usually not feasible to produce or to select rocks of fairly uniform size; and it is questioned whether such selection may be at all necessary. It is believed that a grading from larger to smaller rocks, with the rejection of very small rocks and spalls, and the placing of the fill in low lifts extending completely along the length of the dam, from abutment to abutment, will be advantageous in accomplishing the larger part of the settlement of the fill as construction goes forward, and in diminishing the settlement which is to be expected when the full water load is taken and continues. Even if great care is taken to reject fine material —earth, spalls, and dust—at the quarry, a certain amount of rather fine material will be present in the fill due to chipping and breaking of the rock during transportation from the quarry and handling in the fill.

NOTE.—The paper by J. D. Galloway, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1937, by Messrs. Cecil E. Pearce, and H. B. Muckleston; January, 1938, by Harold K. Fox, M. Am. Soc. C. E.; February, 1938, by Messrs. Charles H. Paul, and A. Floris; March, 1938, by Messrs. Howard F. Peckworth, Oren Reed, Walter L. Huber, Samuel B. Morris, and L. F. Harza; April, 1938, by Messrs. Paul Baumann, O. W. Peterson, and George W. Howson; and May, 1938, by Messrs. John E. Field, John H. Wilson, Frederick H. Fowler, I. C. Steele and Walter Dreyer, and F. Knapp.

[36] Cons. Engr., Los Angeles, Calif.

[36a] Received by the Secretary April 1, 1938.

Sluicing the dumped rock in order to wash this finer material into the fill and to prevent its accumulation in any particular zone, is important; and there may also be a lubricating effect from sluicing, resulting in somewhat better settlement of rocks into position upon their supporting rocks. The performance of San Gabriel Dam No. 2 of Los Angeles County Flood Control District is illustrative in this regard. This high rock-fill dam was built in 1932-1933, the fill being made "in the dry," without sluicing. The specifications provided that the rock used in the main fill should be:

> "* * * a well-graded mixture of sound, angular, quarry-run rock, forty (40) per cent of which shall vary in weight from quarry chips to one thousand (1000) pounds, thirty (30) per cent shall vary in weight from one thousand (1000) pounds to three thousand (3000) pounds, and thirty (30) per cent shall vary in weight from three thousand (3000) pounds to fourteen thousand (14000) pounds."

The rock was not to contain more than 3% of total weight of quarry dust. Although the placing of the laminated concrete slab facing of the dam was delayed until the fall of 1933, when the fill was practically completed, the sluicing effect of very heavy rainfall during the following winter months produced such large settlement of the fill and accompanying deformation of the up-stream face of the dam that the facing was seriously damaged. Subsequent sluicing during the summer of 1934 through openings drilled through the slabs resulted in further settlement, but to a lesser degree. The condition of the facing was such, however, that its complete removal became finally necessary; and the dam has since been provided with a laminated timber facing. It is believed to be clearly indicated that generous sluicing of the fill during the construction of this dam would have resulted in such settlement at that time as would have made possible the building of a concrete face and its maintenance without the failure that occurred.

F. J. SANGER,[37] ASSOC. M. AM. SOC. C. E. (by letter).[37a]—In his able summary of the historical development of rock-fill dams, the author shows that large rock-fill dams are rare outside the Western States, and it is thought that the record of such a dam in Australia which partly failed, and the measures taken to remedy the fault would be of interest. Rupert Grenville Knight[38] describes the case fully, and only a very brief précis of his paper is given.

The Eildon Reservoir and Dam were completed in 1927. The dam, including a spillway section (734 ft), is 3 529 ft long. It partly failed in 1929, after the water level had been lowered 46.5 ft, and remedial measures required six years and cost the equivalent of $2 000 000.

The core-wall is of reinforced concrete, 137 ft high, and from 6 ft to 2 ft thick; it is reinforced with 0.5-in. bars at 12-in. centers in grids and has expansion joints at 50-ft intervals. The site is composed of variable shales with some slates and sandstones, with about 20 ft of clay above for two-thirds of the length of the dam. On the up-stream side of the wall a mass of clay was deposited, of base thickness from 27 ft to 37 ft and with a batter of 1 on 6. The rock-fill is of material similar to the bed-rock and weighs 90 lb per cu ft. The slopes were 1 on 2.

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[37] Head, Dept. of Eng. and Building, The Lester School and Henry Lester Inst. of Technical Education, Shanghai, China.

[37a] Received by the Secretary April 9, 1938.

[38] "The Subsidence of a Rockfill Dam and the Remedial Measures Employed at Eildon Reservoir, Australia," by Rupert Grenville Knight, *Journal*, Inst. of Civ. Engrs., March, 1938.

Failure began with a slip, exposing the core-wall as well as deflecting it for a length of 700 ft in the middle of the dam, and then extended to a 1 200-ft width when the top 26 ft of the wall was exposed and the fill moved 55 ft into the reservoir; subsidence continued slowly for some weeks afterward, and the maximum wall deflection was 4 ft 8 in., causing many serious cracks. Borings showed that the clay had been squeezed out into the fill. In the discussion of the paper several theories were advanced for the failure, and the most probable reason seems to have been a clay slip in the foundation material.

Remedial measures included adding rock-fill on the toe to prevent sliding and extra deflection of the wall, repair of cracks, drainage, and the reconstruction of the spillway, outlets, etc. In addition, the frictional resistance of the down-stream side was increased by adding rock-fill and by under-drainage. An additional 700 000 cu yd of fill were needed, making the total in the dam 2 000 000 cu yd.

C. S. Jarvis,[39] M. Am. Soc. C. E. (by letter).[39a]—Examination of rocky bars intruding into or across stream channels will disclose the fact that these are rock-fill dams with slopes naturally adjusted to withstand over-flow. Many of the rapids prevailing in torrential stream channels are only the down-stream faces of such natural barriers. As might have been expected, such sites have been chosen for intakes of power or other diversion canals. Favoring such locations are the ease and effectiveness with which the current can be deflected by merely piling stones to form wing-walls, then stowing brush, straw, or litter to close the interstices; both initial construction cost and cost of maintenance were reduced thereby.

The development of the basic idea of rock-fill dams may be traced through many centuries of agricultural practices, particularly in relation to terracing on hillside slopes. Dry rubble walls with pronounced batter provided the required stability; rock spalls or stony soil strewn against the inner face of the wall formed both a matrix to hold the stones in place, and a porous screen to retain the fertile soil while permitting adequate drainage.

As the need became apparent for conserving nearly all rainfall in place on these terraces, to bridge over the periods of deficiency so far as practicable, the terrace walls were raised and trench-like pockets similar to those provided by modern contour listing were formed to increase water storage capacity. To prevent wholesale loss of stored water through overtopping and drainage from one basin to another, the terraces were divided transversely into sections of moderate length, each with considerably greater capacity than that ordinarily required to take care of local rainfall and resulting pondage.

Other progressive steps in the use of rock-fill dams provided mill ponds and stock-watering ponds, and larger storage basins behind either combination rock-fill and earth dams, or rock-filled timber cribs. Where suitable soil for earth-fill was scarce or required long haulage, and durable rock was abundant, the natural result was a puddled core in an earthen fill, retained between loose rock shoulders, consisting of either dry rubble walls or more voluminous fills of quarry-run stone dumped to natural slope.

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[39] Cons. and Hydr. Engr., SCS, Washington, D. C.

[39a] Received by the Secretary April 22, 1938.

There are in existence to-day, in widely scattered locations, within the United States and neighboring countries, examples of minor rock-fill dams with either earth cores or earth aprons to serve as the relatively impervious element. Among those stock-watering, farm, mill, or wild-life ponds located in regions subject to accelerated erosion, sedimentation has reduced storage volume and necessitated expensive maintenance or enlargement from time to time—or ultimate abandonment. Many of these ponds antedate the legislation requiring approval by the State Engineer's Office; some even date from early colonization by Europeans; but they have served as important factors in the settlement and development of American frontiers.

The writer has never ceased to wonder at the volume of dry rubble and other stone masonry construction, backed by rock-fill, or by a combination of earth and rock, along the network of navigation canals, logging channels, dams, and spillways of the last century or more, including the canal constructed by George Washington around Great Falls of the Potomac River, near Washington, D. C. Some of these structures are still in service, and many others could be restored to working condition at moderate cost, if sufficient demand arose for such uses as they originally served. However, it seems improbable that the nation will ever revert to such extensive projects to serve slow methods of transportation, now that rapid transit seems to be the main objective and has been so widely attained.

There is ample opportunity for future use of low rock-fill dams on minor projects. For greater storage depths the factors of safety should likewise increase, because of the danger of high velocities and sudden widespread destruction as a result of overtopping or failure from any other cause.

Advancement in the art of dam construction has apparently awarded preference to some form of cohesive integral masonry structure for great storage depths. In order to displace masonry structures from what is admittedly their own province, as may doubtless be advisable at times, proponents of the competing type should be required to show overwhelming advantages in first cost, maintenance, or both, with no sacrifice of necessary safety factors. Small differences should be waived in favor of the type which could withstand over-flow and could continue to function even in cases of catastrophic floods, far beyond the probable limits as contemplated by the design.

The position of the water-tight element of a rock-fill dam may be anywhere within the limits of the up-stream face, in the writer's opinion (see Fig. 25), provided the design of cross-section conforms to the position so chosen. The pavement, diaphragm, or apron used as a water-seal may or may not be parallel to the up-stream surface. A core-wall, if installed at all, should stand vertically with either stepped or battered faces to account for thinning toward the top. Certainly, a well-constructed core-wall with its footings properly established in bed-rock, and the superstructure well reinforced against cracking, sandwiched between the up-stream third and the down-stream two-thirds of the entire fill, should be a satisfactory water-seal, if properly encased or cushioned against irregular concentrations of pressure (as from large, angular boulders during the inevitable settlement and compaction of materials). Likewise, an equal volume of masonry or durable mastic material, spread over the greater area presented by the up-stream slope; or well-fabricated sheet metal in a similar position, but protected from both corrosion and abrasion by suitable covering, may satisfactorily serve the same purpose, and may both permit and require inspection, repair, or replacement more frequently than the core-wall.

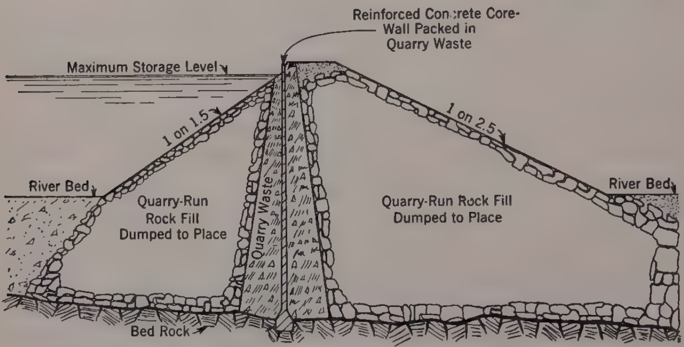
Reinforced Concrete Core-Wall Packed in Quarry Waste
Maximum Storage Level
1 on 1.5
1 on 2.5
River Bed
Quarry-Run Rock Fill Dumped to Place
Quarry Waste
Quarry-Run Rock Fill Dumped to Place
River Bed
Bed Rock

FIG. 25

The author has contributed much of both historical and scientific interest relating to the design of rock-fill dams and to fundamental principles, considerations, and requirements influencing the final choice of structural type for a given location. His definition of rock-fill dams departs somewhat from that accepted in general practice, in that it would apparently exclude such structures as depend on either puddle cores or earth aprons as the impervious face. Thus, his definition would exclude the Castlewood Dam (Table 1, Item No. 5), which had an "earth embankment added to upstream face, 8 feet wide at crest with 3:1 slope faced with 12-inches of riprap."[40] Likewise the Avalon Dam, in New Mexico, both the Milner and Minidoka Dams, in Idaho, and the Zuni Dam, in New Mexico, listed as "rock-fill."[41] would have been excluded by the author's restrictive definition. Moreover, it would exclude a number of rock-fill dams with puddled earth cores described by the writer[41] in 1915.

Whether the impervious element is in the form of timber, metal plate, masonry, or dense plastic material used, respectively, as facing or as diaphragm core, cut-off, or up-stream apron, a structure dependent on a loose rock-fill for stability seems to qualify as a rock-fill dam. When the earth-fill for either the core-wall or the up-stream apron comprises a volume nearly equal to that represented by the rock-fill, it would appear fitting to include mention of this material in the designation, as "earth and rock-fill dam." However, when the earth apron represents only a minor percentage of the total volume, and is placed mainly to increase the resistance to seepage through bed-rock and abutment formation, by adding to length of travel and thus reducing the pressure gradient, the structure may still be designated as a rock-fill dam. There would be fully as great justification for mentioning the timber, the steel plates, the concrete facing, or the masonry core-wall as to mention the earth-fill used for the same specific purpose, namely, as a seal against seepage.

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[40] American Civil Engineers' Handbook, Fifth Edition, 1930, p. 1547.

[41] "Provo Reservoir Company's Irrigation Project," by C. S. Jarvis, *Engineering News*, August 26, 1915, p. 394.

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DISCUSSIONS

SOLUTION OF TRANSMISSION PROBLEMS OF A WATER SYSTEM

Discussion

By Messrs. F. Knapp, and Weston Gavett

F. Knapp,[20] Esq. (by letter).[20a]—In 1936, the writer was called upon to investigate the pressure and flow conditions of the water supply and fire protection system of an important Brazilian chemical plant where interruption of the flow of water would lead to disastrous results. It was especially desired to know the effect of shutting off, quickly, certain flow demands for production purposes. As a prerequisite to this extensive surge investigation (by the graphical method), the flow and the resulting head losses in the system for steady conditions had to be found, and a graphical method, first published by Spiess,[21] was used and extended for the requirements of this specific problem. As far as the author's Case V is concerned, the graphical solution as proposed is far simpler and more general than the method used by the writer, a combination of graphical and analytical procedures.

The author is to be commended heartily for his timely paper which is of considerable interest to all hydraulic engineers engaged in problems of water flow in pipes. It is only too true that the design of the transmission and distribution system receives little attention in spite of the great sums of money invested in such installations. The rational method proposed by the author makes it possible also to decide at once whether certain cross-connections are essential and of decided benefit. In the investigation made by the writer, considerable savings were effected by removing several long pipes connecting points of about equal, or slightly different, pressure.

At first glance, the paper seems more complicated than it really is. The discharge-head relations for the solution of the several fundamental cases are plotted, using tables or any other appropriate means. Furthermore, it is not

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Note.—The paper by Ellwood H. Aldrich, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1938, by Messrs. Lynn Perry, Charles M. Mower, Jr., and Thomas R. Camp; February, 1938, by Messrs. Harold E. Babbitt, and C. Maxwell Stanley; and March, 1938, by Messrs. Warren E. Wilson, and Harold K. Palmer.

[20] With São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

[20a] Received by the Secretary February 18, 1938.

[21] *Journal für Gasbeleuchtung und Wasserversorgung*, 1887, p. 513; also treated in "Hydromechanik der Druckrohrleitungen," by R. Winkel, München, 1919.

necessary to make Table 1 as complete as the form presented by the author. It is sufficient and facilitates the work to record only the results of the simplified layout, such as, for example, "Total, *D* to *J*."

As far as the solution of the case presented in Fig. 12 is concerned, the author did not make it clear that, actually, four diagrams of the type shown in Fig. 10, drawn for the various flow demands in the system, are required; and only when the writer realized this point, did he understand the solution.

Except for this minor criticism, the paper deserves to be mentioned as essential to the art of hydraulics, and the writer hopes that its contents will come to the attention of authors of books on applied hydraulics, who too often do not pay enough attention to professional papers representing an advance in the art.

WESTON GAVETT,[22] ASSOC. M. AM. SOC. C. E. (by letter).[22a]—In so far as it demonstrates the application of Mr. Freeman's method to the solution of complicated pipe systems, the paper by Mr. Aldrich is most interesting. By showing the application of the graphical method, first to simple typical problems and then to a more complicated system, the author clearly demonstrates the method and advantage of this procedure. Its advantage is the use of natural co-ordinates for head and flow, permitting the graphical summation of these values.

Celluloid sheets described by the author are convenient and may be easily prepared by rubbing the surface of commercial celluloid with an ink eraser.

Although the author mentions the hydraulic slide-rule as an easier means of solving simple problems than the graphical method, he gives no details as to the slide-rule procedure so that the reader may compare the two methods. In stating that "a mathematically accurate determination * * * is impossible of attainment from a practical standpoint," he is correct. This statement, however, tends to encourage rather than dispel the current opinion that, until 1935, it was impossible to design a distribution system. To assume that for years water-pipe systems have been designed by guesswork is extreme.

The computation of the losses in a distribution system that may be checked by actual tests to an accuracy greater than the precision of the basic data should be sufficiently accurate for all except academic purposes. Such computations may be made and have been made with the hydraulic slide-rule at least since 1905.[23]

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[22] San. and Hydr. Engr., Clyde Potts, New York, N. Y.

[22a] Received by the Secretary March 8, 1938.

[23] "Hydraulic Tables," by Williams and Hazen, 1905.

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DISCUSSIONS

ECONOMICS OF THE OHIO RIVER IMPROVEMENT

Discussion

BY C. L. HALL, M. AM. SOC. C. E.

C. L. HALL,[33] M. AM. SOC. C. E. (by letter).[33a]—Assumption (1) of the paper was stated in the form of a rule to be adopted in the writer's accounting for the Ohio River improvement. The inaccuracy of this rule was conceded, but it was presented as a condition that tended to become true in time. Professor Grant mentions one situation in which the assumption would not be true in a free economy, and hence it does not tend to become true in time, namely, if the producers favored by the waterway are marginal. As the principal producers of water-borne freight on the Ohio River are the great Pittsburgh steel companies and the Kanawha Valley coal mines, it may be conceded that they are not marginal. Professor Grant also shows that the full difference between rail and water costs cannot be claimed as a benefit if the traffic would not move by rail in the absence of water routes. This statement is correct; but it is moot because, except for a little river sand and gravel, all Ohio River traffic would have to move somehow—even if the waterway were to disappear from the face of the earth—unless the economic level of the region were to fall.

In preparing the financial summary (Table 5) the writer assumed a balance at the end of 1904—with everything paid for at that time. In General Kutz's official unpublished report[34] he states that there is no doubt that the savings effected by the Ohio River during the period 1875–1904 greatly exceeded the expenditures. However, this assumption, although it may be entirely justified, gave an unduly rosy picture of the situation from 1905 to 1910. Incidentally, the cost of Ohio River improvement prior to 1905 is such a small share of the total expenditures that research into commerce before that date is unjustified.

NOTE.—The paper by C. L. Hall, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: December, 1937, by Eugene L. Grant, Assoc. M. Am. Soc. C. E.; February, 1938, by Messrs. Fred Lavis, O. Slack Barrett, and Edmund L. Daley and Forrest E. Byrns; March, 1938, by J. E. Willoughby and W. D. Faucette, Members, Am. Soc. C. E.; and May, 1938, by W. E. R. Covell, M. Am. Soc. C. E.

[33] Col., Corps of Engrs., U. S. Army, Commanding 1st Engrs., U. S. Army, Fort Du Pont, Delaware.

[33a] Received by the Secretary May 21, 1938.

[34] Rept. of June 30, 1926.

It may be added that comparisons of benefits prior to 1905 are of very slight value. The Chesapeake and Ohio Railroad, for instance, was not built until 1905. Before that time, therefore, the river was the normal method of carrying freight between river towns, and there is no base upon which comparisons of costs with competing carriers could be computed adequately.

The main point discussed by Professor Grant raises a question that extends far beyond the Ohio River. He declares that public works are socially unprofitable unless they give at least as great a return as the private investments displaced by the diversion of taxpayers' funds. In its literal form the principle is impossible of application because it would rule out, completely, structures like the Lincoln Memorial. What the statement would mean in practice is that public works competing with privately owned facilities should not be constructed unless they could produce revenues far greater than those accounted for by non-competing public works; that is, a lock and dam must show much greater benefits than a new post office. Professor Grant's complaint that profits during the period of regulation are not a proper charge against losses after canalization was begun, has a certain measure of justification. He establishes his case quite neatly in Table 7. However, the fact that there were profits for 1930–1933, when most other activities in the United States were losing money, is itself proof of a rather wise investment.

The writer must dissent from the view that the life of a prospective public investment should be calculated in commercial terms. Experimental commercial investments are only justified by large prospective profits. The Government, however, can afford to experiment (or pioneer) on a public project with an undoubtedly long engineering life, subject to the hazards of a shorter economic life. If the Government refuses to do this kind of experimenting, nobody will. Will the country be better off if such pioneering enterprises are never undertaken?

Mr. Lavis' distinction between internal and external waterways also could not be applied in practice. If it were made absolutely rigid the enormous public benefits caused by the improvement of the St. Mary's River, for instance, would never have been accomplished. If the distinction were not made absolutely rigid, it would assuredly be disowned by any American Government within the range of practical politics. Moreover, even on Mr. Lavis' principle, the secondary local benefits from channels to the ocean, obtained on streams such as the Upper Hudson River, would cause a subsidy to certain types of internal waterway traffic.

The writer does not believe that he has overlooked the advantages in distance of the usually shorter land route. In the preparation of Table 4 the comparison was between terminals, and the benefits were developed on a basis of ton-miles measured by water. The writer hoped that Table 6 would have made that point entirely clear.

In Mr. Lavis' discussion, as in some others, there seems to be a feeling that the costs given in the paper are based on distances between main-line termini. This is not true. The comparison was for rail rates between producer and consumer on the one hand, and net costs plus profit to the competing shipper (rail rates, water costs, and terminal charges) on the other. These values were reduced to ton-miles measured by water in order to give a unit by which typical studies could be applied generally; but the very methods used gave the all-rail carrier complete credit for his shorter mileage.

Mr. Barrett's opinion that there is no difference, in final result, between benefits derived by producers and those derived by a necessarily limited number of consumers has considerable logic behind it. It seems probable, however, that public funds will not be provided for waterway improvement unless it can be shown clearly that a direct financial saving is obtained by a large number of consumers. Of course, the benefits from a public route of transportation cannot be distributed uniformly over the entire nation; but they must be open to all the people of the region affected, and not merely to stockholders of great corporations.

The remarks of Colonel Daley and Mr. Byrns are much appreciated. The writer did not cover the general regional benefits that have accrued to upper river ports from the Ohio River improvement, because he could find no method of making a statistical comparison. The point that real new wealth has been produced by Ohio River development, however, is a vital one, and should have been discussed.

Messrs. Willoughby and Faucette attack the improvement of the Ohio River on two general grounds, as follows: (1) That the improvement, in so far as it does permit cheaper shipment, merely diverts money from rail carriers to water carriers; and (2) that if the Federal costs were charged against the shipper there would be no cheaper shipment. If the net total of goods moved in the United States were constant (that is, if the country were static), the first complaint would probably be valid. Along that line, every new ocean vessel (except a replacement for a scrapped one) would reduce the revenues of existing ocean carriers. The nation has always refused to make any such melancholy admission, however. It has always claimed that the reduction of distances in money or time caused by technical improvements in means of transit has increased total consumption, and total freight moved, by a far larger amount than the losses caused to pre-existing cargo carriers. Beyond dispute this was the effect of the substitution of railroads for mule-drawn wagons in the Middle West seventy years ago.

If true waterway costs are greater than rail costs, Messrs. Willoughby and Faucette are correct in their second complaint. The aim of the writer was to show that the costs of shipment by the Ohio River were, on the whole, less than those by competing carriers. His proof was based on certain carefully stated assumptions, followed by an investigation of all the data that he could procure. The assumptions are those used in the study of proposed waterways for many years. Perhaps more valid ones could be prepared, which would be concurred in by those responsible for authorizing public expenditures; but the writer ventures to doubt it. The only assumption criticized constructively is the fifth, in which the writer excluded, for accounting purposes, any allowance for unpaid taxes. The question of charging unpaid taxes as a constructive expense of Government enterprises is a very difficult one. If full interest and depreciation are charged, and if the Government "polices" the work with maintenance funds, it is difficult to show that taxes are an equitable charge against the property. On the other hand, there is a certain degree of statistical error in comparing tax-free carriers with taxed ones. Every one should welcome a rule to correct the rather unsatisfactory state in which the present situation leaves the student of governmental enterprises.

The discussers are wrong in believing that the Ohio River pools are not beneficial to sand-diggers. Sand is also more readily carried in pools of fixed minimum depth. From his experience as a District Engineer, the writer can assure them that the improvement does not prevent the formation of new bars.

Allowance has already been made for greater circuity of water routes. In the preparation of Table 4, as stated in replying to Mr. Lavis, the basic comparisons were between movements from origin to destination by different routes. No question of relative lengths of haul entered into this matter until a final benefit per ton per route was determined. This benefit was then distributed as commercial benefit per ton-mile measured by water, so that all rail advantage of shorter haul was already credited; for example, in the movement of steel pipe from Youngstown, Ohio, to Memphis, Tenn., the shipper's cost is:

By rail (tariff charges)		$14.60
By water:		
Rail rate to river terminal	$0.335	
Line cost of river movement; 1 188 miles at 3 mills per ton-mile	3.56	
Terminal charges	0.75	
Total water cost		$4.645
Commercial benefit		$9.955
Unit ton-mile commercial saving		$0.0084

It will be noted that this commercial saving, although calculated in mills per ton-mile measured by water as the only convenient method of generalization, gives the railroad full credit for its shorter haul.

As to the criticism of Table 6, the journey from Cincinnati, Ohio, to Louisville, Ky., was selected because these two river cities are separated by a distance of about the length of the average haul (119 miles). The object of the table was to check Table 4 by an independent calculation. The writer was astonished at his own results. Incidentally, the rail and water distances from Cincinnati to Louisville are about identical. It is believed that the criticism of this comparison between rail and water methods of moving a typical ton a typical mile would have been more useful if data had been given for making the same comparison over some other trip between major river ports.

Colonel Covell's very interesting discussion presents a highly valuable cost analysis of steamboat transportation. He shows that the values derived from confidential reports by the big river operators are inherently reasonable, and he thus supports the general accuracy of the statement of savings (see Table 4). The writer has made no claim that the improvement is justified, wholly or in part, by its regulatory effect on rail rates. Probably it is true, as Colonel Covell states, that the existence of the improvement has inhibited a general increase in coal rates; but it is believed that internal improvements constitute a rather expensive means of rate regulation. In that respect, the writer tends to agree with Messrs. Willoughby and Faucette.

During the progress of this discussion the commercial data for the calendar year 1937 have become available. They give a total commerce of 23 356 676 tons, an increase of 25% over 1934; and of 2 671 926 225 ton-miles, an increase of 50% over 1934. The average haul for 1937 was 114.39 miles.

Undoubtedly, the discussions have clarified the points upon which differences as to matters of principle are held. There is no certainty as to what profits to the public should be required from public works competing with privately owned ones. There is no certainty as to whether taxes not paid by the Government should nevertheless be arbitrarily added to public costs for purposes of comparing accounts. There is no certainty as to the minimum distribution of private advantages, secured from public works, required to establish such advantages as public benefits. These questions, of course, are political in their nature; but the political decision should follow a crystallization of engineering opinion. Needless to say the final decision should apply to all public works competing with commercial investments. It was the writer's aim to use this paper on the Ohio River as a case study of a completed competing project, which would bring out all disputes on principle, besides contributing to technical literature pertaining to the Ohio River. His aim seems to have been accomplished—at least in part.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

LABORATORY INVESTIGATION OF FLUME TRACTION AND TRANSPORTATION

Discussion

BY MESSRS. E. W. LANE, AND JOE W. JOHNSON

E. W. LANE,[39] M. AM. SOC. C. E. (by letter).[39a]—A very valuable contribution to the science of the transportation of solids by flowing water is presented in this paper. It seems to the writer that the reason for some of the lack of agreement of different experimenters on the values of critical tractive force for the movement of the various sizes of material is probably that initial movement is not a function of tractive force alone but of a combination of tractive force and turbulence. The traction is an average force over a considerable area, but the forces that move the first particles are probably intense local forces, acting only over a small area of the bottom, having a magnitude greater than the average force acting. The initial movement, therefore, would depend not only on the magnitude of the average force but also on the range above the average of the variable forces, due to the turbulence in the water. Therefore, initial movement would depend on both tractive force and turbulence. These two factors would also influence other degrees of movement, but it does not seem likely that turbulence would be of relatively as great importance in them as in the initial movement.

In his summary of Part 1, Item (5), the author concludes that the "resistance to motion of particles on a horizontal bed is nearly double their resistance on a sloping bed having the same inclination as the water surface." This would indicate that the motion of bed load is very sensitive to changes in the bed slope. The bottom of a stream where bed load is in motion is usually rough, with slopes both upward and downward in the direction of flow. Even in a laboratory flume there are sand waves such that the sand moves up slopes considerably steeper upward in the direction of flow than the water surface slopes downward in this direction. Sand waves were observed in the

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NOTE.—The paper by Y. L. Chang, Esq., was published in November, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by Hans Kramer, M. Am. Soc. C. E.

[39] Prof. of Hydr. Eng.; Associate Director, in Chg. of Laboratory, Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

[39a] Received by the Secretary April 2, 1938.

Mississippi by the late J. B. Johnson, M. Am. Soc. C. E.,[40] with an average height of 5 ft, and an average length of about 330 ft, indicating an upward slope of 1 in 66. The surface slope was probably only about one two-hundredths of this inclination. The depths of water ranged from 13 to 30 ft, with movement apparently throughout this range. It would be interesting to extend the analysis that led to the foregoing conclusion to see if it would indicate any motion at all up the slopes of the Mississippi sand waves. The effect of bottom slope on the bed load carried in a stream may be a factor that will make it very difficult to correlate field and laboratory results.

The author shows that a change in the law of bed-load transportation occurs at a grain size for quartz of about 0.5 mm, and states that this is best explained by turbulence. Since the law governing the settling rate of quartz grains changes at about the same size, may the change in the bed-load law not be related to the change of settling rate? There is good evidence that the law of transportation of material in suspension shows a similar sharp break at about this size.

The author concludes (see "Summary": Part III, Item (5)) that "silt transportation tends to decrease the mean velocity of flow." This conclusion is the opposite of that reached by A. B. Buckley, Jr.,[41] from observations in the Nile Valley. Surprisingly low roughness values have been obtained on some of the silt-laden rivers of North China by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., and others, which support Mr. Buckley's conclusions. This is apparently a field which justifies much further research.

Joe W. Johnson,[43] Jun. Am. Soc. C. E. (by letter).[43a]—In this paper the author has presented an excellent review of the problem of transportation by flume traction. He has also presented original data and conclusions that are welcome contributions to the knowledge of this phenomenon. Of particular interest to the writer is the section of the paper treating the formulas for rate of transportation. Therein is given a review of the published formulas and a new formula which is of the du Boys[14] type.

The writer reviewed a large number of formulas representing the data obtained from various experiments. Perhaps the most common type of formula is that of du Boys (Equation (26)), in which rate of movement is plotted against a function of tractive force. Also very common is the Schoklitsch-MacDougall type (Equation (41)), in which rate of movement is assumed to be proportional to excess power.[22] Authorities undertaking flume studies have usually presented their data in one of the common types of formulas, but, in many instances, with slight modifications. The result of such studies has been that there are approximately as many formulas describing the rate of sand movement as there are authorities on the subject. It was with this situation in mind that an investigation was made in 1937 to determine, for a certain set of published flume observations, whether a particular method of presenting data offered distinct advantages over other methods. In order to obtain an unbiased opinion in this determination, recourse was made to the methods of statistical analysis.

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[40] Chf. of Engrs'. Rept., 1879, Pt. III, p. 1967.

[41] "The Influence of Silt on the Velocity of Water Flowing in Open Channels," by A. B. Buckley, Jr., *Minutes of Proceedings*, Inst. C. E., Vol. 216, p. 1813.

[43] Asst. Hydr. Engr., SCS, U. S. Dept. of Agriculture, Washington, D. C.

[43a] Received by the Secretary April 19, 1938.

[14] "Le Rhône et les riviéres a lit affouillable," par P. du Boys, *Annales des Ponts et Chaussées*, Vol. 2, 1879.

[22] "Bed Sediment Transportation in Open Channels," by C. H. MacDougall, *Transactions*, Am. Geophysical Union, 1934.

Flume-study observations published in United States Waterways Experiment Station *Paper No. 17* were used in the computations. These data are for U. S. Waterways Experiment Station Sand No. 1 which was tested in a concrete-lined flume, 2.416 ft wide and with slopes of 0.0010, 0.0015, and 0.0020. Only those data in which "general movement" occurred were used in this study. From the results of these measurements, computations were made to permit the data to be plotted according to the criteria of du Boys,[14] Chang, Meyer-Peter,[20] U. S. Waterways Experiment Station,[10] MacDougall,[22] and O'Brien.[44] It is noted that in Fig. 11 the author plots $\frac{G}{n_s}$ versus $T\ (T - T_0)$, whereas in the general statement of his formula (Equation (33)), $\frac{G}{n}$ is adopted as the dependent variable. In the discussion to follow, the parameters in Fig. 11 have been used. Figs. 17, 18, and 19 show the results of plotting the computed data according to various authorities. The curves were fitted by the method of least squares and their "goodness of fit" was tested by the χ^2-test. The equations for the various groups are, as follows:

The U. S. Waterways Experiment Station (Fig. 17):

$$G_1 = \frac{29\,400}{n}\,(d\,S - d_0\,S_0)^{1.41} \qquad \text{(76a)}$$

Du Boys (Fig. 18(*a*)):

$$G_1 = 20\,860\,[T\,(T - T_0)]^{0.87} \qquad \text{(76b)}$$

Chang (Fig. 18(*b*)):

$$G_1 = 480\,600\,n_s\,[T\,(T - T_0)]^{0.74} \qquad \text{(76c)}$$

O'Brien, for $S = 0.0010$ (Fig. 19):

$$G_1 = 2.4 \times 10^{-5}\left(\frac{V}{R^{\frac{1}{3}}}\right)^{14.4} \qquad \text{(76d)}$$

O'Brien, for $S = 0.0015$ (Fig. 19):

$$G_1 = 1.8 \times 10^{-6}\left(\frac{V}{R^{\frac{1}{3}}}\right)^{16.0} \qquad \text{(76e)}$$

and O'Brien, for $S = 0.0020$ (Fig. 19):

$$G_1 = 3.5 \times 10^{-6}\left(\frac{V}{R^{\frac{1}{3}}}\right)^{14.3} \qquad \text{(76f)}$$

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[20] "Neuere Versuchsresultate über den Geschiebetrieb," von E. Meyer-Peter, H. Favre, und A. Einstein, *Schweizerische Bauzeitung*, No. 103, 1934, *Professional Paper 86*.

[10] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," U. S. Waterways Experiment Station, *Paper 17*, 1935.

[44] "Notes on the Transportation of Silt by Streams," by Morrough P. O'Brien, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Geophysical Union, Pt. II, 1936, p. 431.

The MacDougall data resulted in curves having the following equations (illustrations not included):

For $S = 0.0010$ (G_1 plotted against discharge per foot width of flume):

$$G_1 = 40.47\,q - 4.78 \quad \text{(77a)}$$

For $S = 0.0015$:

$$G_1 = 73.20\,q - 6.18 \quad \text{(77b)}$$

and, for $S = 0.0020$:

$$G_1 = 98.62\,q - 6.26 \quad \text{(77c)}$$

The general formulas of du Boys, Chang, U. S. Waterways Experiment Station, and MacDougall have been reviewed by the author and further discussion is not necessary. Perhaps it is desirable, however, to discuss briefly the Meyer-Peter formula (Equation (36)) inasmuch as it was developed from measurements on material of uniform granular size. The observations on material ranging from 3.17 mm to 28.8 mm were obtained by the laboratory at Zurich, Switzerland, and from the data of Gilbert.[17] The result of these studies, stated generally by Equation (36), is:

$$\frac{G_2{}^{\frac{2}{3}}}{D'} = 1.323\,\frac{q^{\frac{2}{3}}\,S}{D'} - 0.335 \quad \text{(78)}$$

in which G_2 is the cubic feet of bed material transported per foot width of channel per second, and D' is the diameter of bed material, in feet. A plot of this equation results in the formula $\left(\frac{q^{\frac{2}{3}}\,S}{D'}\right.$ plotted against $\frac{G^{\frac{2}{3}}}{D'}$; graph not included$\left.\right)$:

$$G_2 = (1.66\,q^{\frac{2}{3}}\,S - 0.147\,D')^{\frac{3}{2}} \quad \text{(79)}$$

Equation (79) was intended primarily for use in movable bed-model studies. The advantage of this relation is obvious when it is noted that dividing both sides by $g^{\frac{1}{2}}$ gives a resulting equation that is non-dimensional and in agreement with the Froude law of similarity.

In flume tests on Columbia River material, Professor O'Brien[44] found that experimental data showed considerable variation when rate of movement was plotted against the following parameters: Velocity at a constant distance from the bottom, mean velocity, the U. S. Waterways Experiment Station method, or the method of Schoklitsch. In all cases the disagreement was greater than could be accounted for by inaccuracy of measurement. Good results, however, were obtained by plotting rate of movement versus $\frac{V}{R^{\frac{1}{3}}}$, as shown in Fig. 19. Professor O'Brien[45] also reported a second method, namely, plotting $\frac{G}{f}$ against $\frac{V}{R^{\frac{1}{3}}}$, that showed slightly less variation than the first.

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[17] "The Transportation of Débris by Running Water," by G. K. Gilbert, *Professional Paper 86*, U. S. Geological Survey, 1914.

[45] "Laboratory Study of Transportation of Columbia River Bed Materials," *Technical Memorandum No. 11*, U. S. Tidal Model Laboratory, Berkeley, Calif., January 28, 1936.

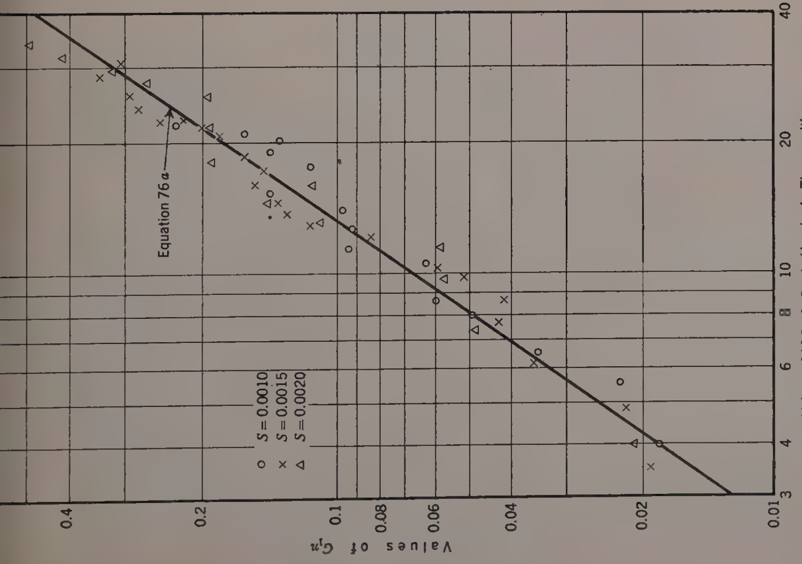

Fig. 17.—Data on Transportation by Traction, According to U. S. Waterway Experiment Station; Equation (39) or Equation (76a)

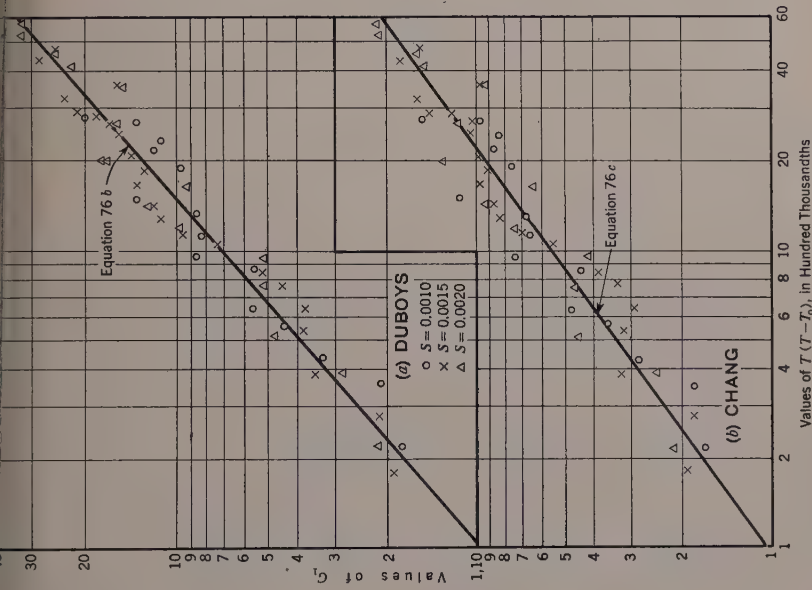

Fig. 18.—Data on Transportation by Traction; Equations (26) and (33)

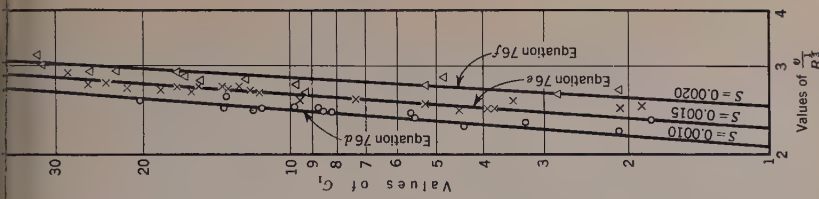

Fig. 19.—Data on Transportation by Traction, According to O'Brien; Equations (76d), (76e), and (76f)

From a glancing examination of the curves shown in Figs. 17, 18, and 19, it is difficult to determine which type of plotting gives the best fit; however, by the application of the χ^2-test, personal opinion is eliminated and an unbiased measure of the goodness of fit is obtained. Table 7 shows values of χ^2 computed for the various criteria. An interpretation of the significance of the computed values of χ^2 requires the use of tables from which the probability of fit can be obtained. Reference to probability tables[46] shows that in 99 out of 100 trials, in random sampling, one should obtain a fit as bad as or worse than that observed, if the real distribution is normal. For these various plots which show such excellent fit, it is to be noted that the probabilities of occurrence are so near the same magnitude that it cannot be truthfully stated that a particular method of plotting is superior to any one of the other methods. The reason for favoring a particular form of plotting, therefore, appears to be only in the ease and accuracy with which certain variables can be measured.

TABLE 7.—COMPARATIVE VALUES OF χ^2

Reference No.	Authority	Number of observations	Values of χ^2	Reference No.	Authority	Number of observations	Values of χ^2
1	U. S. Waterways Experiment Station	52	16	4	Chang	52	23
2	du Boys	52	18	5	Meyer-Peter	52	25
3	MacDougall	52	20	6	O'Brien	52	46

In flume studies there are three sources of error which, in many instances, have seriously limited the value of the results. These sources of error are: (*a*) The use of the water-surface slope instead of the slope of the energy gradient; (*b*) the neglect of the retarding effect of the channel side-walls; and (*c*) the use of relatively short collection periods during which the bed material is trapped. An accurate determination of the slope of the energy gradient requires the measurement of the velocity distribution at each end of the experimental reach. Owing to the relatively short time of a run, this observation is often eliminated and the error involved in slope determination is fairly large for the usual experimental conditions. It is of interest to note that Gilbert[17] was undecided as to the proper value of slope to use and stated:

> "I do not find it easy to decide which slope should be regarded as the true correlative of capacity for traction, but as all our laboratory data include the débris slope, while the determinations of water slope were relatively infrequent, the discussion of the results has adhered almost exclusively to the former. If the water slope is the true correlative, then the use of the débris slope involves a systematic error."

Professor O'Brien and Lieut. B. D. Rindlaub[47] support Gilbert's selection in the statement,

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[46] "Statistical Methods for Research Workers," by R. A. Fisher, Oliver & Boyd, Lond., 1936.

[47] "The Transportation of Bed Load by Streams," by M. P. O'Brien and B. D. Rindlaub, *Transactions*, Am. Geophysical Union, June 1934, Pt. II, p. 593.

"It is to be noted that the slope of the bottom is more nearly equal to the slope of the energy-gradient than is the slope of the water-surface and partly for this reason, the data of G. K. Gilbert show less scattering than the data of more recent experimenters who have criticized Gilbert for not measuring the slope of the water-surface in all of his experiments."

Consideration of the retarding effect[48] of the channel side walls is necessary in order to correlate the data from flumes of various widths or to correlate accurately the data from a particular flume whose width to depth ratio is relatively small. A striking illustration of the importance of side-wall resistance is afforded by Fig. 20, which shows rates of transportation of Gilbert's Grade *B* sand in flumes of various widths. It is obvious that consideration of side-wall resistance is essential if a general expression for describing rate of transportation by traction is expected. MacDougall[22] attempted to eliminate the effect of the side walls mechanically by placing diversion walls on the experimental flume. Meyer-Peter[20] in his formulation of Equation (36) eliminated side-wall disturbances on the basis of the method given by Einstein.[49] The latter method gives that part of the total discharge which is supposed to be acting upon the bed. The results from Einstein's method are similar to the Schoklitsch[7] method defined by the author in Equation (12). In the data herein discussed, a reduction in scattering of plotted points was not effected by the introduction of α into any of the various parameters used in Figs. 17, 18, and 19.

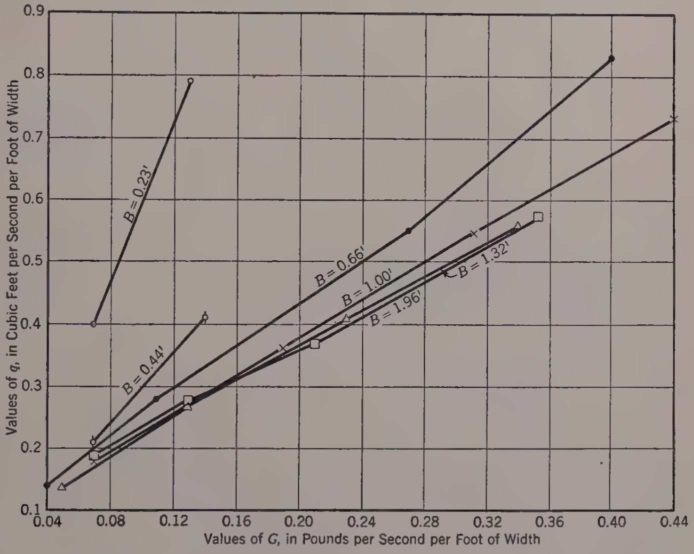

FIG. 20.—DATA ON TRANSPORTATION OF GILBERT'S GRADE *B* SAND ON FLUMES OF VARIOUS WIDTHS; SLOPE = 1 PER CENT

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[48] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 519.

[49] "Der Hydraulische oder Profil-Radius," by A. H. Einstein, *Schweizerische Bauzeitung*, No. 8, Vol. 103, February 24, 1934.

[7] "Wasserbauliche Strömungslehre," by P. Nemenyi, Leipzig, 1934; or, "Über Schleppkraft und Geschiebewegung," by A. Schoklitsch.

The use of relatively short collection periods when observing rates of bed-load movement probably accounts for most of the scattering of experimental data. For example, during calibration tests on bed-load traps for use on the Rhine, Einstein[50] observed the rates of bed-load movement shown in Fig. 21.

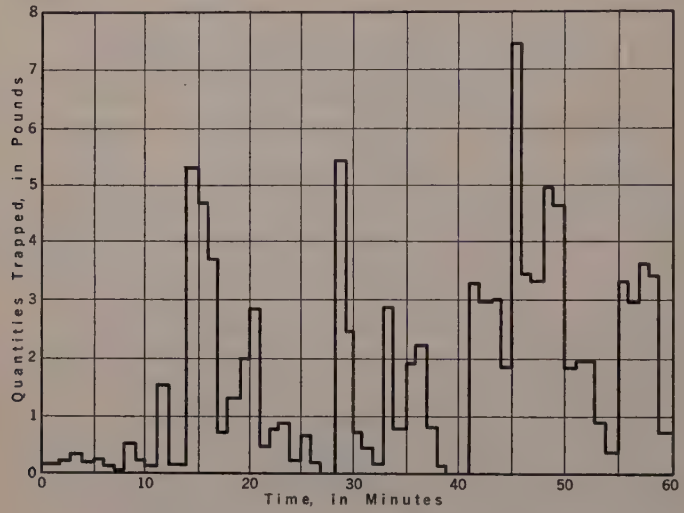

Fig. 21.—Trend of Trap Results in the Course of One Hour, with Constant Hydraulic Conditions

It is obvious from an examination of this diagram that relatively long collection periods and recourse to statistical analysis are perhaps necessary to obtain reliable and consistent data on the movement of granular material by flowing water.

Corrections for Transactions: In Equation (4), change "$n_g^{\frac{3}{2}} d$" to read "$2\, n_g^{\frac{3}{2}} d$"; and, change Equation (36) to read "$G = (a\, S\, Q^{\frac{2}{3}} - b\, D)^{\frac{3}{2}}$." (See also *Proceedings*, Am. Soc. C. E., March, 1938, p. 604.)

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[50] "Calibrating the Bed-Load Trap as Used in the Rhine," by A. H. Einstein, *Schweizerische Bauzeitung*, Vol. 110, No. 14.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

GRAPHICAL REPRESENTATION OF THE MECHANICAL ANALYSES OF SOILS

Discussion

BY MESSRS. E. W. LANE, F. J. SANGER, AND F. KNAPP

E. W. LANE,[30] M. AM. SOC. C. E. (by letter).[30a]—In this brief but very enlightening paper, Mr. Campbell has discussed three principal points: (1) The use of semi-logarithmic paper for plotting mechanical analyses; (2) a method of indicating the variability of the size graduation; and (3) a suggested division of soils into classes according to the size of the particles of which they are composed, with class names. In the following paragraphs the writer will discuss each of these three points in turn.

The use of the semi-logarithmic type of gradation curves advocated by the author has much to commend it and, particularly in the field of earth-dam material, it is much superior to any other method of graphical representation of particle sizes with which the writer is acquainted. Another method extensively used, particularly by geologists, is the so-called histogram, which is simply a bar graph, the length of each bar of which represents the percentage of the total material between two sizes. The number of bars depends on the number of fractions into which the sample was divided. Another type is a summation curve similar to that advocated by Mr. Campbell, but for which a Cartesian scale of co-ordinates is used for the grain diameter in place of a logarithmic one.

The semi-logarithmic plotting of a summation curve is much superior to either of these methods. The disadvantages of the histogram method are that it is not standardized since the widths of the bars are all the same, but each bar may represent any size range selected by the investigator. The shape of the histogram also depends upon the number of fractions into which the sample is divided. It is not possible to determine the percentage of the

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NOTE.—The paper by Frank B. Campbell, Assoc. M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1938, by Messrs. Donald M. Burmister and A. J. Weinig, Jr.; March, 1938, by Messrs. Carl H. Kadie, Jr., Carlton S. Proctor, T. T. Knappen, Jacob Feld, and Howard F. Peckworth; and May, 1938, by Messrs. Joel D. Justin, L. B. Olmstead, T. A. Middlebrooks, and Frank E. Fahlquist and Waldo I. Kenerson.

[30] Prof. of Hydr. Eng.; Associate Director, in Chg. of Laboratory, Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

[30a] Received by the Secretary March 28, 1938.

material smaller or greater than any specified size, except such sizes as the limits of a particular fraction, and then only by a process of adding the lengths of the bars between that point and one end of the diagram. In the semi-logarithmic plotting, the curve will be practically the same regardless of what limits are used for the various fractions, and it is possible to determine the percentage greater or less than any specified size and also to determine the median size or size for which 50% is greater and 50% smaller. This size forms a convenient single value to express the size of the material.

In the form of summation curves using the Cartesian co-ordinates for the grain diameter, unless the chart is very long, the smaller sizes are difficult to read and plot accurately. Very fine sizes particularly come so close to the zero size line that it is very difficult, if not impossible, to read them. In the semi-logarithmic plotting, the scale of the smaller size is enlarged, and it is possible to read accurately down to the finest particle used. For purposes of comparison, it is possible, using the semi-logarithmic paper, to plot on the same sheet, using the same co-ordinates, the size distributions of a number of samples with much less confusion than is possible using the other methods.

In the plotting of sizes of material moved by flowing water there is frequently considerable uniformity of particle size and the curves often take the shape of Mr. Campbell's Sample (*c*). They frequently have a distinct S form, showing that the greater part of the material is of nearly uniform size, but that a small part has a considerable size range larger than the main body and that another small part has likewise a considerable range smaller than the principal part. As these small fractions may prove to have a considerable bearing on the transportation of the material, it is desirable that the method of graphical representation used indicate their proportions accurately. A system of co-ordinates which would expand the extremities of the "per cent larger" scale would aid in this desirable end. The writer is experimenting on the development of such a form of paper.

Mr. Campbell's comparison of the various classifications—clay, silt, sand, etc.—serves to emphasize the lack of agreement among the workers in this field. Mr. W. W. Rubey[31] has mentioned several others, including those in Table 2, Columns (1) (2), and (3), each of which has had considerable use. Still another classification is that used by L. G. Straub, Assoc. M. Am. Soc. C. E. It is the same as that of Mr. C. K. Wentworth except that it divides silt into four classes. Mr. Campbell's classification has much to recommend it, but the writer fears that no classification recommended by an individual, regardless of how eminent he may be, will be universally accepted. It seems probable that the advantages of any reasonable classification, if widely accepted, would more than offset any disadvantages that might occur in case the standard was not the best possible one, and, therefore, considerable advantage would result from the general adoption of a single standard. That classification should be adopted which seemed to be most generally acceptable to all groups, and if no agreement can be reached with other groups, it is suggested that civil engineers at least adopt one standard. Any one who had definite opinions on the superiority of another classification would still be free to use the grouping of his choice. Perhaps an agreement on certain features of the method of plotting could also be standardized, such as whether the size range should increase or decrease toward the right, and whether the percentage smaller than or larger than a given size should be used for the ascending scale of ordinates.

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[31] *Professional Paper 165A*, U. S. Geological Survey, p. 23.

TABLE 2.—COMPARISON OF SOIL CLASSIFICATION STANDARDS
(Units are Millimeters)

Udden* (1)	Atterberg† (2)	Wentworth‡ (3)	Straub§ (4)
	Boulders	Boulder	Boulder gravel
256	2 000	256	256
Boulders	Blocks	Cobble	Cobble Gravel
16	200	64	64
Gravel	Pebbles	Pebble	Pebble Gravel
1	20	4	4
Sand	Gravel	Granule	Granule Gravel
1/16	2	2	2
Silt	Sand	Very Coarse Sand	Very Coarse Sand
1/256	0.2	1	1
	Powder Sand	Coarse Sand	Coarse Sand
....	0.02	1/2	1/2
	Silt	Medium Sand	Medium Sand
....	0.002	1/4	1/4
	Clay	Fine Sand	Fine Sand
....	....	1/8	1/8
		Very Fine Sand	Very Fine Sand
....	....	1/16	1/16
		Silt	Coarse Silt
....	....	1/256	1/32
		Clay	Medium Silt
....	....	....	1/64
			Fine Silt
....	....	....	1/128
			Very Fine Silt
....	....	....	1/256
			Clay

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* "Mechanical Composition of Clastic Sediments," by J. A. Udden, *Bulletin*, Geological Soc. of America, Vol. 25, 1914, pp. 655–744.

† "Die rationelle Klassifikation der Sande und Kiese," by A. Atterberg, *Chem. Zeitung*, Jahrgang 29, 1905, pp. 195–198.

‡ "A Scale of Grade and Class Terms for Clastic Sediments," by C. K. Wentworth, *Journal of Geology*, Vol. 30, p. 382, 1922.

§ "Missouri River," H. R. Doc. No. 238, 73d Cong., 2d Session, Appendix XV, February, 1934.

The standard of gradation or uniformity suggested by Mr. Campbell, the writer believes to be a very good one. Although it has a purely empirical basis, so far as known no classification on a more rational basis has been suggested. It has the great advantage of being easy to obtain. No such standard will be widely used which for each sample requires the expenditure of considerable time for its determination. The writer has long recognized the desirability of some easily determined coefficient of variability for convenience in expressing in numerals, roughly, the composition of samples of granular material. He believes that the two numbers, median size (50% of the weight composed of larger particles and 50% of smaller particles) and the Campbell variability coefficient, together make possible a fair representation of the size and gradation of a granular material.

It is believed, however, that it would be possible to improve the proposed gradation coefficient. Mathematically, the value of this coefficient is 10/6 of the logarithm (Base 10) of the ratio of the "80% smaller" size to the "20% smaller" size. To use 6/6, the logarithm of this ratio, instead of 10/6, would have several advantages. The coefficient would then have a simpler physical meaning, which is always an advantage, especially in defining it. Knowing the 80% and 20% sizes, it would be possible to determine approximately the coefficient by mental calculations, and an accurate computation could be readily made with a slide-rule, requiring one less operation than the form proposed. Graphically, it would be the number of cycles of the semi-logarithmic scale between the 20% smaller and 20% larger point. This distance could be measured directly without drawing the slope line on the graph. The value of this proposed revised coefficient in every case would be 0.6 of that of the one proposed by Mr. Campbell. Some thought should also be given to the possibility that the use of, say, the 15% and 85% points on the 10% and 90% might be preferable to the 20% and 80% points.

F. J. SANGER,[32] ASSOC. M. AM. SOC. C. E. (by letter).[32a]—It is a well established fact that the only way to describe the grading of a soil is graphically, and the best chart is probably the one commonly used, having the log scale at the bottom for grain sizes. The terminology used in describing grains of particular grain sizes cannot be justified. To take existing, well understood terms describing natural soils and to use them for more precise meanings very near, but very different from, their previous meanings, is unscientific and misleading. The exact divisions are artificial and hence can never be standardized to agree with all points of view. Why not, then, discard these confusing divisions? If it really is necessary to subdivide the horizontal scale according to present practice, other terms should be used, and probably a numbering system is best. The grain-size distribution of a particular natural soil, as specified in the chart used by the U. S. Bureau of Public Roads, can better be shown by an area bounded by two curves, inside which a soil of the particular class must lie. Engineers are not in the habit of defining structures and machines by lengthy descriptions; but rather by the use of drawings. Why not describe the grain-size distribution that way and not bother about printing percentages?

With regard to grain shape, is it not probable that the moisture and plasticity tests, as given in A. S. T. M. publications, take full account of the important effects?

This means that to describe a natural soil, a chart and some moisture and plasticity data are necessary—not a list of percentages, which are difficult to appreciate as they stand. Natural moisture content, specific weight of soil particles, and natural voids ratio from which density of the mass can be calculated, are also desirable, of course. The point is that the list of percentages should be superseded by the curve and that the existing terms for sub-divisions should be discontinued. The importance of the geologic history of the soil is now being appreciated and a description of the soil should include such a history, where it is known.

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[32] Head, Dept. of Eng. and Building, The Lester School and Henry Lester Inst. of Technical Education, Shanghai, China.

[32a] Received by the Secretary April 9, 1938.

F. KNAPP,[33] ESQ. (by letter).[33a]—The proposal of the author to designate a particular soil by means of the grain size and the grade is most helpful and, in combination with Fig. 1, serves to give an immediate idea as to the type of the soil. Furthermore, this proposal makes generic designations, such as clay, sand, etc., superfluous. The writer would suggest that a more complete diagram of the several hundred samples mentioned by the author be included in the closing discussion.

The graphical representation of analyses of soils and their definitions by the mean grain size and grade should be helpful in permitting a more logical interpretation of a hydraulic problem, namely, the permissible maximum velocity in earth canals in order to prevent erosion. Although other factors still enter into the problem (such as the cross-section of the canal and the velocity distribution) it is clear that the properties of the soil are of outstanding importance.

Russian engineers[34] have published a table of permissible velocities in clayey soil, subdivided into four classes according to porosity. From the few indications given in regard to grain size, the writer plotted the distribution as proposed by the author. The product of mean grain size and grade multiplied by 10^3 may be plotted as abscissa, the velocities as ordinates, and the porosities as parameters. With some imagination it was found that a curve could be fitted to the various points, considering especially that the properties of soil in Nature do not change abruptly. Such a representation serves to indicate more clearly the inter-relation of the permissible velocity and the properties of the soil.

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[33] With São Paulo Tramway, Light & Power Co., Ltd., São Paulo, Brazil.

[33a] Received by the Secretary April 29, 1938.

[34] "Die zulaessigen Hoechstwerte der mittleren Geschwindigkeit in Gerinnen," by A. Schoklitsch, *Wasserkraft und Wasserwirtschaft,* 1937, No. 21.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PRELIMINARY DESIGN OF SUSPENSION BRIDGES

Discussion

BY GLENN B. WOODRUFF AND NORMAN C. RAAB,
MEMBERS, AM. SOC. C. E.

GLENN B. WOODRUFF[19] AND NORMAN C. RAAB,[20] MEMBERS, AM. SOC. C. E. (by letter).[20a]—Even to the experienced designer, the deflection theory for suspension bridges has heretofore been largely a matter of following formulas without a complete conception of their meanings. The authors are to be congratulated upon their success in presenting a clear picture of the interaction of the various parts of the suspension system. The paper is so complete and the results so convincing that, within its scope, there remains but little to be said. In their "Conclusion," the authors have emphasized several subjects for further investigation. The following is submitted, not as an answer to these problems, but rather as a suggested basis for these investigations.

Before commenting on these problems, reference is made to Fig. 7 which, it is believed, neglects the fact that unless other changes in design are made, changing the truss stiffness would also affect the dead load, which, in turn, would affect the deflections. Assuming that the designers of the San Francisco-Oakland Bay Bridge considered an increase in rigidity necessary, the question arises as to the most economical means of securing such increase. For the present, only the methods of increasing the dead load of the suspended structure, increasing the moment of inertia of the stiffening trusses, or a combination of the two, are considered. To keep the dead load of the structure as low as practicable, light-weight aggregates were used for the 6-in. upper-deck paving slab, at an increased cost for the light-weight as compared to standard aggregates. The cost of the paving, as well as its supporting floor steel, could have been reduced by using standard aggregate. Still further savings in the floor construction could have been made by using a 9-in. paving slab with standard aggregate.

NOTE.—The paper by Shortridge Hardesty and Harold E. Wessman, Members, Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. A. W. Fischer, and Jacob Karol.

[19] Engr. of Design, San Francisco-Oakland Bay Bridge, San Francisco, Calif.

[20] Senior Designing Engr., San Francisco-Oakland Bay Bridge, San Francisco, Calif.

[20a] Received by the Secretary April 19, 1938.

Assuming that the quarter-point deflection is a satisfactory index of rigidity, calculations have been made for the following cases: (I) Original design; (II) 50% increased I (no increase in dead load); (III) 50% increased I (additional weight of trusses and cables included); (IV) changing pavement slab to standard concrete (no increase in I); (V) changing paving slab to standard concrete (30% increased I); and (VI) changing paving slab to 9-in. standard concrete (no increase in I).

The results for these cases, shown in Table 3, indicate that, for this particular structure, increasing the moment of inertia of the stiffening trusses is, in general, the most economical means of increasing rigidity. They do indicate, however, that there would be no economy in extra expenditures for an extremely light-weight floor and adding truss metal to regain the rigidity lost by decreasing the dead load.

TABLE 3.—EFFECT OF MOMENT OF INERTIA ON RIGIDITY

Case	Dead load, in pounds per linear foot of cable	Area of cable, in square inches	Quarter-point moment, in foot-kips	Deflection, in feet	Decrease in deflection	Cost change	Cost per foot, decrease
I	9 200	500	110 000	10.10	....	....	....
II	9 200	500	142 500	8.92	1.18	....	....
III	10 000	530	137 600	8.52	1.58	$1 150 000	$730 000
IV	9 800	525	107 900	9.68	0.42	294 000	700 000
V	10 200	540	124 900	8.94	1.16	976 000	840 000
VI	11 100	570	100 000	9.03	1.07	978 000	910 000

Certain of the additional studies proposed by the authors lead to a specification for the design of stiffening trusses which, in the present stage of suspension-bridge design, is desirable. For this purpose, it is necessary to specify the intensity and distribution of live load, a measure of the required rigidity of the stiffening truss, and the permissible unit stresses. Considering only vehicular loadings, the probabilities of several of the loading conditions assumed in the calculations of maximum stresses in stiffening trusses are so remote that they may be safely discounted to a large extent. The most probable loading, for a bridge subjected to heavy traffic, is to have the roadway filled with passenger cars interspersed with comparatively few trucks and buses. With traffic having an unusually high percentage of trucks and buses, and assuming that the roadway is completely filled, bumper to bumper, the results per 100 vehicles would be approximately as follows:

80 passenger cars @ 3 200 lb = 256 000 lb @ 16 ft = 1 280 ft
5 buses @ 16 000 lb = 80 000 lb @ 26 ft = 130 ft
15 trucks @ 20 000 lb = 300 000 lb @ 26 ft = 390 ft
100 vehicles @ 6 360 lb = 636 000 lb @ 18 ft = 1 800 ft

With traffic moving slowly, the average space between cars would be about 12 ft, or a total of 30 ft per vehicle. With traffic at a standstill, a load of 35 lb per sq ft is possible. With traffic moving at slow speed, this reduces to 25 lb per sq ft.

It is recognized that traffic is not distributed uniformly and that there may be a sequence of heavy trucks. The probability of such a sequence on one part of the bridge with no loading on the other parts is small, but should be considered a possibility. The probability of such a critical (as used in stiffening truss analysis) loading on more than one lane is more remote. Unless deliberate efforts are made to achieve such a result, the writers do not believe there is any chance of a loading heavier than that of an H-20 truck in the lane adjacent to the stiffening truss and 250 lb per lin ft in the other lanes. As applied to customary practice, the foregoing rule would have the effect of increasing the design loads for two-lane bridges and decreasing those for wider bridges. For the Triborough Bridge, it would produce a live load of 1 500 lb per ft of truss in comparison with the 2 000-lb design load.

In the writers' opinion, the question of omitting the stiffening truss (see "Conclusion") should be reframed to read, "How flexible may the stiffening trusses safely be made?" A certain degree of vertical stiffening is always desirable to spread concentrations over several hangers; otherwise, severe distortion may occur at the stringer connections and at the cable bands. The case of a damaged hanger should be considered. Chords are always required for the wind truss and, as a minimum, the material required therefore should be disposed so as to afford as much vertical stiffening as practicable. Vertical stiffness is also desirable to dampen the vertical waves induced by wind gusts.

A suggestion toward a specification for the degree of vertical stiffening has been made by Sterling Johnston, M. Am. Soc. C. E.[21] The writers, however, suggest that the angular change at any panel or suspender point, rather than the change of grade, be considered the criterion for the required stiffness of the system. Further data are required as to the effects of angular changes on the cable.

If a definite specification based on rigidity is established, there will be so many variables in any specific case (such as economical panel arrangement, back-stay conditions, dead loads, etc.), that each structure will be a separate study and no definite rules as to economic truss depths will cover the large range of possibilities.

The writers believe that efforts should be made to reduce the participation of the floor system in stiffening truss action as much as practicable. Usually, it is not economical to combine direct and bending stresses in any member. In most designs, floor expansion joints are introduced to reduce the floor participation. Such joints interrupt the continuity of the pavement and generally involve considerable maintenance. Efforts should be made to reduce the number of such joints to a minimum. Participation from vertical loads may be eliminated by keeping the plane of the stringers at the neutral axis of the truss.

On the basis of the usual calculations, stresses in stiffening trusses are complete reversals. Under these conditions, W. M. Wilson, M. Am. Soc. C. E., has shown[22] that the usual alloy steels have strengths no greater than carbon steel; however, the loadings that produce these stresses are of rare occurrence. Silicon steel costs only slightly more than carbon steel, and for occasional over-stresses, should have an increased strength in the ratio of their yield points. It is believed, therefore, that silicon steel should be generally used for the chords of stiffening trusses.

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[21] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1210.

[22] Rept. on Fatigue Tests of Riveted Joints, by Wilbur M. Wilson, presented at the meeting of the Structural Division, New York, N. Y., January 20, 1938. (Not published.)

The authors have furnished a method whereby comparative studies of any variable can be quickly made. It is better engineering to make these comparisons than to give any direct statement as to the economies of light-weight floors or other detail problems. As long as the designer considers the rigidity furnished in both vertical and horizontal planes by the dead load, the solution can be reached quickly.

The writers trust that this paper will become the basis for a specification for the design of stiffening trusses for vehicular suspension bridges.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

STRUCTURAL BEHAVIOR OF BATTLE-DECK FLOOR SYSTEMS

Discussion

BY H. N. HILL, JUN. AM. SOC. C. E., AND R. L. MOORE, ESQ.

H. N. HILL,[5] JUN. AM. SOC. C. E., AND R. L. MOORE,[6] ESQ. (by letter).[6a]—Having clearly recognized the essential elements of this complicated problem, the authors have proceeded logically toward an experimental study of each element. It seems to the writers, however, that certain phases of the experimental approach and the subsequent analysis of the data are open to question. The writers also feel that a word of caution should be offered against the general use of design methods empirically formulated from a limited number of tests without the benefit of a rational analysis of the problem.

The determination of the load distribution along and between stringers in a panel of battle-deck flooring, subjected to a concentrated load, is indeed a complex problem. Assuming that deflections are produced entirely by bending, the method of approach used by the authors (that of measuring slopes along the stringers and differentiating to obtain moments and shears) is theoretically sound. It is extremely sensitive, however, both to experimental inaccuracies and to slight variations in the manipulation of the slope data in obtaining the derivatives. From the nature of the level-bar readings, it was evidently not possible to obtain the slopes at the extreme ends of the stringers. As a result, the end moments and shears, which were the critical values in determining the end-fixation and load-distribution factors, were obtained from the derivatives of extrapolated curves. Fig. 7(*a*) suggests that the liberties taken in sketching in the slopes at the ends of the panels may have had considerable influence upon the accuracy of the results obtained.

That the bending moments and load-distribution factors determined in this manner are not altogether satisfactory is indicated by certain incon-

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NOTE.—The paper by Inge Lyse, M. Am. Soc. C. E., and Ingvald E. Madsen, Jun. Am. Soc. C. E., was published in January, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[5] Research Structural Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[6] Research Structural Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[6a] Received by the Secretary April 29, 1938.

sistencies found between the loads carried by the different stringers and the corresponding center moments, as shown in Table 1. It would seem from the nature of the problem that the load on the stringers not carrying the wheel-load concentration would be distributed along the span according to the relative deflections of adjacent stringers. Table 1 shows, however, that for the full-sized floor the computed center moments for Stringers *B* and *D*, on either side of the loaded stringer, were greater than would have been obtained had the load computed for these stringers been concentrated at the center of the span.

Although the question of load distribution between stringers is of interest from the standpoint of analysis, the designer is usually concerned only with the maximum intensity of load and the corresponding maximum stresses that may be produced. In every case but one, given by the authors, the computed stresses in the bottom flange of the loaded stringers were less than the measured values, the difference in three cases being more than 20 per cent. As long as load-distribution factors are to be determined experimentally, it would seem that an attempt should be made to make them as consistent as possible with the maximum stresses observed.

Table 1 shows that in every case the computed stresses in the top flange of the loaded stringers (plate stresses, considering T-beam action) were about twice the measured values. Although these stress differences are probably not important from the standpoint of design, the writers believe that they may be attributed very definitely to the small widths of plate computed to be effective in T-beam action. Since the end shears were determined from the second derivatives of the slope curves, multiplied by $E I$, any errors in the slopes, of course, would be reflected in a greatly magnified manner in the I-values. Any tendency to assume greater negative moments at the ends of the stringers than were actually present, as appears to be the case at the left end of Stringer *A* in Fig. 7(*a*), would result in high values for the second derivative of the slope curve and correspondingly low values of $E I$.

The authors conclude that "the width of plate acting with the stringer varies with the stringer spacing." For certain conditions of loading this statement is undoubtedly true; but it should be borne in mind that the effective width is dependent to a much greater extent on other variables. Professor S. Timoshenko[7] has shown by mathematical analysis that for certain types of loading on a wide-flanged T-beam, in which the flange thickness is small compared to the depth of the beam, the effective width of flange is a function of the span length alone. In general, however, the effective width depends not only upon the span length and type of loading, but upon the plate thickness and the elements of the stringer section as well. It may be shown from an extension of Professor Timoshenko's analysis that the theoretical effective width of plate acting with a stringer in the full-sized floor, assuming a concentrated load at the center and a span of 16.75 ft, is about 45 in. Such a width is greater than the stringer spacing, but as long as the loads on all the stringers are not of the same magnitude, there seems to be no reason for placing an arbitrary limit on effective width.

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[7] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill, p. 156.

Table 4 gives section elements for Stringer C of the full-sized floor, assuming effective widths of plate in T-beam action of 17.5 in. and 45.0 in., as computed by the authors and writers, respectively. It is clear from the differences shown that a great increase in the effective width of the plate would have little effect on the calculated stresses in the bottom flange of stringers, but would decrease those at the top considerably.

TABLE 4.—SECTION ELEMENTS FOR STRINGER C OF FULL-SIZED FLOOR

Description	Width of Plate in T-Beam Action, in Inches		Percentage increase
	17.5*	45.0†	
Moment of inertia, in inches^4	427	509	19
Section modulus, in inches^3:			
Top	137	280	104
Bottom	44.7	46.8	5

* From authors' Table 2. † Computed by writers.

Using the value for computed bending moment, given by the authors in Table 1, and an effective width of 45 in., the calculated stress for the top of Stringer C of the full-sized panel would be 2 200 lb per sq in., as compared to a measured stress of 2 100 lb per sq in. The authors' calculated top flange stress for this stringer, using an effective width of 17.5 in., was 4 400 lb per sq in. The curve in Fig. 14, pertaining to the length of plate affected by T-beam action, can scarcely be considered accurate since it was based on values for effective width of plate that are so greatly in disagreement with the top flange stringer stresses, the only measured values which are sensitive to effective widths.

In computing the deflections shown in Table 1, the authors assumed the load on each stringer to be concentrated at the center of the span, despite the fact that the actual load distribution was presumably known from the slope measurements and the computed values of center moments given in the same table did not correspond to such a distribution. It is also apparent that shearing deformations were omitted from the values of computed deflections. The effect of shear on the deflections would not be very great, of course, producing an increase of only about 7% for the full-sized floor. The addition of this percentage, however, would produce a greater discrepancy between measured and computed values for the loaded stringer than that shown. The computed deflection of Stringer C, for instance, would be increased from 0.179 to 0.192 in., as compared to a measured value of 0.170 in. If an effective width of plate of 45 in. rather than 17.5 in. had been used, the computed deflection, including shear, would have been 0.163 in.

During the past seven years (1931–38) the writers have had occasion to make tests on several bridge-floor panels of the battle-deck type, and on numerous other panels in which the action was somewhat similar. All the panels tested were fabricated from high-strength aluminum alloy plates (or sheets) and structural shapes.

From measurements of the bearing areas of dual-tired wheels on large trucks, used to provide load in several field tests, the writers arrived at a laboratory loading to simulate H-20 plus 35% impact. In this loading a total load of 21 600 lb is distributed over two parallel bearing pads which are 8 in. wide and $11\frac{3}{4}$ in. in the direction of traffic. These bearing pads are of felt, 1 in. thick, backed up by steel plates 1 in. thick, and are spaced 12 in., center to center, so that the total width is 20 in. It is believed that this type of loading more nearly simulates the true wheel load than the application of a uniform pressure over a continuous bearing area.

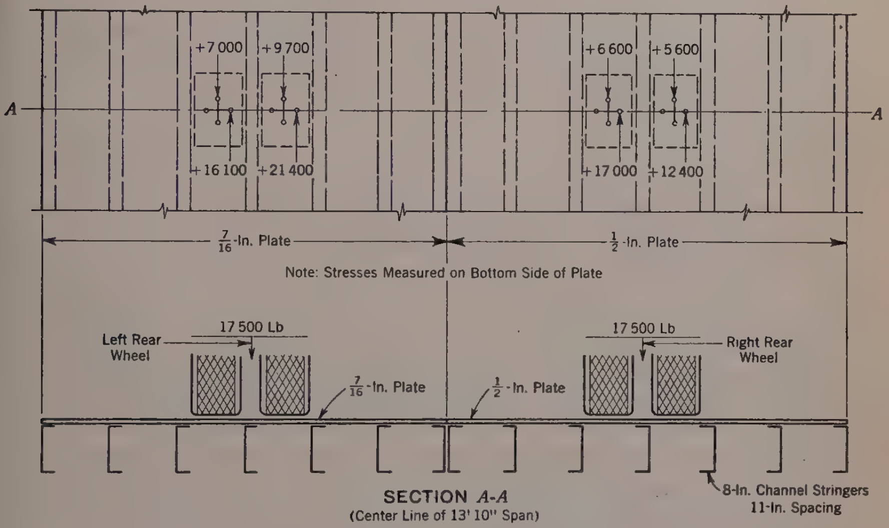

FIG. 15.—MEASURED STRESSES IN PLATES OF BATTLE-DECK BRIDGE FLOOR

It is questionable whether Equation (3), recommended by the authors, is adequate for the design of battle-deck floor systems of proportions different from those tested. The writers have tested floor panels composed of plates $\frac{1}{2}$ in. and $\frac{7}{16}$ in. thick on 8-in. channel stringers, spaced 11 in. apart, loaded with a truck having dual rear wheels and carrying 35 000 lb on the rear axle. With the truck so placed that the tires on each of the rear wheels straddled a stringer, the plate stresses shown in Fig. 15 were measured. It will be noticed that the transverse stresses found under the inside tires were considerably higher than those measured under the outside tires. This was true to about the same degree for both thicknesses of plate. The test was made so that the panel with the $\frac{1}{2}$-in. plate was loaded by the right rear wheel of the truck, whereas the left rear wheel was placed on the panel with the $\frac{7}{16}$-in. plate. This unequal distribution of the wheel load between the inside and outside tires may have been influenced by a number of factors, such as the deflection of the rear axle under load, the air pressures in the tires, and the extent of wear on the tires. Whatever the reason, the fact remains that this unequal distribution was obtained for a typical case of a truck in service and the existence of such a condition should be recognized.

If the wheel loads in Fig. 15 are distributed between the tires in the same proportion as the ratio of the measured transverse stresses, it is found that the inside tires carried about 57% of the total, which is equivalent to a load of 10 000 lb on a bearing area of 8 in. by $11\frac{3}{4}$ in. According to Equation (3), this load would produce maximum transverse stresses of 22 800 lb per sq in. in the $\frac{1}{2}$-in. plate and 30 400 lb per sq in. in the $\frac{7}{16}$-in. plate, which values are 35% to 40% higher than those actually measured. As far as the effect of the load on the action of the plate is concerned, the writers' tests differed in two essential details from the tests described by the authors:

(1) In the writers' tests the load was distributed in such a manner that the plate was loaded on both sides of a stringer, whereas in the authors' tests the load was carried entirely between two stringers. It would seem that this difference might produce the effect of a greater degree of fixation in the plate at the stringer.

(2) The length of the bearing area (in the direction of traffic), expressed in terms of clear distance between stringers, was about three times as great in the writers' tests as in those of the authors'. This greater relative length of bearing area would produce a distribution of longitudinal stress that would be without a sharp peak, such as that shown in Fig. 13(*a*).

The writers do not feel justified in attempting to derive an equation for maximum plate stress that would be generally applicable to this type of construction on the basis of the meager test data available. Because of the small deflections in the writers' tests (as in those of the authors') the load may be assumed to be carried entirely by bending of the plate. In the case of thin plates, however, in which the deflection might be several times the thickness of the plate, a part of the load will be carried by direct stresses, and any equation derived on the basis of bending resistance only would obviously be inadequate.

In discussing the manner in which a load applied over the center stringer is distributed laterally to the other stringers, the authors state (see "Summary," Item 6) that "the distribution factor varies with the thickness of the plate and the distance between the stringers." Nevertheless, in Fig. 14, which is recommended as a basis for design, the percentage of the total load carried by the loaded stringer is plotted against stringer spacing, with no regard to plate thickness. There are at least three other variables which affect the distribution of the load among the stringers: (1) The stiffness of the stringer (or composite plate and stringer section); (2) the width of the panel (or the number of stringers); and (3) the span length of the panel.

Since, as the authors state, the load transferred from one stringer to an adjacent stringer is a function of the difference in the deflections of the two stringers, it follows that if the stiffness of the stringers is increased the deflections will decrease and the condition of very stiff stringers and a relatively thin plate will be approached. In such a case the total applied load would be carried by the loaded stringer, regardless of the stringer spacing.

The effect of the panel width on the part of the load carried by the loaded stringer can be demonstrated by considering a panel with a large number of stringers and of such proportions that the center stringer carries 35% of the applied load, and the two stringers adjacent to the center each carry 15% of the load. The remainder of the load would then be carried by the stringers beyond the center three. If the width of the panel is decreased so that it contains only the center three stringers (which previously carried 65% of the load) they must carry the entire load.

The importance of the span length in determining the manner in which a centrally applied load is distributed among the various stringers can be demonstrated by data obtained by the writers from tests on a bridge-floor panel of the interlocking-channel type. The action of this panel was similar to that of the battle-deck type since the panel was essentially composed of a series of ribs connected by flat plates. The panel (which was 11 ft 6 in. wide) was tested on simple spans of 3-ft, 4-ft, and 5-ft lengths. Load was applied over a bearing area which had a dimension of 12 in. in the direction of the width of the panel. With a rib-spacing of only 7.25 in. and the load centered on the middle rib, it is evident that the loaded area extended almost to the two ribs adjacent to the middle. The proportion of the load carried by each rib for the different span lengths is given in Table 5. These percentages were determined from measured deflections on the basis that the load carried by each of the ribs was proportional to its deflection. Stresses measured on the bottom of each rib were substantially in agreement with such a distribution of the load. It will be noticed in Table 5 that for all spans the outside ribs were unloaded and that the center rib carried about 50% more load with the panel on a 3-ft span than when the panel was tested on a 5-ft span.

TABLE 5.—LOAD DISTRIBUTION IN PANEL OF INTERLOCKING CHANNEL

Span, in feet	PERCENTAGES OF TOTAL LOAD CARRIED BY THE FOLLOWING RIBS, FOR LOAD CONCENTRATED ON CENTER RIB AT MID-SPAN																		
	LEFT OF CENTER									CENTER	RIGHT OF CENTER								
	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9
3..........	0.0	0.0	0.3	0.6	1.3	2.2	4.4	7.9	18.9	31.4	18.6	7.9	3.5	1.6	0.9	0.6	0.0	0.0	0.0
4..........	0.0	0.5	1.0	1.5	2.1	2.6	4.4	9.1	17.8	24.8	18.3	8.3	4.3	2.0	1.5	1.0	0.7	0.2	0.0
5..........	0.0	0.4	1.0	1.6	2.2	2.9	5.0	10.1	17.1	21.6	17.6	8.8	4.7	2.4	1.9	1.3	0.9	0.4	0.0
5*.........	0.1	0.8	1.6	3.2	4.4	6.1	7.3	8.6	11.0	12.2	11.2	9.1	7.8	6.7	4.6	2.7	1.8	0.6	0.1

* Distributor beam at center of span.

In some of the tests made by the writers, one or more "distributor" beams were fastened to the ribs in a direction normal to that of the ribs. The function of these beams was to produce a more favorable distribution of the load among the ribs. The degree in which this was accomplished on the panel of the interlocking channel can be seen from the values in Table 5. With the distributor beam at the middle of the 5-ft span, the center rib carried only about 12% of the total load (as determined from measured deflections), whereas about 22% of the load was carried by the same rib without the distributor beam. Obviously, the effectiveness of the distributor beam will depend on its stiffness.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

WATER-HAMMER PRESSURES IN COMPOUND AND BRANCHED PIPES

Discussion

BY MESSRS. K. J. DEJUHASZ, AND HAROLD A. THOMAS

K. J. DEJUHASZ,[16] ESQ. (by letter).[16a]—For the purpose of analyzing complicated inter-relationships, the graphical method is far superior to numerical computations. The writer's experience with water-hammer is based not on water conduits, but on fuel-injection systems of Diesel engines. These types also involve complicated water-hammer phenomena for which numerical computations are scarcely practicable, although they can be analyzed readily by graphical methods. Therefore, the writer fully endorses the author's preference for the graphical method.

The examples given in this paper treat important practical cases and are highly instructive. In the writer's opinion, however, their clarity would be enhanced by a supplementary diagram, with time as the abscissa and distance as the ordinate, showing when and where the v and h-flow conditions exist. Indexing the v and h-points, as was done by the author, is helpful, but does not tell the entire story. Such distance *versus* time diagrams are easily constructed if the acoustic velocity for the various pipe subdivisions are known. It is admitted, however, that for knowledge of the absolute magnitude of the surge pressure (and this is the most important question), the v, h-chart alone is sufficient.

In dealing with fuel-injection phenomena the writer has also been confronted by the question as to how to represent the pipe friction in the diagram. The author's approach appears workable, but it is open to the objection that the friction is assumed to be localized at definite points in the pipe, whereas actually it is distributed all along the pipe. As yet the writer cannot suggest any definite procedure that would more closely approach the actual conditions. It seems, however, that the friction could be taken into account by varying the slopes of the directrix, that is, using a directrix of greater slope when pres-

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NOTE.—The paper by Robert W. Angus, Esq., was published in January, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

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sure is converted into velocity (because the frictional resistance absorbs part of the pressure energy), and using a directrix of smaller slope when velocity is converted into pressure (because the frictional resistance absorbs part of the velocity energy). Admittedly, such a method would be handicapped by some complications that the author's simplified procedure does not involve, and it is offered solely as another line of attack along which the solution of this question might be attempted.

Transient phenomena in elastic columns constitute a wide group, including, in addition to water-hammer, spring surges, surges in gas columns, and electric transients, all having important applications in engineering. The graphical analysis appears to be a highly useful tool. It is to be hoped that Professor Angus' scholarly paper will help to direct attention to the advantages of the graphical method and serve as an incentive to its application in related engineering fields.

Correction for Transactions: Delete two sentences (and the included footnote) beginning the third line after Equation (10*b*): "Equations (10) have been * * * by Norman R. Gibson, M. Am. Soc. C. E."; and, in the last sentence of the paragraph following Equation (12), omit "and also in the illustration given in the Appendix."

HAROLD A. THOMAS,[17] M. AM. SOC. C. E. (by letter).[17a]—American hydraulic engineers have been rendered a notable service by the publication of this clean-cut and workable graphical method for solving problems of water-hammer in branching and compound pipes. While studying this method the writer has noted that Professor Angus is not correct in stating that the solution of these problems by analytical means is almost impossible (see "Synopsis"). The fundamental equations underlying the method lead to a convenient analytical solution which gives the same results as the graphical one.

Since numerical methods of computation have certain advantages over graphical ones, it seems desirable to present a brief outline of the analytical method for solving problems of the type under consideration. Among the advantages of the analytical method may be mentioned the possibility of obtaining high accuracy by the use of computing machines, the convenience of checking by another person, the keeping of the work on sheets of convenient size for filing and incorporation in reports, freedom from troubles due to paper shrinkage and wrinkling, and freedom from trouble due to having the work extend beyond the boundaries of the paper. In important cases, each method should give a valuable check on the other.

In general, the analysis is applicable to problems involving a valve or gate of variable opening, connected to a group of uniform pipes of various lengths and diameters, these pipes being joined together by two-way or three-way junctions. Before starting the computation, a sketch of the system should be made, each end of each uniform pipe designated by a letter, and an arrow marked on each uniform pipe to indicate the assumed positive direction of flow in that pipe. This direction may be assumed arbitrarily, except that in the case of two pipes in series both arrows must point in the same direction, and in the case of a three-way junction all three arrows must not point either toward or away from the intersection. In certain pipes of the system the flow will have a known direction under a condition of steady discharge with the gate or valve open, and it is convenient to make this the positive direction of flow for those pipes.

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Notation Pertaining to a Frictionless Uniform Pipe.—The notation to be used herein will follow that of the author, with some necessary amplifications. The flow is assumed to be frictionless except where otherwise mentioned. The analytical work is materially simplified by presenting the equations in terms of discharge rather than velocity. A = area of cross-section; V_w = velocity of acoustic wave; T = time to any given instant; $T' = T - \frac{L}{V_w}$ = time to an instant, $\frac{L}{V_w}$ sec earlier; X designates that end of the pipe at which the head and discharge are unknown at a given instant, T; X' designates the other end of the pipe where the head and discharge are known at the earlier instant, T'.

In the following notation for head and discharge the simpler or abbreviated symbols can be used in cases where no ambiguity results:

Symbol	Definition
H, H_X, or H_{XT} =	head at Section X, at time, T.
Q, Q_X, or Q_{XT} =	discharge at Section X, at time, T.
H', H'_X, or H_{XT}' =	head at Section X', at time, T'.
Q', Q'_X, or Q_{XT}' =	discharge at Section X', at time, T'.

The term, "head," at any section of a pipe is used herein to designate the elevation of the center of the pipe above the datum, plus the pressure head at the center of the pipe.

Finally, N = a factor whose value is $+1$ or -1 according to whether the positive direction of flow in the given uniform pipe is toward X or away from X; $S = \frac{N\,V_w}{g\,A}$ = slope of a line in graphical construction (see Fig. 23); and, C = a water-hammer constant, such that,

$$C = H + S\,Q = H' + S\,Q' \quad \text{(54)}$$

Notation Pertaining to a Valve or Gate.—If the gate discharges into the open air, take the datum plane through the *vena contracta* of the jet, and let H and Q be the head and discharge, respectively, just above the gate, whose coefficient of discharge is C_d and whose area of opening is A_g. Then, $Q = C_d\,A_g\,\sqrt{2\,g\,H} = k\,\sqrt{H}$, or,

$$H = \frac{Q^2}{k^2} \quad \text{(55)}$$

in which k is a known function of the time, T.

If the gate discharges under water, assume the datum plane arbitrarily, and let H_X and H_Y be the heads just up stream and down stream from the gate, respectively. In cases where $H_X - H_Y$ is positive, Equation (55) then becomes,

$$Q = k \sqrt{H_X - H_Y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

In order to provide for cases in which the flow through the gate may be reversed so that Q and $H_X - H_Y$ become negative, the relation may be generalized to read: $Q = \frac{k\,(H_X - H_Y)}{\sqrt{|\,H_X - H_Y\,|}}$ or,

$$H_X - H_Y = \frac{Q\,|\,Q\,|}{k^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

in which the symbol, | |, enclosing a given quantity designates the absolute (positive) numerical value of that quantity.

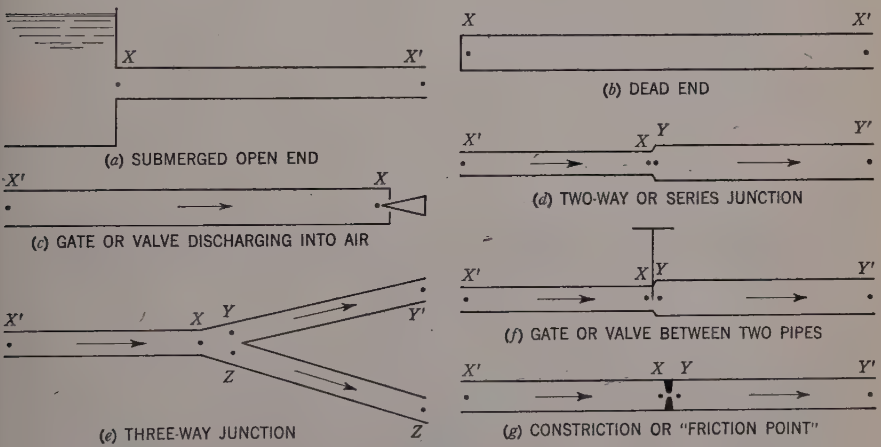
General Relation Between Head and Discharge in a Frictionless Pipe in Which Waves of Pressure Change Are Present.—According to the principles derived in Professor Angus' paper the head, H_{XT}, and the discharge, Q_{XT}, are related as follows to each other and to the head, $H_{X'T'}$, and discharge, $Q_{X'T'}$:

$$H_{XT} + \frac{N\,V_w}{g\,A}\,Q_{XT} = H_{X'T'} + \frac{N\,V_w}{g\,A}\,Q_{X'T'} = \text{constant} \ldots\ldots (58)$$

In the foregoing abbreviated notation, this relation may be written:

$$H + S\,Q = H' + S\,Q' = C \ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

The following paragraphs give the application of this general formula to a variety of special cases.



(*a*) SUBMERGED OPEN END

(*b*) DEAD END

(*c*) GATE OR VALVE DISCHARGING INTO AIR

(*d*) TWO-WAY OR SERIES JUNCTION

(*e*) THREE-WAY JUNCTION

(*f*) GATE OR VALVE BETWEEN TWO PIPES

(*g*) CONSTRICTION OR "FRICTION POINT"

FIG. 21

Submerged Open End; Fig. 21(*a*).—In this case since the head in the open water of the tank or reservoir is always equal to H_0 the head at X at any instant, T, is given by $H = H_0$. Moreover, by Equation (59), $H_0 + S\,Q = C$. Therefore, the discharge at Section X at any instant, T, is given by,

$$Q = \frac{C - H_0}{S} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (60)$$

Dead End; Fig. 21(*b*).—In this case the discharge past the dead end is zero at all times. Therefore, the discharge at Section X at any instant, T, is given by $Q = 0$; also, by Equation (59), the head at X at the instant, T, is given by:

$$H = C \qquad (61)$$

Valve or Gate Discharging into Air; Fig. 21(*c*).—The datum plane is taken through the *vena contracta* of the jet. In this case by Equation (59) and Equation (55):

$$H + S\,Q = C \qquad (62a)$$

and,

$$H = \frac{Q^2}{k^2} \qquad (62b)$$

Therefore, the discharge at Section X at any given instant, T, is given by:

$$Q = \frac{S\,k^2}{2}\left(\sqrt{1 + \frac{4\,C}{S^2\,k^2}} - 1\right) \qquad (63a)$$

and the corresponding head is given by,

$$H = C - S\,Q \qquad (63b)$$

Two-Way or Series Junction; Fig. 21(*d*).—Neglecting the effects of velocity head and energy losses due to sudden contraction or expansion, the head at the junction is the same whether the junction is considered a part of Pipe $X'\,X$, or of Pipe $Y\,Y'$. Therefore, let $H = H_X = H_Y$. Assuming that the positive direction of flow has been chosen the same in both pipes, the law of continuity of flow may be written: $Q = Q_X = Q_Y$. Also, by Equation (59),

$$H + S_X\,Q = C_X \qquad (64a)$$

and,

$$H + S_Y\,Q = C_Y \qquad (64b)$$

Therefore, the discharge and head at the junction at any given instant, T, are given by:

$$Q = \frac{C_X - C_Y}{S_X - S_Y} \qquad (65a)$$

and,

$$H = C_X - S_X\,Q \qquad (65b)$$

Three-Way Junction; Fig. 21(*e*).—Neglecting the effects of velocity head and energy losses due to expansion or contraction, the head at the junction is the same whether the junction is considered a part of Pipe $X'\,X$, Pipe $Y\,Y'$, or Pipe $Z\,Z'$. Therefore, let $H = H_X = H_Y = H_Z$. Assuming that all three arrows do not point either toward or away from the junction, and that the points, X, Y, and Z, are chosen so that branches Y and Z are the two in which the positive directions of flow with respect to the junction are the same, the law of continuity of flow takes the form: $Q_X = Q_Y + Q_Z$. By Equation (59), $H + S_X\,Q_X = C_X$; $H + S_Y\,Q_Y = C_Y$; and, $H + S_Z\,Q_Z = C_Z$. Therefore, at any given instant the head at the junction is given by,

$$H = \frac{\frac{C_X}{S_X} - \frac{C_Y}{S_Y} - \frac{C_Z}{S_Z}}{\frac{1}{S_X} - \frac{1}{S_Y} - \frac{1}{S_Z}} \qquad (66)$$

and the discharges in the three branches are given by,

$$Q_X = \frac{C_X - H}{S_X} \qquad (67a)$$

$$Q_Y = \frac{C_Y - H}{S_Y} \qquad (67b)$$

and,

$$Q_Z = \frac{C_Z - H}{S_Z} \qquad (67c)$$

Gate or Valve Between Two Pipes; Fig. 21(f).—In this case it is considered that the positive direction of flow has been assumed the same in both pipes, and that X and Y are so chosen that the positive direction of flow is from X to Y. Then, at any instant, T, the discharge through the gate is $Q = Q_X = Q_Y$. It follows from Equations (57) and (64) that, at any instant, T, the discharge through the gate is given by:

$$Q = \frac{k^2 (S_X - S_Y)}{2} \frac{(C_X - C_Y)}{|C_X - C_Y|} \left(\sqrt{1 + \frac{4 |C_X - C_Y|}{k^2 (S_X - S_Y)}} - 1 \right) \qquad (68)$$

and the heads just above and below the gate are given by,

$$H_X = C_X - S_X Q \qquad (69a)$$

and,

$$H_Y = C_Y - S_Y Q \qquad (69b)$$

Friction Point; Fig. 21(g).—The approximate friction loss, H_f, in any uniform pipe is considered to be $j Q^2$, in which Q is the discharge at the middle of the pipe at the given instant and j is the coefficient of Q^2 in the ordinary pipe-friction formula,

$$H_f = f \frac{L}{D} \frac{V^2}{2g} = f \frac{L}{D} \frac{Q^2}{2 g A^2} \qquad (70)$$

For purposes of computation the loss is assumed to be concentrated in a very short distance, $X Y$, at the mid-section of the pipe, the letters being so chosen that the positive direction of flow is from X to Y. Therefore, $Q = Q_X = Q_Y$ and $S = S_X = -S_Y$. In order to provide for possible reversal of the direction of flow in the pipe, the relation between lost head and discharge is written in the generalized form, Equation (57), $H_X - H_Y = j Q |Q|$. By Equation (64), $H_X + S Q = C_X$ and $H_Y - S Q = C_Y$. Therefore,

$$Q = \frac{S (C_X - C_Y)}{j |C_X - C_Y|} \left(\sqrt{1 + \frac{j |C_X - C_Y|}{S^2}} - 1 \right) \qquad (71)$$

$$H_X = C_X - S Q \qquad (72a)$$

and,

$$H_Y = C_Y + S\,Q \qquad (72b)$$

It is to be noted that the effect of a "friction point" is exactly the same as that of a constriction at the mid-section of the pipe. If it is desired to compute the friction loss in a uniform pipe with more refinement than this, the pipe may be regarded as divided into several segments and a "friction point" located at the middle of each. The value of j for each segment will then be based on the length of that segment. If two segments of equal length are used, the two constrictions producing the friction loss will thus be located at the quarter-points of the original pipe. Professor Angus' method of finding the friction head for this case, as shown in Fig. 20, is slightly different from the foregoing, his two constrictions being at the one-third points instead of at the quarter-points. Where several segments are used, the method outlined herein is more accurate than that used by Professor Angus, since it makes the stepped hydraulic grade line conform more closely to the true linear hydraulic grade line. However, in many problems, the difference of the results obtained by the two methods would be inappreciable.

Solution of Problems.—The foregoing equations are sufficient for the solution of all ordinary problems of water-hammer in compound and branching pipes. The work is greatly facilitated by tabulating the numerical quantities in a systematic manner. A suitable arrangement for the computations is suggested in the following illustrative problem. In general, a separate table must be arranged for each of the following kinds of points occurring in the given pipe system: submerged open end, dead end, valve or gate, series junction, and three-way junction. If the effects of friction are to be considered, one or more "friction points" are placed in each uniform pipe, and a table is included for each of these points.

When the blank tables have been prepared and the initial values of head and discharge have been written into the tables in their proper places, the remainder of the computation becomes automatic. The values of head, H, and discharge, Q, computed in one table for a given instant, T, at one end of one of the uniform pipes, can always be inserted in another table as the values of H' and Q' for the abutting end of an adjacent uniform pipe. Repetition of this process solves the problem.

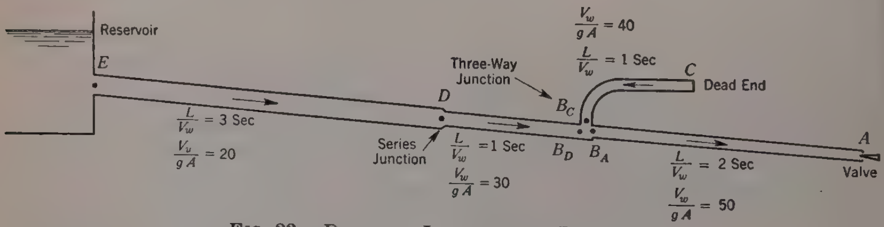


FIG. 22.—DATA FOR ILLUSTRATIVE PROBLEM

Illustrative Problem.—For this problem (see Fig. 22) $H_0 = 100$ ft; and $Q_0 = 10$ cu ft per sec; therefore, $k_0 = \dfrac{Q_0}{\sqrt{H_0}} = 1$. The valve is closed at a uniform rate in 10 sec; and, therefore, $k = 1 - \frac{T}{10}$. Values of head and discharge are to be computed at 2-sec intervals at the various sections designated by letters in Fig. 22. Friction is neglected. For this problem the foregoing general formulas reduce to the following special ones:

For Section A, a valve discharging into air, $X = A$; $X' = B_A'$; $N = 1$; $S = 50$; $C = H' + 50\,Q'$; and $k = 0.1\,(10 - T)$:

$$Q = 25\,k^2\left(\sqrt{1 + \frac{C}{625\,k^2}} - 1\right) \qquad (73a)$$

and (see Equations (63)),

$$H = C - 50\,Q \qquad (73b)$$

Equations (73) are solved in Table 1.

TABLE 1.—SOLUTION OF EQUATIONS (73); SECTION A, FIG. 22; A VALVE DISCHARGING INTO AIR

Item No.	Section-instant X' T'	B_A 0	B_A 2	B_A 4	B_A 6	B_A 8
1	Discharge, Q'	10	10	8.97	7.53	5.28
2	$50\,Q'$	500	500	448.5	376.5	264.0
3	Head, H'	100	100	117.5	153.3	198.4
4	C (Item No. 3 plus Item No. 2)	600	600	566.0	529.8	462.4
5	$k = 0.1\,(10 - T)$	0.8	0.6	0.4	0.2	0
6	$25\,k^2$ (see Equations (73))	16	9	4	1	0
7	$625\,k^2$	400	225	100	25	0
8	$C \div 625\,k^2$	1.500	2.667	5.66	21.19	....
9	$\sqrt{1 + \text{Item No. 8}} - 1$	0.582	0.915	1.581	3.70	....
10	Q = Item No. 9 × Item No. 6 (see Equation (73a))	9.31	8.23	6.32	3.70	0
11	$50\,Q = 50$ × Item No. 10	465.5	411.5	316.0	185.0	0
12	$H = C - 50\,Q$ = Item No. 4 minus Item No. 11	134.5	188.5	250.0	344.8	462.4
Item No.	Section-instant X T	A 2	A 4	A 6	A 8	A 10

For Section B, a three-way junction (see Fig. 22):

$X = B_A$; $Y = B_C$; $Z = B_D$; $C_X = H_X' - 50\,Q'_X$;
$X' = A$; $Y' = C$; $Z' = D$; $C_Y = H_Y' + 40\,Q'_Y$;
$N_X = -1$; $N_Y = 1$; $N_Z = 1$; $C_Z = H_Z' + 30\,Q'_Z$;
$S_X = -50$; $S_Y = 40$; $S_Z = 30$; and (see Equations (66) and (67)):

$$H = \frac{C_X}{-3.916} + \frac{C_Y}{3.132} + \frac{C_Z}{2.349};$$

$$Q_X = \frac{C_X - H}{-50}; \qquad Q_Y = \frac{C_Y - H}{40}; \qquad \text{and} \qquad Q_Z = \frac{C_Z - H}{30}.$$

The foregoing equations are solved in Table 2.

TABLE 2.—SOLUTION OF EQUATIONS FOR THREE-WAY JUNCTION

Item No. (1)	Section-instant (2)	$X' T'_X$ (3)	$Y' T'_Y$ (4)	$Z' T'_Z$ (5)	A 2 (6)
1	Discharge, Q'	Q'_X	Q'_Y	Q'_Z	9.31
2	S	−50	40	30	−50
3	$S Q'$ (Item No. 2 × Item No. 1)	$-50 Q'_X$	$40 Q'_Y$	$30 Q'_Z$	−465.5
4	Head, H'	H'_X	H'_Y	H'_Z	134.5
5	C (Item No. 4 + Item No. 3)	C_X	C_Y	C_Z	−331.0
6		3.916	3.132	2.349	3.916
7	Item No. 5 ÷ Item No. 6	....	....	....	−84.6
8	Head, H	H	H	H	117.5
9	Item No. 5 minus Item No. 8	$C_X - H$	$C_Y - H$	$C_Z - H$	−448.5
10	Discharge, Q (Item No. 9 ÷ Item No. 2)	Q_X	Q_Y	Q_Z	8.97
Item No.	Section-instant	$X T$	$Y T$	$Z T$	B_A 4

TABLE 2.—(*Continued*)

Item No. (1)	C 3 (7)	D 3 (8)	A 4 (9)	C 5 (10)	D 5 (11)	A 6 (12)	C 7 (13)	D 7 (14)
1	0	10	8.23	0	9.30	6.32	0	7.72
2	40	30	−50	40	30	−50	40	30
3	0	300	−411.5	0	279.0	−316.0	0	231.6
4	100	100	188.5	135.1	114.0	250.0	171.3	145.6
5	100	400	−223.0	135.1	393.0	−66.0	171.3	377.2
6	3.132	2.349	3.916	3.132	2.349	3.916	3.132	2.349
7	31.9	170.2	−57.0	43.2	167.4	−16.9	54.7	160.6
8	117 5	117.5	153.3	153.3	153.3	198.4	198.4	198.4
9	−17.5	282.5	376.8	−18.2	239.7	264.4	−27.1	178.8
10	−0.44	9.41	7.53	−0.45	7.98	5.28	−0.68	5.96
Item No.	B_C 4	B_D 4	B_A 6	B_C 6	B_D 6	B_A 8	B_C 8	B_D 8

For Section C, a dead end (see Equation (61)): $X = C$; $X' = B$; $N = -1$; $S = -40$; $Q = 0$; and $H = C = H' - 40\,Q$. The solution is arranged for convenience in Table 3.

TABLE 3.—SOLUTION OF EQUATIONS FOR DEAD END

Item No.	Section-instant $X' T'$	B_C 4	B_C 6	B_C 8
1	Discharge, Q'	−0.44	−0.45	−0.68
2	$-40\,Q'$	17.6	18.0	27.2
3	Head, H'	117.5	153.3	198.4
4	Head, H (Step (3) plus Step (2))	135.1	171.3	225.6
5	Discharge, Q	0	0	0
Item No.	Section-instant $X T$	C 5	C 7	C 9

For Section D, a series junction:

$X = D_E$; $Y = D_B$; $C_X = H'_X + 20\,Q'_X$;
$X' = E$; $Y' = B_D$; $C_Y = H'_Y - 30\,Q'_Y$;
$N_X = 1$; $N_Y = -1$;
$S_X = 20$; $S_Y = -30$; and (see Equations (65)):

$$Q = \frac{C_X - C_Y}{50}; \qquad \text{and,} \qquad H = C_X - 20\,Q.$$

The solution of these formulas is given in Table 4.

TABLE 4.—SOLUTION OF EQUATIONS FOR SERIES JUNCTION

Item No.	Section-instant...	$X'\ T'_X$	$Y'\ T'_Y$	*E* 2	B_D 4	*E* 4	B_D 6	*E* 6	B_D 8
1	Discharge, Q'....	Q'_X	Q'_Y	10	9.41	10	7.98	10	5.96
2	S..............	20	−30	20	−30	20	−30	20	−30
3	$S\,Q'$ (Item No. 2 × Item No. 1)..	$20\,Q'_X$	$-30\,Q'_Y$	200	−282.3	200	−239.4	200	−178.8
4	Head, H'........	H'_X	H'_Y	100	117.5	100	153.3	100	198.4
5	C (Item No. 4 plus Item No. 3).....	C_X	C_Y	300	−164.8	300	−86.1	300	19.6
6	$C_X - C_Y$ (difference in Item No. 5).............	$C_X - C_Y$	....	464.8	....	386.1	....	280.4	....
7	Discharge, Q (Item No. 6 ÷ 50)....	Q	....	9.30	....	7.72	....	5.61	....
8	$20\,Q$ (20 × Item No. 7)..........	$20\,Q$	....	186.0	....	154.4	....	112.2	....
9	Head, H (C_X, Item No. 5 − Item No. 8).........	H	....	114.0	....	145.6	....	187.8	....
Item No.	Section-instant...	$X\ T$	....	*D* 5	....	*D* 7	....	*D* 9	....

For Section E, a submerged open end, at reservoir: $X = E$; $X' = D$; $N = 1$; $S = -20$; $C = H' - 20\,Q'$; $H = 100$; and (see Equation (60)), $Q = \dfrac{C - 100}{-20}$. The solution is arranged as in Table 5.

TABLE 5.—SOLUTION OF EQUATIONS FOR A SUBMERGED OPEN END, AT RESERVOIR

Item No.	Section-instant $X'\ T'$......................	*D* 5	*D* 7	*D* 9
1	Discharge, Q'...........................	9.30	7.72	5.61
2	$-20\,Q'$................................	−186.0	−154.4	112.2
3	Head, H'...............................	114.0	145.6	187.8
4	C (Item No. 3 plus Item No. 2)............	−72.0	−8.8	75.6
5	C (Item No. 4 − 100)....................	−172.0	−108.8	−24.4
6	Q (Item No. 5 ÷ − 20)...................	8.60	5.44	1.22
7	Head, H................................	100.0	100.0	100.0
Item No.	Section-instant $X\ T$......................	*E* 8	*E* 10	*E* 12

Tables 1 to 5 are arranged so that the known values, Q' and H', appear at the top of each column and the unknown values, Q and H, appear at the bottom. The term, "section-instant," is used to denote a given place and time. Thus, in Table 5, at the section-instant D 5, the discharge, Q', is 9.30 cu ft per sec. The letter at the bottom of the first numerical column in each table designates the given section, and the numeral designates the instant 2 sec after the first impulse from the valve movement reaches the given section. For example, at Section E the initial conditions remain unchanged for 6 sec after the valve movement begins. Therefore, in the table for Section E (Table 5) the symbol, E 8, is written at the bottom of the first numerical column. The numerals at the bottom of the remaining columns (or groups of columns) of each table increase by 2-sec increments. The letters and numerals at the top of each column are determined from those at the bottom. Thus, in Table 5, the symbol, D 5, is written at the top of the first numerical column, because Section D is at the far end of the uniform pipe terminating in Section E, and an impulse requires 3 sec to travel from D to E.

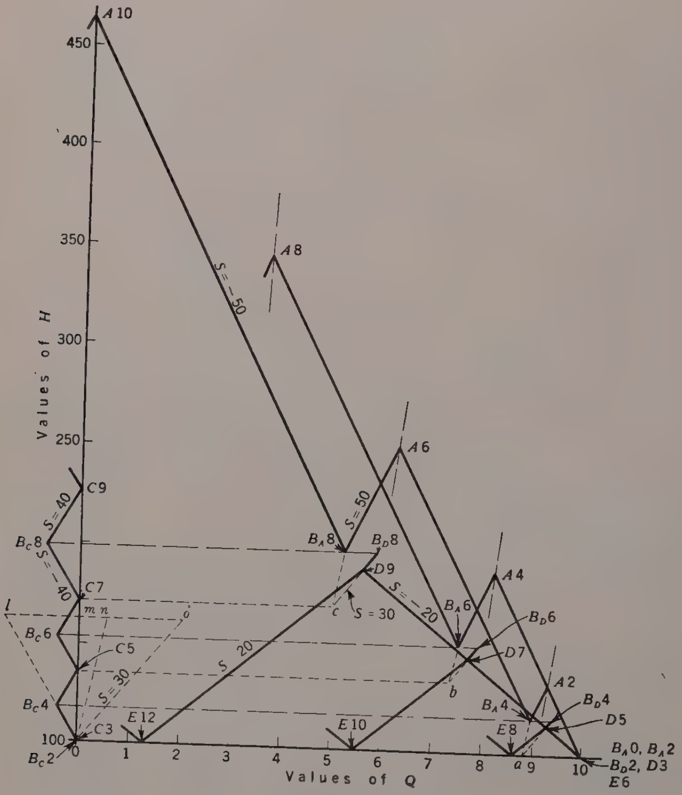


FIG. 23

The computation is started by writing into the tables the known initial head of 100 ft for section-instants, B_A 0, B_A 2, C 3, D 3, E 2, E 4, E 6, E 8, E 10, and E 12; the known initial discharge of 10 cu ft per sec for section-instants, B_A 0, B_A 2, D 3, E 4, and E 6; and the known initial discharge of 0 cu ft per sec for instant-points, C 3, C 5, C 7, and A 10.

The values of head and discharge are now computed for all the remaining section-instants in the following order:

A 2
A 4
A 6
B_A 4
B_A 6
B_A 8
B_C 4
B_C 6
B_C 8
B_D 4
B_D 6
B_D 8
C 5
C 7
C 9
D 5
D 7
D 9
E 8
E 10
E 12, etc.

A schedule for each individual computation is indicated in the tables. The numerical work in this example has been extended to the tenth second at Section A, and by means of the same process it can be carried to any desired subsequent even second.

Comparison with Graphical Solution.—A solution of the foregoing problem by Professor Angus' graphical method, slightly modified, is shown in Fig. 23. It is seen that the analytical and graphical methods give identical results. In this diagram the abscissas are plotted in terms of discharge rather than of velocity, in conformity with the principle given in Equation (59). This device of expressing the flow in terms of discharge rather than velocity is very helpful in clarifying the general analysis of water-hammer problems in compound and branching pipes, since it eliminates the use of complicated velocity relationships such as those described by the author in connection with Equation (37). However, it does not change the shape of the final graphical figure.

Professor Angus is to be complimented especially on the elegance of his graphical solution for the problem of the three-way junction. Practice in the use of this device suggests modifications for cases in which the construction falls beyond the boundaries of the paper. Such a modification is shown in Fig. 23. On the arbitrarily located horizontal line, *ol*, lay off *on* equal to *ml* and draw a line from C 3 to n. The direction of the latter defines the direction of the lines which solve the three-way junction problem by joining a to B_A 4, b to B_A 6, and c to B_A 8.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

STREAM POLLUTION IN THE OHIO RIVER BASIN

A SYMPOSIUM

Discussion

By Messrs. W. H. Wisely, F. C. Dugan, G. M. Ridenour, Arthur W. Baum, Jr., G. R. Scott, and Don E. Bloodgood

W. H. Wisely,[39] Assoc. M. Am. Soc. C. E. (by letter).[39a]—One of the authors, Mr. Streeter, does well to place emphasis on the need for consideration of stream usage in the planning and interpretation of stream studies to be used as bases for pollution-control programs. It is often also advisable to give consideration to the possibility that a stream may be developed in the future for functions not utilized at present. Rock River, in Northern Illinois, for example, is potentially a fine recreational stream, within 100 miles of Chicago. At present (1938), however, it is not extensively developed for this purpose because several other streams offering vacational advantages are closer to Chicago. In consideration of present pollution abatement requirements in the Rock River Basin, the Illinois Sanitary Water Board recognized this probable future recreational development and any stream studies will be planned with this in mind.

The appropriate engineering analogy between the river system under effective pollution control and the well-designed bridge, which is used illustratively by Mr. Streeter, can be extended somewhat further. Determination of the degree of treatment required at each source of possible pollution may be likened to the design of the structural members of the bridge, under-treatment representing a weak member affecting the usefulness of the entire structure, and over-treatment representing wasteful design.

Note.—The Symposium on Stream Pollution in the Ohio River Basin was presented at the meeting of the Sanitary Engineering Division at Pittsburgh, Pa., on October 14, 1936, and published in January, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1938, by Messrs. D. E. Davis, L. S. Morgan, C. A. Holmquist, and Robert Spurr Weston; and May, 1938, by Messrs. Karl Imhoff, Henry D. Johnson, Jr., Charles M. Reppert, Edwin K. Morse, James H. Le Van, and John C. H. Lee.

[39] Asst. San. Engr., State Dept. of Public Health, Div. of San. Eng., Springfield, Ill.

[39a] Received by the Secretary April 8, 1938.

Where industrial pollution is of major importance in a river study, it is advisable to determine the actual operating times of the various industries during the year previous to the sampling period, in order to evaluate, properly, the residual pollution in the form of sludge deposits that may be present when sampling is begun. During sampling of the Illinois River below Peoria, Ill., and Pekin, Ill., in July and August, 1936, industrial pollution representing a combined population equivalent of 789 000 entered the river. However, several industries had operated during the winter and spring preceding the sampling period but suspended operations prior to July 1. Wastes from the latter represented an additional population equivalent of 375 000. Accordingly, it was necessary to modify computations involving the river data to allow for sludge deposits in the river during the sampling period not accountable to the pollution actually discharged during July and August.

Mr. Streeter's discussion of criteria for permissible limits of pollution of streams used for sources of water supply, commercial fishing, and industrial development is highly practical. However, there is a need for similar practical criteria to govern streams used for recreation, particularly bathing, as this usage requires much more rigid pollution control than any other.

Bacterial studies on the Upper Fox River in Illinois, in a reach used extensively for bathing and recreation, revealed that *coli-aerogenes* concentrations of 10 per 100 ml to 28 700 per 100 ml were present at some of the most popular bathing places. The program of restoration of this stream involved the requirement of complete treatment, with bathing-season effluent chlorination, at all sources of human sewage pollution so that the foregoing maximum value has been greatly reduced. Nevertheless, there may still be some question regarding the absolute protection of bathers due to combined sewer overflows and similar sources of intermittent pollution, evidencing a need for definite interpretations of bacterial analyses of the waters from this stream.

The following facts must be given consideration in the development of criteria for permissible pollution limits of streams used for bathing:

(1) A stream offering recreational advantages by virtue of its proximity to metropolitan areas and natural environment will be used for bathing almost entirely without regard for the actual existing pollution (it is very doubtful that policing of such streams as a public health measure would be practicable);

(2) It is not possible, from a practical standpoint, to maintain the water, continuously, at natural bathing places, in such condition that it will meet the usual requirements for swimming-pool waters; and,

(3) In addition to the intestinal bacteria hazard, the presence of bacteria capable of producing skin, ear, eye, and throat infections warrants consideration.

Appreciating the need for practical interpretations of bacterial concentrations in stream waters used for bathing, the Great Lakes and Upper Mississippi River Boards of Public Health Engineers have adopted by resolution, the following tabulation, indicating the "Significance of the *B. Coli* Index as a Measure of Water Pollution":[40]

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[40] Rept. on Coastline Pollution Surveys of Michigan, Michigan Stream Control Comm., June, 1933.

Significance of Index	*B. Coli* Index, or *B. Coli* per 100 cu cm of sample
Water sample free of pollution	0
Treasury Department standard for drinking water	Not more than 1
Indicative of good water—normal for inland and Great Lakes, free of sewage pollution	10 to 100
Normal for inland streams, free of detrimental sewage pollution, might be attributed to land wash	100 to 500
Suspicious—generally indicates mild pollution in natural waters, but dangerous in proximity to fresh sewage pollution	1 000
Definite evidence of fresh sewage pollution—menace to health	10 000
Heavy sewage pollution—definitely dangerous	100 000
Normal sewage	1 000 000, or more

The foregoing classification is based on analysis of considerable data assembled by the Michigan Department of Health and the Michigan Stream Control Commission.

F. C. DUGAN,[41] M. AM. SOC. C. E. (by letter).[41a]—Stream-pollution control in the Ohio River Basin cannot be accomplished by any one State acting alone; it can result only from co-operative action by all of the fourteen States that lie partly in the Ohio River drainage area.

The *B. Coli* index of the raw water determined at the various water-works intakes along the Ohio River clearly shows that there is, at times, a very thin line of defense against outbreaks of water-borne diseases. At Ashland, Ky., it is noted that during the twelve years, 1926–1938, there were only a few months during which the average *B. Coli* index of the raw water was not greater than that which is considered as the upper limit of pollution that a water filtration plant should be required to handle. During and preceding the outbreak of gastro-enteritis, which occurred during the great drought of 1930–1931, there was not the slightest indication from the bacteriological analysis of water supplied to the inhabitants of several cities in Kentucky that the outbreaks resulted from impure water. However, it is now believed by many that the water supplies were responsible and that the causative agent was some toxin, of either chemical or bacteriological origin, which resulted from the excessive pollution by sewage and industrial wastes and which was not removed by any known method of water purification.

It is possible that the acid wastes, especially in the Pittsburgh area, have prevented local nuisances from occurring, but certainly the discharge of acid wastes is causing serious economic loss because of damaging effects on steel hulls of boats, pipe lines, concrete, and other structures, and because of the increased cost of water treatment and destruction of fish life. It is known that the bulk of these acid wastes come from abandoned coal mines, and it is essential that the mine-sealing program now (1938) in progress in several States be continued until these acid wastes are reduced to a minimum.

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[41] Chf. Engr., State Board of Health, Louisville, Ky.

[41a] Received by the Secretary April 23, 1938.

The canalization of the Ohio River, although it is of benefit in some respects, has intensified the pollution problem and has increased the difficulty of treating the water satisfactorily for domestic and industrial uses.

Several of the States, acting under the authority granted by a Joint Resolution passed by the Seventy-fourth Congress, have appointed commissioners who are now (1938) at work drafting a compact which will eventually lead to definite plans for pollution control in the Ohio River basin.

G. M. Ridenour,[42] Assoc. M. Am. Soc. C. E. (by letter).[42a]—So thoroughly has Mr. Streeter covered the mechanics of stream-pollution control surveys that little seemingly remains to be said in this respect. To the writer the paper clearly emphasizes five important facts: (1) That effective pollution-control results can be secured only by general control of water-shed areas rather than by local sections; (2) the extensive scope necessarily implies the establishment of large-scale technical organizations for each specific drainage area; (3) the eventual establishment of regulation by either intra-State compacts or by Federal bodies; (4) the important part that both applied and pure research must of necessity play in such stream-pollution surveys; and (5) the question as to when stream-pollution surveys become necessary.

Control by water-shed area is evidently the only logical method for securing effective, consistent, and economical solution of stream-pollution problems. Broad control of this kind seems to be without doubt the only fundamental method of attack. It is common experience to all those interested in stream pollution to observe cases of over-treatment and under-treatment of sewage and wastes in the same river basin within comparatively short distances, with attendant economical loss in both cases. Such situations have arisen not through intent, but through a combination of lack of adequate knowledge of river-purification capacities and long-range planning for the specific water-shed under consideration. Fundamental knowledge of river-purification characteristics and capacities is the first requisite of intelligent planning and must be obtained through more comprehensive and embracing information than can be secured from over-burdened local sections. Treatment can be distributed economically only through a combination of knowledge of river-purification capacities and long-range planning in waste treatment. Mr. Streeter's plan of pollution-survey control meets both these requirements.

The maintenance of complete technical organizations for each specific drainage area or river basin is also an integral part of the program. Each basin will require the same routine control measures, regardless of the degree of pollution, and will likewise embrace the same basic branches of the science.

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[42] Asst. Prof., Dept. of Water Supplies and Sewage Disposal, Rutgers Univ.; Research Engr., Dept. of Water and Sewage Research, Agri. Experiment Station, New Brunswick, N. J.

[42a] Received by the Secretary April 28, 1938.

The establishment of intra-State or Federal regulatory bodies in river pollution is only a question of time. Pollution control can only be limited, temporarily, to overburdened parts of a drainage area. The ultimate direction for most successful accomplishment of the stream pollution survey control outlined by Mr. Streeter appears to lie in a national policy-making organization and local intra-State or Federal regulatory bodies applying to each specific water-shed area. The problem cannot escape a national scope without simply producing a geographical exaggeration of present local conditions.

A phase of the program which the writer feels could be more fully stressed is the important part that research should play in such stream-pollution control surveys. This research can be directed, very profitably, to increasing certain fundamental knowledge of stream-purification agencies and principles, which can be obtained only in conjunction with large-scale programs of the kind outlined.

Of greater practical significance and of vital necessity in any stream-control survey is the part that research could play in determining, more accurately, the permissible limits for pollution control. The interest in this phase of the problem has occupied second place in relation to treatment plant devices and efficiencies, with the result that wide divergencies in stream-pollution criteria now exist under different regulatory bodies. These different requirements are sufficient to make wide differences in waste-treatment requirements, under these different regulations. This applies particularly to the maximum bacterial pollution permissible for natural bathing and recreational areas, and also to minimum dissolved oxygen requirements to prevent nuisance and maintain fish life in the stream.

The pollutional indices for drinking water supplies might also be profitably subject to further scrutiny. Under growing quantities of industrial and domestic wastes, the use of the *coli* index alone for determining the suitability of a water supply may prove inadequate for determining the quality of the water. From the health standpoint increasing concentrations of both industrial and domestic wastes in the water supplies open questions as to pathological consequences of increasing quantities of organic and inorganic toxins and their limiting tolerances. This may apply particularly in cases of the more heavily industrialized and populated sections where, by continuous re-use of water, the organic and inorganic salt contents are being continuously built up in spite of present purification methods. Indices for palatableness of water have also become an important part of water standards and promise to become increasingly so in the future. Taste, odor, and physical appearance are receiving almost equal attention to bacterial qualities under present standards of most water plants, and future improvement in this non-pathological direction will depend entirely on the development of facilities, through research, to attain these ends. The source of water supply is of equal importance to the water purification process in securing better physical and sensual qualities.

Of further practical value, research may show ways and means of improving the stream as a purification unit, depending on the fullest utilization of natural local conditions of the drainage area. The receiving stream should properly be conceded as an integral unit in the waste disposal system and as occupying as important a place in the system of waste treatment as any other treatment plant unit; and as such, it deserves the same attention toward improving operation efficiency as other plant units.

The same program research can yet play a profitable part in the design and operation of waste treatment devices to secure more economical methods of treatment. This is particularly true with respect to industrial wastes, which (economically, at the present time) appear to offer the largest unsolved problem in waste treatment. There are yet many local "sore spots" within the treatment devices themselves and it seems the doubtful part of wisdom to fail to consider inefficiencies in waste treatment processes as other than an integral problem in river pollution control. The ultimate economical solution of pollution control will not be realized until the stream, on the one hand, has been made to perform its maximum work as described by accurate pollution criteria; and, on the other hand, when the methods of waste treatment are developed to their maximum efficiency.

A final question which the term, "stream pollution control," naturally suggests is, "When does a pollution control survey, as outlined, become necessary?" The first evident reason for control measures is found naturally in those cases of grossly over-polluted streams. A second, less evident, reason is found in the border line case where the conditions are approaching pollutional limits. Probably the least evident reasons are apparent in those cases of streams that are still in satisfactory condition for normal water uses, but which by potential development possibilities may be subject to large pollutional loadings in the future. From the magnitude of the problems mentioned by Mr. Streeter in the case of the Ohio River and the tremendous simultaneous economic problem involved in the case of cleaning up of grossly over-polluted streams, it would seem that the future direction of stream control surveys should be not only toward the correction of streams already grossly over-polluted, but also toward proper maintenance of river basins that are still below such conditions.

ARTHUR W. BAUM, JR.,[43] ESQ. (by letter).[43a]—The statements by Mr. Wolman that "the intensity of stream pollution is not nation-wide. It is concentrated largely in five or six of the heavy industrial States," are debatable. The relativity of intensity varies with opinion and many people of the Pacific Northwest believe that their pollution situation is also intense. For example, Oregon certainly is not an industrial State and is surely not over-populated—with slightly more than a million people in 96 000 sq miles; but it also has its pollution problems, which are of major importance.

In the industrial States the main pollution problems center around the use of the larger polluted rivers as sources for a water supply for localized population. In Oregon, the pollution problem is studied primarily for the harmful effects it has on the fishing industry (both commercial and sport) and on the public health. With ample sources of water supply available for practically every city in Oregon, it is seldom necessary to use a known polluted stream for a water supply source.

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[43] Senior in Civ. Eng., Oregon State Coll., Corvallis, Ore.

[43a] Received by the Secretary April 13, 1938.

The greatest difference in the pollution problem in one part of the United States as compared to another is represented in one's definition of "intensity." In the East, where the salmon runs have disappeared, the pollution problem is not intense to-day unless it interferes with a water supply source. The Pacific Northwest has great natural resources, especially from the viewpoint of those interested in sports and recreation, and derives a large income from this asset. More and more people are going there to enjoy the recreational facilities and the fishing offered by the streams, and as the present pollution situation is harmful to fish life and is damaging one of the region's best assets, as well as a multi-million dollar commercial industry, the pollution problem is certainly intense.

As approximately 64% of the population of Oregon resides in the Willamette River water-shed, and as the Willamette River is one of the largest tributaries of the Columbia River, it is only natural that a pollution problem is centered in this district. This area has been surveyed several times to determine the intensity of pollution present at that period when conditions are such that they should be at their worst.

The pollution situation is not a year-round problem of the same intensity, and it is only during late June, July, August, and early September of average years when there is only a precipitation of about 2 in. (and this is received in light showers), that the situation is critical. During this period the Willamette River has a minimum flow and the canneries and most other industries are running at capacity. From Salem to Portland, the oxygen content of the river gradually decreases until, in Portland Harbor, surveys show the water to have a zero oxygen content and a bacterial count at a point dangerous to the public health.[44] This "critical zone" can be considered with its two most harmful effects. It ruins a recreational area close to the major population by subjecting participants in water sports to disease and not only prevents the anadromous type of fish from passing on to the head-waters of the Willamette, but offers an insurmountable barrier to the young fish trying to reach the ocean and continue their natural cycle. Experiments have shown that a stream must have an oxygen content bordering on 3.5 ppm to sustain game fish life [45] and, if a stretch of water is present in which the dissolved oxygen content is below this point, it means suffocation to those fish trying to pass through this zone. This stretch of water in its present condition will have the similar effect on anadromous fish as cutting the bark completely off the trunk of a tree—the tree soon dies.

G. R. Scott,[46] Assoc. M. Am. Soc. C. E. (by letter).[46a]—The well-written paper by Mr. Streeter, based on detailed studies and observations of the Ohio River during the 24 yr, 1914–1938, gives a complete description of the basic data which should be collected and correlated prior to the establishment of standards of purity or requirements designed to prevent over-pollution of any river system. Instead of attempting any unnecessary restatement of the subject material in his paper, this discussion is presented solely to emphasize the need for such studies, the desirability of making them before serious conditions exist, and a general statement of the work now (1938) under way in the Tennessee Valley.

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[44] "A Sanitary Survey of the Willamette River from Sellwood Bridge to the Columbia River," *Bulletin Series, No. 6*, April, 1936, Eng. Experiment Station, Oregon State Agricultural Coll., Corvallis, Ore.

[45] "Stream Pollution in Wisconsin," Wisconsin State Board of Health, p. 249.

[46] San. Engr., Health and Safety Dept., TVA, Chattanooga, Tenn.

[46a] Received by the Secretary May 11, 1938.

The natural resources of an area, whether they are the raw materials used in making finished products, or the fuel or energy necessary for the conversion, have always been the controlling factors in its development. For most industries water in some form is a necessity. Water used in the process, as steam or as the primary source of power, is usually considered a natural resource; but when used to carry away wastes, it is regarded as a fortunate convenience only—an agent that happens to be present and that is to be used for the purpose of carrying this waste material away from back doors. Many municipalities, depending as they do upon industries for financial support and population development, while at the same time providing them with their source of labor and personnel, look upon these streams with a view not much more enlightened. As a result, it is not long before such streams become little more than open sewers, having lost their valuable resources—those attributes or characteristics natural to fresh, clean streams. The water is still there and proceeds normally down its channel as before. In some cases it can be used again by others down stream after extensive and costly treatment; but in other cases it is so grossly polluted that its re-use is impracticable. This loss to the stream is accompanied by the loss of another quality, formerly considered of little value, but now being appreciated for its potentialities. This is the loss, to the area as a whole, of natural areas affording opportunity for those wholesome recreational activities which are so necessary in this energetic age for rebuilding or reconditioning the human machine so that it may be able to continue the increasing tempo of the "tread-mill." This idea, although it is persistently coming to the light, is being continually darkened by the argument that it is more important for work to be provided for the many, than that a few shall be favored by having full creels. This contention is granted; but, in general, it is quite probable that both arguments can be satisfied.

It seems logical to assume that those who can develop an intricate process of manufacture also should have the ability to devise satisfactory ways and means of disposing of waste products. Methods are already available for the disposal of most wastes and are being adopted by many industries. Others continue to discharge raw untreated wastes into running streams with the excuse that the cost of doing otherwise is prohibitive. It is recognized that the cost of known methods for the treatment of wastes from certain industries is exorbitant, and it is also realized that other industries are so handicapped by their locations that the costs of the treatment of their wastes are abnormally high. These industries should be assisted, rather than discouraged by any sanitation program.

If possible, the streams should be surveyed and studied before over-pollution occurs and the unregulated discharge of polluting wastes becomes an established practice. The data discussed by Mr. Streeter should be gathered and assembled to form a base line of existing conditions and to provide a foundation upon which to build the necessary regulatory measures. Comprehensive plans for the proper use and conservation of this natural resource, for the prevention of its thoughtless destruction, will be far less expensive and much easier of enforcement than the later remedial measures. In the preparation and establishment of these standards and regulations, due regard should be given to the economics of the problem. It should be realized that the true development of the industries and the advancement of the area are mutually dependent upon a well-rounded program in which all factors are given full consideration. It will not be just to require that all wastes be purified to meet certain standards regardless of cost; on the other hand, the uncontrolled discharge of raw untreated wastes should not be tolerated.

For the proper solution of this problem, much research work will be necessary; co-operative studies must be undertaken in which the industries should be willing to assist. By combined effort and co-operation, new solutions may be found. Although industries have made many investigations resulting in benefits to themselves by the recovery of products formerly wasted, and in benefits to the public in the way of cleaner streams, they still have a long and difficult way to go, and a way that can best be traveled at the moderate pace now possible before speed is made necessary by an aroused public through national legislation.

Early in the history of the Tennessee Valley Authority, the need for the collection of these basic data was felt by the Health Departments of the seven Valley States. At their request, the Authority has undertaken the study with the advice of the U. S. Public Health Service and with the co-operation and assistance of the States. Although the area cannot be considered intensively industrialized at present, certain areas are highly developed and the uncontrolled discharge of wastes is seriously polluting a few of the major tributaries. With the expected development of this area, peculiarly rich in a wide range of natural resources, additional streams will be ruined and conditions aggravated unless uniform procedures applying to the entire Valley are effective. This interstate nature of the problem was an added factor which prompted the States to request the Authority to direct the studies and to act as a clearing house for the assembled information.

The work was begun in 1936. All information pertinent to the study was collected from the various State Health Departments. This consisted primarily in the extent of sewers in the various towns in the Valley. No information as to the extent or character of industrial wastes, or on the present conditions of the streams, was available. It was necessary, therefore, to obtain and correlate this information before true conditions could be pictured. At present (1938), an approximate list of industries has been collected; some have been studied in detail; and plans for the future contemplate the examination of a sufficient number of the remainder to complete the necessary data. Four sections of the Tennessee River, from which weekly samples were collected from a sufficient number of stations, to define conditions of pollution and recovery, were studied for a period of one year. These sections were at

Knoxville and Chattanooga, Tenn., and at Decatur and below Wilson Dam, in Alabama. On the completion of these studies, similar studies were undertaken on the main tributaries, some by the States, and others (for example, all field and laboratory work), by the Authority.

As in other areas where such studies have been conducted in co-operation with industries, it has been necessary to develop mutual confidence before the full value of the study can be obtained. Although the program has progressed somewhat slowly, research problems in co-operation with one of the industries will be undertaken this summer (1938).

In the execution of this work, it is the desire of all concerned to use the resulting information co-operatively and constructively in such manner as to be compatible with the achievement of the end in view, namely, the control of the discharge of wastes and sewages to insure a balanced development of this resource; that it will not be rendered unfit for re-use by municipalities and industries located down stream; that the maximum of fish life may develop; and that a sufficient number of areas safe for recreational purposes may be maintained. Although the progress of this program, as indicated by visible results, may be somewhat slower than that resulting from more drastic action, it is expected that its achievements will be more substantial and enduring. In addition, it is hoped that this project, having an early beginning before serious conditions develop and embracing as it does the entire Tennessee Valley, will demonstrate the value of such co-operative policies and action.

DON E. BLOODGOOD,[47] ESQ. (by letter).[47a]—The effect of sunlight upon the dissolved oxygen in the stream is an important factor, often overlooked when samples are taken. Samples taken on White River below the City of Indianapolis, Ind., show that the dissolved oxygen may vary as much as 1.0 ppm from one side of the stream to the other, if the one side is shaded and the other is in the sun. This effect is only noted when the river is in a favorable condition for algæ growths. Samples taken at midday may have as much as 3.0 to 4.0 ppm of dissolved oxygen whereas, at night, it may drop to zero. It can be seen that samples taken during the day may not indicate the extent of pollution in a stream if the plant life in the water is abundant.

For a number of years the movement of the river water has been studied by determining the chloride concentration in the stream. The chlorides that are not removed by the purification plant pass on into the stream and, because they fluctuate from low to high with the peak flow of sewage, they are easily traceable in the stream when the ratio, of effluent to river flow, is low.

The comments by Mr. Streeter on the retardation of re-aeration by the presence of sewage in a stream are very interesting. In some work done in 1935[48] the first indications were that the saturation point of sewage with oxygen was less than that of water. This was disproved later by a further study which showed that the saturation point of sewage was the same as that of water if determined by a modified Winkler method, in which the demand for iodine was first satisfied. Whether the rate of re-aeration was slower was never determined.

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[47] Supt., Dept. of Sanitation, City of Indianapolis, Ind.

[47a] Received by the Secretary May 17, 1938.

[48] Rept. of Dept. of Sanitation, City of Indianapolis, Ind., March 25, 1935.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

RELATIVE FLEXURE FACTORS FOR ANALYZING CONTINUOUS STRUCTURES

Discussion

BY MESSRS. A. FLORIS, D. B. HALL, RALPH W. HUTCHINSON, ARTHUR B. McGEE, AND E. NEIL W. LANE

A. FLORIS, ESQ.[28] (by letter).[28a]—In the proposed method for analyzing continuous beams and rigid frames by means of the relative flexure factors, only the influence of the bending moments upon the deformation of the structure is taken into account. The influence of the shearing forces is neglected. This is the usual procedure in the analysis of continuous structures. However, cases may arise in practice, in which a knowledge of the influence of the shearing deformation is not only desirable but necessary. This is especially true in short bars of considerable depth.

In the following discussion the author's analysis will be modified in such a manner as to include the shearing deformation of structures composed of bars with constant section. This is done very easily, because all the necessary data for this purpose are already available.[29]

In developing his theory the author uses the distances of the centers of gravity of the moment areas from the ends of the bars, the stiffness factors of these bars, and the fixed-end moments. By considering the shearing deformation of the structure the distances of the centers of gravity from the ends of the bars (Fig. 17) are: $x_1 = 2\lambda$; and, $x_2 = 2\zeta$, in which,

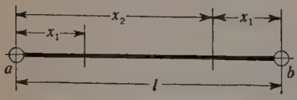

FIG. 17.—CENTROIDS OF JOINT MOMENT AREAS WITH SHEARS

$$\lambda = \frac{l}{6} - \nu \frac{\gamma}{l} \qquad (9a)$$

and,

$$\zeta = \frac{l}{3} + \nu \frac{\gamma}{l} \qquad (9b)$$

NOTE.—The paper by Ralph W. Stewart, M. Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1938, by Messrs. Frederick Shapiro, and Dean F. Peterson, Jr.; and May, 1938, by Messrs. George W. Housner, Homer M. Hadley, Adolphus Mitchell, John B. Wilbur, and Leon Blog.

[28] Dipl.-Ing., Los Angeles, Calif.

[28a] Received by the Secretary April 8, 1938.

[29] "Shearing Deformation in Continuous Beams and Rigid Frames," by A. Floris, *Journal*, Am. Concrete Inst., November–December 1937, p. 165.

In Equations (9), $\gamma = \frac{I}{A}$; I = moment of inertia; A = sectional area; and ν = 3.0 for steel and, approximately, 2.8 for concrete.

The stiffness factors in this case are for unyielding supports,

$$K = I \frac{\zeta}{\zeta^2 - \lambda^2} \qquad (10a)$$

and, for yielding supports,

$$K = \frac{I}{(\lambda - \zeta)\, l} \qquad (10b)$$

In both cases the carry-over factor will be,

$$\beta = \frac{\lambda}{\zeta} \qquad (11)$$

By considering the shearing deformation of the structure, the fixed-end moments are expressed by,

$$S_{ab} = \frac{Q_r\, \zeta - Q_l\, \lambda}{(\zeta^2 - \lambda^2)\, l} \qquad (12a)$$

and,

$$S_{ba} = \frac{Q_l\, \zeta - Q_r\, \lambda}{(\zeta^2 - \lambda^2)\, l} \qquad (12b)$$

in which Q_l and Q_r are the static moments of the moment area of the freely supported bar relative to the left and right supports, respectively. For symmetrical loadings the fixed-end moments in Equations (12) reduce to the ordinary fixed-end moments, found in almost all handbooks on structural engineering.

For a concentrated load, P, at a distance, l_1, from the left and l_2, from the right supports, for instance, these moments are,

$$Q_l = \frac{P\, l_1}{6} (l^2 - l_1^2) \qquad (13a)$$

and,

$$Q_r = \frac{P\, l_2}{6} (l^2 - l_2^2) \qquad (13b)$$

If several concentrated loads, unsymmetrically spaced, are acting on the bar, the statical moments are found by substituting the values in Equations (13). It should be noted in passing that, if the structure is composed of different materials, the stiffness factors should be multiplied by the modulus of elasticity of the material, everything else remaining the same.

In all the foregoing expressions, λ and ζ are taken from Equations (9). By neglecting the last terms of this equation containing ν and γ and showing the influence of the shearing deformation, the expressions used by the author are obtained immediately.

The further analysis of the structure follows the steps that were so ably outlined by Mr. Stewart. For this reason a repetition of this procedure is not necessary.

In conclusion, it should be emphasized that the proposed method, in addition to its theoretical interest, has an intrinsic practical value. Dispensing with the inconvenient simultaneous equations and by the aid of few physical concepts and pure geometrical relationships, the author succeeds in solving, in a natural, speedy, and elegant manner, problems of marked complexity. In certain cases, the method is almost without rival.

D. B. HALL,[30] ASSOC. M. AM. SOC. C. E. (by letter).[30a]—In this paper the author further demonstrates, by new applications, the usefulness of his conception of the elastic curve as a traverse.[2] The first requisite in applying the methods of this paper, or in developing and applying still different methods, is a thorough grasp of this principle. The property of the traverse used in the paper (which makes it at once so simple and yet effective for solving continuous structures, and which has been extensively used but not explicitly stated by the author), is this: In a traverse such as Fig. 18, Angles a and d measure the rotations at the ends of the member; and, Angles b and c measure the moments at the ends of the member. Thus, the simple geometrical relationship among these four angles fully defines the elastic properties of the beam as it acts in a structure.

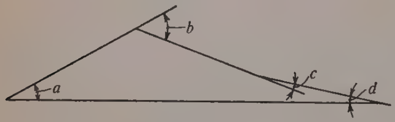

FIG. 18

The author has mentioned the fact that many complicated structures may be analyzed by considering only the members in the vicinity of any particular point, but he has devoted his paper mainly to exact solutions, as was appropriate for purposes of demonstration. From a practical standpoint the approximate solutions may be the more important.

Consider the closed box in Fig. 5. If End B, of Member BC, is given a rotation of 1, regarding it as the free end of a four-span structure, and then the traverse is traced all the way around to Point B in Member AB, the rotation at that point will be 142, and the moment, 1 × 246. Although a theoretical analysis of the structure should take into account the net rotation of Member BA with respect to Member BC, it is evident that, for all practical purposes, the stiffness of Member BA is $\frac{246}{142}$. The stiffness of Member BC found similarly by starting with a unit rotation at Point B in Member BA, is $\frac{2 \times 123}{74.5}$. It may be of academic interest, but it is scarcely worth remembering, that the true relative stiffness factors are $\frac{246}{141}$ and $\frac{2 \times 123}{73.5}$, since the rotation of one member with respect to the other is 1 less than its absolute rotation. Using one less span each way (that is, going around clockwise from C to B, or counter-clockwise from A to B) gives, for Member $B\,A$, $\frac{45}{26}$; and, for Member

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[30] Detailer, Ash-Howard-Needles & Tammen, New York, N. Y.

[30a] Received by the Secretary April 14, 1938.

[2] "Analysis of Continuous Structures by Traversing the Elastic Curves," by Ralph W. Stewart, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105.

BC, $\frac{2 \times 33}{20}$, values which differ from the preceding by about 1 per cent. Thus, although a theoretically exact solution for the closed box required quite a little ingenuity, a practically exact solution presents no special difficulty.

Ralph W. Hutchinson,[31] Assoc. M. Am. Soc. C. E. (by letter).[31a]—The relative flexure method of analyzing continuous structures presents a direct solution of these structures which may be applied efficiently to continuous-span bridges. Since haunched and tapered beams are used freely in continuous bridges, the determination of the properties of the members becomes an important detail of the analysis. Many computers use prepared charts or tables to obtain the properties of these beams for use with the familiar methods of analysis. The lack of such charts prepared for the relative flexure method need not discourage the use of the method as a tool because, by simple inter-relations, the position of the points of inflection and the stiffness factors of haunched and tapered beams may be obtained quickly from the tables and charts in common use for obtaining the properties of these beams, which are used in connection with the method of end-moment distribution.

When a moment is applied to one end of a beam which is hinged at the other end, the ratio of the rotation at the opposite ends of the beam is the carry-over factor, γ. Furthermore, $\frac{x\,l}{l - x\,l} = \gamma$; or, $x = \frac{\gamma}{1 + \gamma}$, which locates the point of inflection. For the same beam the angle of flexure, Δ (Fig. 19), is the sum of the rotations of the two ends of the beam and is equal to $(1 + \gamma)\,\theta_L$ in which θ_L is the rotation at the applied moment.

Inversely, the stiffness for flexure, $K\,F = \frac{K\,C}{1 + \gamma}$, in which the product, $K\,C$, is the stiffness of a free-ended beam as used in end-moment distribution. Furthermore, for symmetrical beams the stiffness, $K\,F = K\,(1 - \gamma)$, in which K is the stiffness of the fixed beam as used in end-moment distribution. For tapering beams there will be two carry-over factors and, therefore, two stiffness factors and the points of inflection will not be symmetrical.

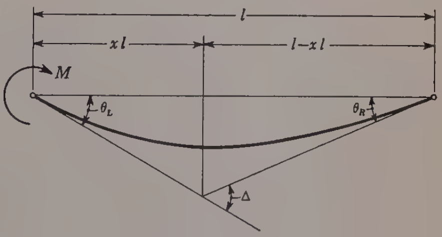

FIG. 19

Using the charts prepared by Thor Germundsson, M. Am. Soc. C. E.,[32] in 1932, the properties of the haunched beams from the author's illustrated example, Fig. 4(*a*), can be readily obtained. From the charts $\gamma = 0.71$; and, $K\,C = 6.2$. The point of inflection is at $x = \frac{0.71\,l}{1.71} = 0.415\,l$, which checks the value of $0.41\,l$ given by the author. The stiffness factor, $K\,F = \frac{6.2}{1.71} = 3.62$. The actual stiffness is $\frac{3.62 \times 2.25^3}{54} = 0.765$ or, as stated by the author, the relative $\Delta = \frac{M}{0.765} = 1.308\,M$, which checks the value of $1.292\,M$ given by the author.

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[31] Associate Engr., Bridge Dept., State Div. of Highways, Sacramento, Calif.

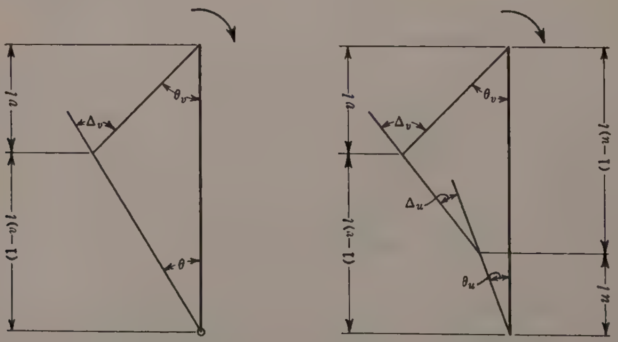
[31a] Received by the Secretary May 2, 1938.

[32] *Civil Engineering*, October, 1932, p. 647.

The relative flexure method is most efficient when applied to a series of continuous spans on flexible piers which are subject to moving loads. For analyzing a structure of this type the method appears to be faster than any other. For continuous spans on stiff piers it requires the introduction of another flexure system, that of the pier, and thus becomes inferior to the end-moment distribution method which gives moments in the piers from the beam moments without additional effort or complications. Hinged beams can also be solved readily by end-moment distribution with no more effort than that required for a solid beam whereas the solution by the relative flexure method appears to be excessively cumbersome.

The case of continuous spans on flexible piers appears to the writer to be the only one in which the relative flexure method has any advantages over the common methods of solution. It is the writer's opinion that only those computers who frequently handle this type of structure should attempt to master the method. Those who work with this type of structure infrequently can best solve it by one of the methods with which they are already familiar.

ARTHUR B. MCGEE,[33] ESQ. (by letter).[33a]—The analysis of partial fixture, in this paper, is very brief. Partial fixture of a member with variable moment of inertia, and treatment of a moment applied to the frame at a point of partial fixture, are not included.



(a) (b)

FIG. 20

The writer prefers to omit the idea of an imaginary member and deal directly with the angle of rotation of the partially fixed end. Fig. 20(a) shows a "traverse of the elastic curve" of a member with variable moment of inertia, hinged at the lower end. Fig. 20(*b*) shows a traverse of the same member with the lower end partially restrained. The terms, u and v, are identical with those same terms in the tables by Walter Ruppel, Assoc. M. Am. Soc. C. E.[34] The fixture factor, r, as used herein, is defined by:

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[33] Structural Engr. in Bridge and Structural Design Div., Bureau of Eng., City of Los Angeles, Los Angeles, Calif.

[33a] Received by the Secretary May 11, 1938.

$$r = 1 - \frac{\theta_u}{\theta} \qquad (14)$$

in which θ is the angle of rotation of the hinged end, and θ_u is the angle of rotation of the restrained end (see Fig. 20). When the end is hinged, θ_u equals θ and $r = 0$. When the end is fixed, $\theta_u = 0$ and $r = 1$. The fixity thus varies from 0 (hinged) to 1 (fixed); 75% fixture means that $r = 0.75$ and the partially fixed end rotates through one-fourth the angle of a hinged condition.

To draw a traverse involving partial fixture and to evaluate the angles, it is only necessary to find the relation of θ_u to Δ_u (Fig. 20(*b*)). From Fig. 20(*a*):

$$\theta = \frac{v\,l}{(1-v)\,l}\,\theta_v = \frac{v}{1-v}\,\theta_v \qquad (15a)$$

from Fig. 20(*b*):

$$\theta_v \times v\,l = \theta_u\,(1-v)\,l + \Delta_u\,[(1-v)\,l - u\,l] \qquad (15b)$$

and,

$$\theta_v = \frac{1-v}{v}\,\theta_u + \frac{1-v-u}{v}\,\Delta_u \qquad (15c)$$

Substituting θ_v in Equation (15*a*):

$$\theta = \theta_u + \frac{1-v-u}{1-v}\,\Delta_u \qquad (15d)$$

and, substituting θ in Equation (14):

$$\frac{\theta_u}{\Delta_u} = \frac{(1-u-v))\,(1-r)}{(1-v)\,r} \qquad (16)$$

Equation (16) is the general formula of the relation between the angles, θ_u and Δ_u. A member with a uniform moment of inertia has values of u and v equal to $\frac{1}{3}$, and Equation (16) is simplified to,

$$\frac{\theta_u}{\Delta_u} = \frac{1-r}{2\,r} \qquad (17)$$

which is the same formula as that given in Mr. Stewart's paper. When a moment is applied at a partially fixed end, a part of the moment is taken directly by the support and may be treated as a fixed-end moment not affecting any other joint in the frame. The part of the moment that is not taken by the support goes back into the frame. It is not necessary to make a traverse starting from the point of partial fixture to distribute this moment. Merely transfer this moment to the far end of the member by multiplying it by $\frac{v}{1-v}$, the usual carry-over factor, and subtracting or adding the result to the existing moment at the far end, depending on whether there is an odd number or an even number (including zero) of points of inflection between the ends.

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[34] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 167–187.

The problem shown in Fig. 21(*a*) will now be worked in detail. Fig. 21(*b*), (*c*), (*d*), (*e*), shows the properties of the members. Each Δ equals the area of an

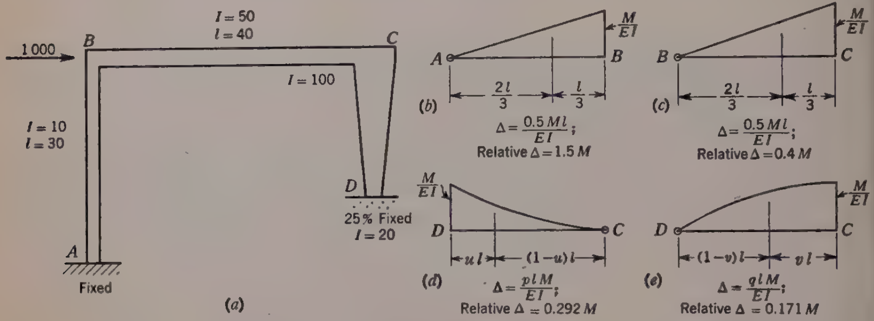

FIG. 21

$\frac{M}{E\,I}$-diagram. Thus, in the traverse an angle represents an area. The formulas for Δ may be found in a discussion by Fang-Yin Tsai, Assoc. M. Am. Soc. C. E.[35] The values of u, v, p, and q are taken from Ruppel's tables.[34] In Fig. 22(*a*) apply an unknown moment at Joint *B* as shown by the arrow. Rotate Joint *B* and start drawing the traverse from that point, progressing both ways to Points *A* and *D*. Angle points occur at each corner and opposite each center of gravity of the $\frac{M}{E\,I}$-diagrams. The lines of the traverse are tangent to the elastic curve at the joints, but the middle section on each side is not tangent to the elastic curve. The numbers in parentheses are moments; the other numbers are angles; and, ratios of Δ-angles to moments are given in Fig. 21(*b*), (*c*), (*d*), (*e*).

Solving Equation (16) gives $\theta_u = 1.691\ \Delta_u$. Now, start at Point *D* letting θ_u, the angle of rotation, equal 1.691 and the next angle, 1. Then, $1 \div 0.292 = (3.425)$; $[1.691 \times l + 1 \times (0.427 + 0.323) \times l] \div 0.427 \times l = 5.717$; $5.717 - 1 - 1.691 = 3.026$; $5.717 \div 0.171 = (33.433)$; $33.433 \times 0.4 = 13.373$; $13.373 \times 1 + 3.026 \times 2 = 19.425$; $19.425 + 13.373 + 3.026 = 35.824$; $35.824 \div 0.4 = (89.56)$; $19.425 \times 2 = 38.85$; $38.85 \div 1.5 = (25.90)$; $19.425 \div 1.5 = (12.95)$; and, $25.90 + 89.56 = (115.45)$.

The traverse in Fig. 22(*b*) is drawn in a similar manner. Begin assigning angle values at End *A* and progress to Corner *C* whose angle of rotation is found to be 2.533. The angles from Point *C* to Point *D* must be proportional to those in Fig. 22(*a*) since $\frac{\theta_u}{\Delta_u} = 1.691$ as before. Therefore, the angles will be $\frac{2.533}{3.026}$ times the values in Fig. 22(*a*).

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[35] *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 129.

Fig. 23(*a*) shows the frame deflected sidewise with the top member straight and both legs fixed at the bottom. The deflection, δ, is the same in each leg so that the relative moments may be found for this condition. End D, however, being only 25% fixed will have a value of 0.25 × 530 = 132.5; 0.75 × 530 × $\frac{v}{1-v}$ = 296, which is carried over to End C, and, 905 − 296 = 609, the moment at End C when End D is 25% fixed. These moments are shown on the frame in Fig. 23(*b*). Now, Member $B\,C$ is allowed to deflect; and, the moment at Joint B is distributed by using $\frac{66.7}{115.46}$ of the moment values in Fig. 22(*a*) and the moment at Joint C is distributed by using $\frac{609}{38.165}$ of the moment values in Fig. 22(*b*). Signs of the moments are readily seen by referring to the arrows on the traverses, clockwise being positive. The arrows are always placed so that they tend to straighten the elastic curves. Shears in the legs of Fig. 23(*b*) now show the moments to be due to a horizontal force of 18.27. Therefore, multiply all moments by $\frac{1\,000}{18.27}$ and find the moment at End A = 3 822; Joint B = 3 994; Joint C = 9 934; and End D = 4 855.

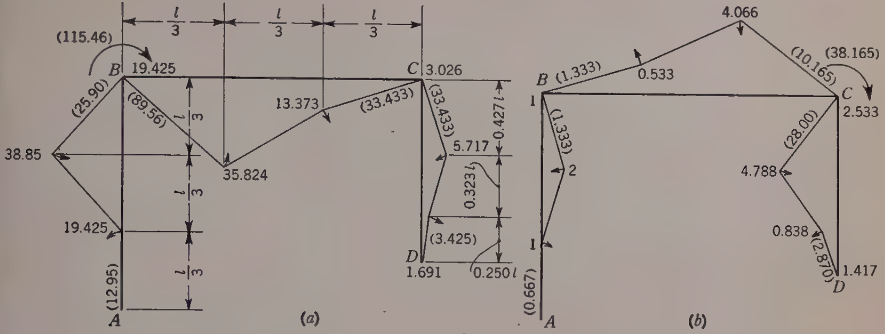

FIG. 22

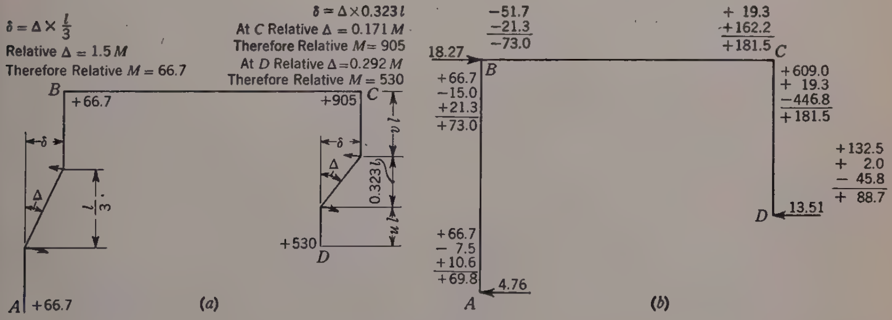

FIG. 23

From preference, the writer has been using the Stewart traverse method since about 1936, for continuous and rigid frame bridges.

E. Neil W. Lane,[36] Jun. Am. Soc. C. E. (by letter).[36a]—The use of relative flexure factors in the analysis of continuous structures, as described by Mr. Stewart, is simple and fundamental in theory. It is a general method for attacking the solution of moments in rigid frameworks that is particularly valuable for use in conjunction with moving and variable load systems. Combining bending moments by superimposing influence diagrams aids in visualizing the elastic behavior of a structure. Moreover, the process is quite time-saving.

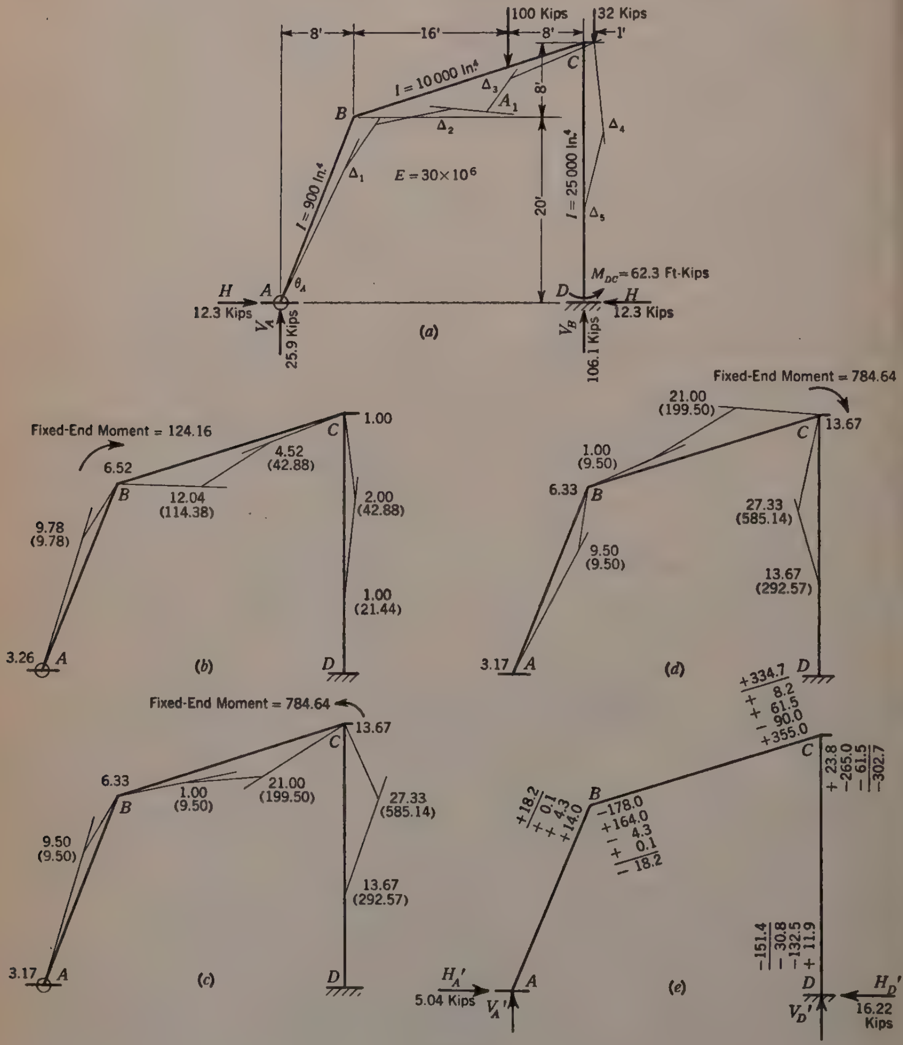

Fig. 24

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[36] Bridge Detailer, State Dept. of Roads and Irrig., Lincoln, Nebr.

[36a] Received by the Secretary May 16, 1938.

The author's method somewhat parallels the steps in the slope deflection method in that the process is indirect. Angles are found first, and then substituted in the original equations to obtain the desired moments. Similarly, the author borrows from moment distribution in his procedure of correcting for side-sway. Trial moments are found first, to which corrections are added to satisfy statics.

In solving for moments under a single fixed condition of loading, the writer prefers the method known as traversing the elastic diagram that was published by the author in 1934.[37] The method of relative flexure factors is an outgrowth and adaptation from the previous paper. It is believed that the evolution of the new method will be of interest. As a means of widening the scope of the author's present work, an example has been chosen to show the solution for moments in a frame with sloping members (see Fig. 24(*a*)).

The solution by the original method requires eight equations, the first of which traverses the elastic diagram:

$$\theta_A + \Delta_1 + \Delta_2 - A_1 + \Delta_3 + \Delta_4 - \Delta_5 = 0 \qquad (18)$$

in which, $A_1 = 0.00307$ radians. Equation (18) is analogous to others frequently used in elastic energy theory.[38] The next two equations are statements of relative stiffness:

$$\Delta_1 \left(\frac{900}{258.5} \right) = \Delta_2 \left(\frac{10\,000}{304.6} \right) \qquad (19a)$$

and, $M_{CB} = 384\,000 + M_{CD}$; or,

$$\Delta_3 = 0.000194 + 2.26\,\Delta_4 \qquad (19b)$$

The vertical deflection of Point A relative to Point C is zero:

$$32\,\theta_A + 26\tfrac{2}{3}\,\Delta_1 + 16\,\Delta_2 - 10\tfrac{2}{3}\,A_1 + 8\,\Delta_3 = 0 \qquad (19c)$$

The horizontal deflection of Point A relative to Point D is also zero:

$$-13\tfrac{1}{3}\,\Delta_1 - 22\tfrac{2}{3}\,\Delta_2 + 24\tfrac{4}{9}\,A_1 - 25\tfrac{1}{3}\,\Delta_3 - 18\tfrac{2}{3}\,\Delta_4 + 9\tfrac{1}{3}\,\Delta_5 = 0 \qquad (19d)$$

Now, to obtain the sixth equation, three member equilibrium equations must be solved simultaneously. These are: $M_{CD} + M_{DC} - 336\,H = 0$, or,

$$10^6\,(\Delta_4 + \Delta_5) - 0.0752\,H = 0 \qquad (19e)$$

$M_{BC} - M_{CB} + 96\,H - 288\,V_A + 96 \times 10^5 = 0$; or,

$$10^6\,(\Delta_2 - \Delta_3) + 0.0485\,H - 0.1455\,V_A + 4\,850 = 0 \qquad (19f)$$

and, $M_{BA} - 240\,H + 96\,V_A = 0$; or,

$$10^6\,\Delta_1 - 1.153\,H + 0.461\,V_A = 0 \qquad (19g)$$

Slide-rule values give $H = 12.3$ kips; $V_A = 25.9$ kips; and, $M_{DC} = -62.3$ ft-kips.

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[37] "Analysis of Continuous Structures by Traversing the Elastic Curves," by Ralph W. Stewart, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 105.

[38] "Elastic Energy Theory," by J A. Van den Broek, M. Am. Soc. C. E. (1931), p. 81, Equation III.

By using relative flexure factors the moment coefficients, as shown in Figs. 24(*b*), 24(*c*), and 24(*d*), are obtained. These are relative moments, immediately adaptable and easily checked for any system of loading. The moments, uncorrected for side-sway, are shown in Fig. 24(*e*). By the foregoing procedure, Equations (18), (19*a*), and (19*b*) have been eliminated and the problem simplified. The true moment at the end of each member will consist of the uncorrected moment, plus or minus some correction. Let these corrections be: X at Point B; Y at Point C; and, Z at Point D. Then, the solution of the remaining five equations will furnish the desired results. Equations (19*e*), (19*f*) and (19*g*) then become,

$$16\,220 - 0.00298\,Y - 0.00298\,Z - H = 0 \ldots\ldots\ldots\ldots (20e)$$

$$60\,440 - 0.01042\,X = 0.01042\,Y + H - 3.0\,V_A = 0 \ldots\ldots (20f)$$

and,

$$910 - 0.00417\,X - H + 0.40\,V_A = 0 \ldots\ldots\ldots\ldots (20g)$$

For convenience, Equations (19*c*) and (19*d*) were replaced. The fourth equation is a statement of a statement of moments about Point D: $M_{DC} + 9\,216\,000 - 384\,V_A = 0$; or,

$$28\,730 - 0.00261\,Z - V_A = 0 \ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

The fifth formula is a statement equating the vertical deflection of Point A when released to the opposite deflection caused by the redundant reaction:

$$1.372 + 0.00484\,X + 0.00278\,Y - 0.00133\,Z = 0 \ldots\ldots\ldots (21b)$$

This is a means of correcting for side-sway in any frame.

It is worth while to note that the elastic energy theory probably furnishes the most simple and direct solution for this type of problem. When used with area moments, Equation (21*b*) becomes:

$$0.212\,V_A - 0.385\,H - 0.00103\,M_{DC} - 2.19 = 0 \ldots\ldots\ldots (22a)$$

Using a statement of horizontal deflections similar to Equation (22*a*):

$$0.288\,V_A - 0.594\,H - 0.00024\,M_{DC} - 1.19 = 0 \ldots\ldots\ldots (22b)$$

The use of Equations (21*a*), (22*a*), and (22*b*) makes a much more desirable form of solution of problems of this nature than the method proposed by the author. The writer does believe the method of relative flexure factors to be an improvement over the procedure published by Mr. Stewart in 1934 for use in analyzing continuous beams and frameworks not subject to side-sway.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ENGINEERING ECONOMICS AND PUBLIC WORKS
A SYMPOSIUM

Discussion

BY MESSRS. H. K. BARROWS, HARRY A. WIERSEMA, J. D. GALLOWAY, E. S. MARTIN, AND K. BERT HIRASHIMA

H. K. BARROWS,[77] M. AM. SOC. C. E. (by letter).[77a]—The papers by Messrs. Riggs and Wilgus contain much that is of interest. The various public works projects, including Passamaquoddy, Fort Peck, Bonneville, Grand Coulee, and the Florida Ship Canal, have all been given thoughtul attention and a clear picture presented of their nature, cost, and economic status.

From the taxpayers' viewpoint, only one of them (Bonneville) appears to have any chance of being classified as a project justifiable in character and as a wise expenditure of public funds.

As has been emphasized, it is the function and duty of Congress to see that proper use is made of Federal funds. In considering these projects the situation was somewhat involved because of their association with relief work. Such large and expensive projects, with their usefulness far in the future, if at all, are obviously not a proper class of undertaking for relief work.

The authors of the Symposium papers have brought the situation to the attention of engineers, and, in so doing, have also reached the public to some extent. It is the latter who should understand the situation, so that the people may express to their Congressmen their disapproval of such unworthy and undesirable projects. This work of education must begin at home, and engineers, who understand what these projects are and mean, have an individual duty in acquainting the layman with the facts, so that such unwise projects may be avoided if possible in the future. As Professor Riggs, in closing, states: "They will fail in their duty if they keep silence."

NOTE.—This Symposium was presented at the meeting of the Engineering-Economics and Finance Division, Boston, Mass., October 7, 1937, and published in February, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: March, 1938, by J. K. Finch, M. Am. Soc. C. E.; and May, 1938, by Messrs. Elliott J. Dent, C. Frank Allen, Bradley G. Seitz, Alfred Allen Stuart, Pierce P. Furber, R. F. Bessey, Donald M. Baker, and Phillip W. Henry.

[77] Prof., Hydr. Eng., Mass. Inst. Tech.; and Cons. Engr., Boston, Mass.

[77a] Received by the Secretary April 6, 1938.

Harry A. Wiersema,[78] M. Am. Soc. C. E. (by letter).[78a]—In his paper in this Symposium, Mr. Mead has given a brief analysis of several Government projects, including the Tennessee Valley Authority (TVA), and has drawn a conclusion regarding the latter which is far from complimentary. He states (see heading "Government Interference as a Permanent Policy: Responsibility of the Engineering Profession") that he has nothing to criticize with regard to the engineering features of the TVA, but gives the principle that "sound engineering cannot make an uneconomical project an economic success." The writer is entirely in accord with this principle, but he is so at variance with the conclusions regarding the economic feasibility of the TVA project that some discussion of this subject seems desirable.

The highest standard of ethical conduct should actuate all engineers in Government service, and to the extent that it does not such engineers are not true to their profession. It is not enough that an engineer render a high quality of technical service, but he also has a responsibility to the public for rendering the same kind of service that a consulting engineer would render to a private client in determining the economic feasibility of the project entrusted to his care. An engineer entering the service of the TVA would be confronted by a situation which might be compared to the relationship between a client and his engineer. In the following theoretical conversation, the United States Government has been personified to represent the client, who is setting out his plans and purposes to his engineer:

"Many years ago, spurred on by the necessity for the manufacture of munitions during the great war, I made an investment in a hydro-electric development at Muscle Shoals which has been a financial failure. In the first place, the actual sum of money expended in the construction of this project was much greater than it should have been, due to many factors, including the abnormally high price levels obtaining during the initial construction period and due to the prolonged and interrupted construction period. In accordance with the direction[79] of the Congress, the Tennessee Valley Authority submitted a valuation of Wilson Dam of $31 300 000 which was approved by the President on May 6, 1937. In the second place, due to the fact that this is an isolated plant and that there are no facilities for marketing the power, the income has been insignificant, represented only by sales to a single public utility company at a figure of 2 mills per kw-hr, which was the best I could do in such a monopoly market. The amounts of power thus sold were entirely at the discretion of the buyer, and only about one-fourth of the possible output was actually disposed of. I did receive several offers for the purchase of this plant, none of which was acceptable.

"I believe that the only way to save any considerable part of this investment is to develop further the resources of the Tennessee River. Congress has already recognized the potential value of the Tennessee River as an inland waterway by approving a plan for providing a 9-ft navigable channel from Knoxville to the mouth of the river, a distance of 650 miles. Wilson Dam forms an integral part of this plan. The Tennessee River and its tributaries have a considerable flow at time of flood, and it is desirable to provide flood-control projects on this river to assist in protection against floods, both locally on the Tennessee River and on the Lower Mississippi River. I believe that it is possible to develop a series of projects on this river system which will provide

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[78] Gen. Office Engr., TVA, Knoxville, Tenn.

[78a] Received by the Secretary April 15, 1938.

[79] TVA Act of 1933, Section 14.

> the waterway desired by Congress, together with a substantial degree of flood control, and at the same time develop considerable of the potential water power of the stream. In an integrated system of dams, the surplus power at each lower river project is increased by the released storage from plants above. If this water power can be converted into electricity and sold, the income from such sales will go far toward returning the cost of the entire development.
>
> "The Army Engineers have already constructed the lock for a dam at Wheeler, the next location above the existing Wilson plant, so that the navigation program is already under way. Now is a most favorable time for public works construction; Government credit is at its peak, and long-term money can be obtained at very low interest rates. There is a distinct advantage in constructing public works during times such as these when there is considerable unemployment. Such a development will not only effect an immediate improvement of economic conditions in a large section of the country but will also result in permanent additions to the nation's wealth. In addition to these considerations, we will have in this project a welcome experiment in integrated control of an entire water-shed, involving conservation of natural resources, broad scale regional planning, and an opportunity to demonstrate the advantages of a wider and better use of electricity."

There are several points of view which might be adopted with regard to the validity of the Authority's projects. For example, that part of the work which is justified for flood control and navigation could be considered as non-income producing public works; or the entire project could be considered as semi-revenue producing public works, and allocation of costs made to the different benefits. However, in order not to confuse the issue as to the economic justification of the Authority's projects by entering into a controversy regarding "yardsticks" and "allocations," it seems desirable to make an analysis similar to that made by Mr. Mead to determine how much of the total investment can be liquidated by the sale of surplus power. The following study is such an analysis of the economic feasibility of the project, based on conservative estimates of revenues from the sale of hydro-electric power, and actual project estimates which are fairly definitely determined. It is based on the system that will be developed when dams now (1938) under construction will be finished, which will be by January 1, 1942, rather than on the 11-dam system used by Mr. Mead. The 11-dam system has not yet been approved by Congress and the estimates are, therefore, approximate; for the 7-dam project, on

TABLE 8.—ESTIMATED COST OF SEVEN PROJECTS, TENNESSEE VALLEY AUTHORITY

Item No.	Project (1)	Initial power installation, in thousands of kilowatts (2)	Estimated cost of initial stage (3)	Item No.	Project (1)	Initial power installation, in thousands of kilowatts (2)	Estimated cost of initial stage (3)
1	Wilson	184	$31 300 000	5	Guntersville	72	$36 334 720
2	Norris	100	36 361 738	6	Chickamauga	81	43 127 903
3	Wheeler	128	36 849 403	7	Hiwassee	60	21 529 579
4	Pickwick Landing	72	33 219 279				
				8	Total	697	$238 722 622

the other hand, Congressional authorization has been fully obtained and estimates of cost and power can be much more accurately determined. The estimated cost of these seven dams[80] is shown in Table 8.

Except for the Wilson Dam (see Item No. 1, Table 8), which is a valuation determined by Congress, these estimates are carefully prepared and contain conservative sums for construction contingencies. Two of the projects, Norris and Wheeler, were finished slightly under these estimates in 1936 and in none of them is there any question of returning to Congress for additional funds to make up deficits, even in the face of increased labor and material costs.

The actual sums spent for transmission lines and sub-stations through the fiscal year 1937, together with the budget estimates for the fiscal years 1938 and 1939, are as follows:

Period	Expenditure
Prior to July 1, 1937	$11 323 182
Budget, fiscal year 1938	5 740 000
Budget, fiscal year 1939	5 560 000
Estimated additions to January 1, 1942	12 500 000
Total (say, $35 000 000)	$35 123 182

An amount has been added for additional investment up to January 1, 1942, which is approximately proportionate to the existing rate of expenditure and which, it is felt, under existing conditions, is adequate to provide a transmission system capable of disposing of the entire power output that will be available as of that date.

When the estimated cost of transmission facilities is added to that of the hydro-electric projects, the total investment as of January 1, 1942, becomes $273 722 622.

It is well to note that the estimate of the total project cost is based on the actual system being constructed. This system is the most desirable from the standpoint of a multi-purpose development, but it is not the most desirable from the standpoint of hydro-electric power alone. If the TVA had planned its projects for power as a single purpose and for the maximum dollar net income, there are other more economical power projects in the Valley which would have been substituted.

The fixed charges constitute by far the largest portion of the total annual cost of such projects, and these charges would reasonably be computed on the basis of the average interest paid by the Government on long-term bonds. Over a long period of years the average interest rate on borrowings by the Federal Government has been less than 3%, and in the period from 1933 to 1938 Government interest rates have been at an especially low level. The Federal Power Commission has determined the weighted average rate on all borrowings of the United States Government from November 1, 1933, to December 31, 1937, to be 1.54271%; but it would be difficult to justify so low a rate of interest in measuring the feasibility of the TVA program. The "Baby Bonds" that are sold to the general public are computed to yield an interest rate of 2.9% over a long period of years, and it appears to the writer that this constitutes a fair basis for amortizing the project.

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[80] Rept. of Hearings on the Independent Offices Appropriation Bill for 1939, 75th Cong., 2d Session, p. 919.

The life expectancy of the various classes of property included in this project varies widely, ranging from less than 20 yr in the case of certain parts of the transmission system to an indefinite life in the case of reservoir lands. Giving consideration to the portion of the total investment represented by reservoir lands and long-life concrete structures (which will be more than one-half the total cost of the project) it is felt that a service life of as much as 75 or 80 yr is not an unreasonable expectancy. However, in order to adopt a conservative value for the purpose of the present discussion, a 50-yr over-all expectancy has been used. Based on a 50-yr life and an interest rate of 2.9%, the sinking-fund method of calculating annual depreciation charges results in a depreciation rate of 0.9%, or a total amortization rate of 3.8 per cent. Applying this to the total investment previously obtained, of $273 722 622, results in an annual cost for fixed expense of $10 401 450.

In order to determine the income from the sale of hydro-electric power, the best estimates available are those contained in the report given to Congress,[81] in which it is estimated that the demand for firm power in the year 1942 will be 480 000 kw, and for secondary power, 98 000 kw. The installed generating capacity in that year in the seven dams under consideration will be more than sufficient to provide for a total demand of this magnitude. A reasonable estimate of the energy sold with this demand would be about 2 600 000 000 kw-hr of primary and 650 000 000 kw-hr of secondary. Analysis of the capability of these seven projects shows that there is energy in excess of the total of the foregoing amounts after allowing for leakage, evaporation, utilization, and transmission losses, as well as efficiency losses in the structures and equipment. It should be noted that, were these projects to be operated for power only, instead of primarily in the interest of flood control and navigation, the potential available energy in these seven plants would be considerably greater than these amounts.

In estimating the annual revenue accruing from the sale of surplus power, the Authority's standard wholesale rate schedule may be used for firm power. Mr. Mead has quoted this schedule correctly with one exception. The schedule provides for a monthly reduction for energy used in excess of 360 times the billing demand of 0.5 mill per kw-hr from the otherwise applicable rate for such excess. The average rate under this schedule, estimated on a basis of weighted averages of the different sizes and types of load from existing customers, would be at least 4 mills per kw-hr, of which about one-half is represented by the demand charge and one-half by the energy charge.

The average rate for secondary power, as determined from the Authority's existing contracts, is somewhat in excess of 2 mills per kw-hr. The total operating revenue based on the sale of 2 600 000 000 kw-hr of primary energy at the minimum of 4 mills per kw-hr and 650 000 000 kw-hr of secondary energy at the minimum of 2 mills per kw-hr, is $11 700 000.

The Act creating the TVA provides for the payment to the respective States of 5% of the gross revenue received from the sale of power generated within their borders. This payment in 1942 would amount to $585 000.

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[81] Independent Offices Appropriation Bill for 1939, 75th Congress, 2d Session, p. 1000.

The operating costs for this 7-dam system will consist of the cost of generation and transmission of electricity, cost of operating the locks, and certain expenses in connection with operating the reservoirs for flood control. The operation of the locks will be in the hands of the Army Engineers, and the estimate as to the cost of this operation is based on their experience. For the purpose of this analysis it appears desirable to make a comparison on two different bases: (1) Using only the cost of generating and transmitting power; and (2) using all the cost of operating the entire system.

Mr. Philip Sporn has given a table[82] of operating costs of hydro-electric plants, in which he shows that for plants of 50 000 to 100 000 kw of installed capacity the operating costs vary from a high of $1.50 to a low of 60 cents per kw, or a median of 75 cents, whereas, for larger plants the median is still lower. It is estimated that an average of 85 cents per kw would be a fair value. To this should be added the cost of operating the transmission system, and also an allowance for the cost of malaria control on the reservoirs (which, on account of the rather complete control that the Authority is instituting, will be a fairly large proportional sum). Adding these estimates, a value of $3.00 per kw of installed capacity appears to be conservative and reasonable, and this sum has been used in setting up the following tabulated estimate of revenue and expenses, as of January 1, 1942 (the estimates for additional expense for operating the system for navigation and flood control, including operation of the locks and certain required dredging, are also given):

Revenue and Expense	Per Annum	
Gross Revenue:		
Primary—2 600 000 000 kw-hr at $0.004 =	$10 400 000	
Secondary—650 000 000 kw-hr at 0.002 =	1 300 000	
Total gross revenue................	$11 700 000	
Less payment to States at 5%..........	585 000	
Total revenue.................................		$11 115 000
Expense of Operating Hydro-Electric Facilities:		
697 000 kw at $3.00 per kw.......................		$ 2 091 000
Net revenue from power sales alone.............		$ 9 024 000
Additional Expense of Operation:		
Navigation facilities....................	$ 400 000	
Water control and land management.....	500 000	
Total.....................................		900 000
Total net revenue..........................		$ 8 124 000

This tabulation shows that, based on operation of the power system alone, the net revenue would be $9 024 000, whereas based on the entire operation of the project the net revenue would be $8 124 000. Applying this revenue against the fixed costs, the revenue produced from the sale of power is 87% of the total fixed cost of $10 401 500 in the first instance, and even when all costs for operation for navigation and flood control are considered, the revenue is still 78% of the total fixed cost.

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[82] *Proceedings*, Am. Soc. C. E., December, 1937, p. 1927, Table 4.

Reverting now to interest during construction, there are considerations that make this project unusual in this regard. For instance, the actual interest rates during the construction period have been extremely low and not much more than one-half the long-time average interest rate assumed, of 2.9%, so that the interest during construction will be a relatively small percentage of the total annual interest after the project is completed. As each dam is completed, it contributes to the power available for sale in an amount greater than necessary to carry its own interest, with a surplus to contribute to the interest on uncompleted dams. The income from the plants as they are completed prior to 1942 will so closely meet the total expense that it is certainly fair to assume that it will approximately offset the interest during construction; so that neither the factors of income nor interest during construction need be considered in this analysis.

As stated previously, the actual cost of capital to the Federal Government during the construction period of the TVA projects is not as high as 2.9% and, in fact, may be considered as low as 1.55 per cent. Accordingly, it would seem a safe conclusion that the TVA will liquidate practically all the Government's investment in the project during the life time of the physical structures through the sale of its surplus hydro-electric power.

This is a far different "picture" from that presented by Mr. Mead and justifies the faith of the Engineering Staff of the Authority in the economic feasibility of the project as a whole.

It might not be amiss to discuss the benefits from navigation and flood protection, intangible though they may be. The opinion expressed by Mr. Morgan,[16] that the project is worth all it will cost for navigation and flood protection alone, is shared by many engineers; and although it is impossible to analyze this benefit in the same way that the benefit from the sale of hydro-electric power can be analyzed, it is scarcely "so extreme that it seems useless to discuss it" (see heading, "Government Interference as a Permanent Policy: Power"). The Miami District of Ohio was designed and built for local protection only and could not be expected to contribute to flood protection on the Ohio River. The TVA System, on the other hand, is designed and built not only for flood protection on the Tennessee River but also for flood protection on the Ohio and Mississippi Rivers; and the Gilbertsville Reservoir, the farthest down-stream project of the TVA System, is especially valuable for this latter purpose. The U. S. Engineer Corps has allocated[83] to the Tennessee Basin a total flood storage of 10 589 100 acre-ft, of which the Gilbertsville Reservoir will provide 4 600 000 acre-ft with a possible addition of 1 450 000 acre-ft for extreme floods. The storage of this reservoir will reduce the discharge at Cairo by from 184 000 to 220 000 cu ft per sec, and Gen. J. L. Schley, U. S. Corps of

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[16] Testimony before Appropriations Committee, 1936, *Proceedings*, Am. Soc. C. E., February, 1938, p. 253.

[83] House Doc. No. 259, 74th Cong., 1st Session.

Engineers, M. Am. Soc. C. E., has reported that this reservoir will reduce stages on the Mississippi River by 2 ft.[84] The value of this amount of reduction in providing protection to lands and improvements in the upper parts of the Mississippi River Basin has been estimated by Carl A. Bock,[85] M. Am. Soc. C. E., to be approximately $90 000 000. The estimate of $100 000 000 benefit for flood control of the entire TVA System, as made by Mr. Morgan,[16] would seem, therefore, to be fairly well justified.

The benefits from navigation are also intangible, but there is reason to believe that these benefits may be more than double the amount that Mr. Mead has estimated as justifying the expenditure of about $90 000 000.

From the foregoing analysis it is shown that the direct revenue from sale of hydro-electric power will liquidate all but 13% of the cost of the total investment. Applying this percentage to a total project cost of $500 000 000 would leave only $65 000 000 which is only one-half the sum that Mr. Mead himself estimates for the benefits of navigation and flood control, and only a small fraction of what may be the actual value of these benefits.

Whether or not the navigation and flood-control benefits are as great as Mr. Morgan has stated, and even if they are as low as Mr. Mead estimates, there can be no question that they are worth very much more than the actual unliquidated cost. Surely this is a thorough justification of the TVA program, and one of which its engineers may well be proud.

J. D. GALLOWAY,[86] M. AM. SOC. C. E. (by letter).[86a]—Although it would be possible to bring additional facts to support the data given in the able papers of this Symposium, it is believed that sufficient information is included in the four papers to enable conclusions to be drawn. In the words of Col. Wilgus (see heading, "A Challenge") they "constitute a terrific arraignment of those who are intrusted by the people of the United States with the handling of their affairs." However, the question naturally arises, why has such a condition come about and who is responsible? Many uneconomic projects have been undertaken, money by the hundreds of millions of dollars has been expended, and economic conditions are as bad as, or worse than, before. What is wrong with the program? The writer believes that the underlying causes are inherent in the nature of such programs and will examine the subject from that standpoint.

Advance Planning.—This theory has a superficial appearance of merit and Mr. Fay has outlined the best aspects of the subject. The fundamental defect of such planning lies in the fact that it is impossible to determine in advance what the needs of the future will be, or to anticipate the changes that only a few years will bring about. A few examples will make this point clear.

In the early years of the Nineteenth Century, a system of canals was projected and partly built in the Eastern States. Politics, "log-rolling," and much dishonesty characterized the work. Then railroads were invented and practically the entire system of canals was abandoned. The canals were a complete loss and the one remaining in use—the Erie—has been an excellent example of the losses resulting from Government in business. Of what value would have been a plan of canals over the country, when in a few years the entire system as built was abandoned? The writer has seen, in Indiana, remains of one of these canals that never even contained water.

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[84] *Waterways Journal*, November 27, 1937.

[85] *Engineering News-Record*, April 7, 1937.

[86] Cons. Engr., San Francisco, Calif.

[86a] Received by the Secretary April 18, 1938.

The railroads of the country are another example. Fifty years ago they were the principal means of transport, and branch lines were built all over the land. The automobile was invented and in the past few years one of the jobs of the engineers of railroads has been the tearing out of abandoned branch lines. Of all the railroads, the interurban lines, generally operated by electric power, furnish an example of the type largely rendered obsolete by the automobile.

Modern highways furnish another example of the inability of engineering works to last. Due to the rapid development of automobile traffic, roads that were built as late as five years ago (1933, say) are now inadequate. An able engineer of the California State Highway Commission recently stated that an expenditure of at least $100 000 000 was required to make over roads built in the past few years.

Examples of this phase of the subject could be cited without number. It is a matter of common knowledge among engineers that change is the order of Nature and that what is built to-day will be found inadequate tomorrow. It is the price paid for progress. When Americans are satisfied with the works that their fathers built, the nation will have reached a state of stagnation that the writer, for one, does not desire to contemplate.

It will be contended by the advocates of advanced planning that general plans only should be laid out by the planners of to-day. The answer to this contention is that the men of tomorrow will not follow the plans made to-day. In any single piece of work, plans made by one set of men are usually ignored by their successors. The writer has in mind electric transmission lines in California. Lines built under one set of conditions become obsolete by changes in voltage, changes in the location of load center, and various other factors. The amalgamation of systems has altered the entire program. As long as growth continues, the changes that come about preclude any advance planning that would be of value in an emergency.

The most important factor in the case of advanced planning is that the best of plans will not be followed because the decision to say what program shall be followed in the assumed emergency does not rest with engineers. That decision is made by politicians. The United States has been governed in the past by politicians, is now in their hands, and as long as human nature remains as it is, the country will be governed by politicians. It matters little by what label they are designated, they run true to form. The Columbia River Basin is an excellent illustration of this fact. In reality, advanced plans had been made on that large river. For eight years prior to 1933 the river was made the subject of studies and reports and they were of the general tenor that economic development was not feasible at the present time. The entire river was covered. What was the result of all this study? The politicians gave into the hands of the President enormous sums of money to be expended at will and immediately became active to divert as much of the money as possible to the regions they represented. That was the determining factor in causing the adoption of Bonneville and Grand Coulee. All the advance planning availed as nothing against the chances to obtain slices of "pork."

Electric Power.—Closely connected with such decisions as are mentioned is the plan of the present Administration to place the National Government in the business of developing and selling electric energy. Whenever a dam of any size is projected, it becomes (except in special cases) an economic necessity to make the energy from the falling water available for human use. As a rule the energy in the form of electricity is a by-product of the development. However, in the cases mentioned in the Symposium and in another project on the Tennessee River, power has become the main objective of the installation. The President has stated in no uncertain terms that four projects in different sections of the country were being built as "yardsticks" to determine the cost of power. In addition, funds have been, and are being, donated by the National Treasury to various municipalities for the purpose of installing municipal plants that will drive privately owned plants out of business. At first, the sums advanced were 30% of the cost of the works, which sum was later increased to 45 per cent. In addition, the National Government purchased, or will purchase, the securities of the municipality. The result is an outright gift or subsidy offered by the Government for the purpose of wrecking the private industry and driving the owners out of business.

The same effect is produced by the construction of power plants at the various dams recently completed or nearing completion. In complete disregard as to whether there is a market for the power, expensive hydro-electric installations have been made. This is especially true in the Tennessee Valley where the numerous dams serve practically no purpose except as units in a power development. Some benefit will accrue in flood control but the operation of the system will determine the extent to which the reservoirs will be used for the latter purpose. There is an inherent opposition between the requirements for flood control and those of power generation. Flood control demands that reservoirs be largely or wholly emptied in anticipation of a possible flood. Power demands that the reservoirs be kept full so that a maximum fall of the water be obtained. It may be claimed that flood-control space is left in the upper part of the reservoir, but this again emphasizes the fact that power development is the major element in the project.

There is no deeper seated economic fallacy embedded in the public mind than that power costs as developed by private companies are excessively high. The corollary follows that in every power development there is an unlimited source of revenue, which will carry the burden of all the charges on any project, however lacking in merit it may be. Demagogic newspaper writers, and politicians, have "harped" upon the subject until the public has been completely bewildered by their claims. The development of systems of power generation and distribution either from fuel or by water, in the past fifty years, has had an effect upon modern life in America second in benefits only to the great transportation systems. A cheap source of energy that could be distributed in large or small quantities to all parts of the land has been brought into being by the American genius for invention and for organization. The power thus placed at the service of every one can only be measured by that of billions of men.

The great enterprises have been developed by private initiative operating under the laws of the country. There are numerous instances where some of the enterprises have been misused, and corporation piled on top of corporation, for which the public has been asked to pay. No engineer should defend such practices and few do. On the other hand, there are many States where governmental supervision has prevented such practices and where, as in California, an able commission has worked with the power corporations, to protect the public and also the corporations. As a whole, the power corporations have acted wisely and have reduced rates in accordance with increased business and increased efficiency of larger systems.

As, throughout the country, power systems generally extend over two or more States, it becomes the function of the National Government to provide the necessary regulation. That is the proper function of Government. On the contrary, it is not the function of Government to use its power in building power plants and by use of Federal subsidies, drive the private corporation out of business. There was no need for any "yardsticks" to determine the cost of electric energy. One honest engineer and one honest accountant could determine this cost at any location.

In the Pacific Northwest, where the Bonneville and Grand Coulee power developments are situated, the private and public plants already in existence have brought about a per capita use of electric energy greater than elsewhere in the country. The existing systems have been built to supply immediate needs and are capable of extensions which can supply all the electric energy now required or that may be needed in the future. The Government plants were unnecessary. There has recently appeared[87] a statement that at Bonneville the Government investment on June 30, 1938, amounted to \$53 188 800 and that power would be charged with 32.5% of this sum. When the project is completed, the estimated cost will be \$74 144 600, of which power will be charged 57%, or \$42 191 175. The remainder of the costs (two-thirds in one case and nearly one-half in the other) is charged to navigation. It is with such juggling of costs that the "yardstick" will determine the costs of power. It will be noted that the 1934 estimate of \$31 250 000 for this project, stated by Professor Riggs as \$51 000 000 in 1936 (see "Significance of Contingent Expense"), has now (1938) grown to an ultimate of \$74 144 600.

He also states (see "Power in the Northwest") that the Grand Coulee project now has an estimated cost of \$437 000 000. There will be water to irrigate about 1 000 000 acres of land. The experience with the Reclamation Service projects, with few exceptions, shows that farmers cannot, and will not, pay the costs of such projects. They must compete with other lands not subject to the handicap of costly irrigation works and, furthermore, there is a widespread opinion that the Government should furnish the land and water free. The result is that no political management can make them pay. The favorite method has been to write off large sections of the cost of the project, to reduce the interest rates, and to extend the time of payment to a date far in the future. In California, irrigation districts having debts ranging from $15 to $50 per acre have gone insolvent because the farmers could not, or would not, pay their taxes. It is idle to expect that farmers under the Grand Coulee project will act otherwise, especially when Government gifts of all kind are being scattered broadcast. As a possible source of revenue to meet the charges on the Grand Coulee project, the farm lands may be dismissed from consideration. It is doubtful whether the lands could stand the cost of the required canal system. If any revenue is obtained, it will be in the distant future as the country is relatively undeveloped and now amply supplied with power.

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[87] *Western Construction News*, March, 1938.

Tendencies of Government.—It is recognized that there is a widespread belief throughout the United States in the principles of socialism, the fundamental basis of which is that all means of production should be owned and operated by Government. Parallel to this idea is the one that private ownership of property is an evil and Government should extinguish private rights by confiscation. Since the present Administration came into power it is the belief of the writer that a consistent course has been pursued by the Administration tending to destroy private property and private initiative. The most violent assault has been made upon the various power companies. Some of the means to this end are the several projects discussed in the papers of this Symposium. By apportioning a large proportion of the cost to such items as flood control or navigation, the power plant costs are reduced far below what they should be. This arrangement, together with outright gifts to municipalities written off by the Federal Treasury, may produce low-cost power, but as a measure of the correct costs of electric energy, the so-called "yardsticks" are designedly incorrect and without value as such. In the event that the policy of State socialism is made permanent, it would seem advisable to have the correct costs of any project determined, and not have the projects undertaken with the expressed purpose of altering the true costs by subsidies, direct or indirect, from the National Treasury.

The most glaring examples of uneconomic power projects among those cited in the Symposium, is Passamaquoddy. In this case there was lacking the usual excuse of navigation, flood control, or irrigation. All the costs, if properly made in an honest system of accounting, would have had to be charged to the project. The fallacy that energy under modern conditions of demand can be obtained from the varying systems of tides was also present. Hence, a fuel power plant was required of equal capacity to the tidal power plant. The fact that the location was such that there was practically no market for the energy made the entire project a monumental failure from the beginning. However, the politicians at Washington succeeded in spending several millions of money before it fell of its own weight.

In the case of the Wilson Dam and power project, started during the World War, there is another example of unwise Government planning and execution. The country was engaged in a desperate war that required all its energies; yet some one of the politicians secured money from the Treasury to begin the Wilson Dam on the Tennessee River, the power from the plant to be used during the war for the manufacture of nitrates. Years after the war ended, work on the dam was still going forward. When the project was started it was recognized that on account of the variable flow of the river, a steam plant of a capacity about equal to the water-power plant was required, and it was built. It was merely another example of the governmental inaptitude and mismanagement in an emergency. To any one except a politician, it should have been apparent that in a time of a great war, to start a dam and hydro-electric power plant that could not be completed, that could not function, and that required a steam plant of practically equal capacity, was opposed to all ideas of the right thing to do; yet tens of millions of dollars were poured into the project that was obsolete from the start as a producer of nitrates for war purposes.

Opposed to the prevailing tendency toward socialism, there is the individualism of the past. Under whatever term of collectivism it may appear; socialism, communism, Government or municipal ownership, the present tendency is in direct and irreconcilable conflict with the individualism that has built the nation. It is admitted that all over the world at the present time, the doctrine of collectivism is in the ascendency. The moves of the present Administration at Washington are in line with the world tendency. The glaring examples of the adoption by Government of unsound, costly, and uneconomic projects, as set forth in the Symposium, merely express the retrogression into which this country is sliding. In the past, in spite of obvious errors, individualism and personal liberty have accomplished the greatest good for the human race. Compared to the blundering and hampering action of Government, the contrast is a vivid one; and yet, with the other nations of the world, the United States has parted from the system that has made it great, and with the others, is imitating the actions of a certain group of animals of old, which, possessed of devils, rushed down a steep place into the sea. The country is in danger and, in the words of Professor Riggs (see "Conclusions"), "engineers will fail in their duty if they keep silence."

E. S. MARTIN,[88] ASSOC. M. AM. SOC. C. E. (by letter).[88a]—This Symposium directed to the economic phase of the recently enlarged Federal activity in public works plainly pertains to the exigent character of this activity, and to the competence and propriety of Government in business. The abrupt cessation of the flow of investment funds into private works following 1930 forced the Government unpreparedly into the breach or, failing that, forced a complete moratorium upon the accumulation of investment funds. Professor Riggs' intimation that, as with the Panama Canal, Congress should have deliberated for three years before taking action on the great projects now under way, seems, to the writer, without force in the circumstances. It is certainly a debatable point that Congress should decide engineering questions in any circumstances. It is for Congress to consider and determine the sum, time, and purpose, but not generally to consider and determine technical details which its members are not usually trained to comprehend. This point is the origin and strength of the manager type of municipal government.

The economic aspects of any undertaking mean the trading relation of the undertaking to its economic environment. Everything in the business world is a link in a chain. To study the link alone and to presage how and which way it will jerk the chain along is somewhat bizarre. The condition of the chain demands consideration in determining the safety and service of the link. The writer is offering some considerations herein which the proponents of this Symposium have neglected regarding the will of the people to use new works or even those they have. He pleads that the authors take entirely too narrow a view of the problem. They view public works from the banker's and the investor's standpoint; whereas the purpose of this activity is to use some of the available capacity for service, otherwise idle, to serve the nation, and thereby to command service in return—in the language of the street, to make a living.

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[88] Secy.-Treas., James A. Wickett, Ltd., Toronto, Ont., Canada.

[88a] Received by the Secretary May 18, 1938.

Professor Riggs again states (see heading, "Counting the Cost"), "the same sound business principles should control the selection of public construction projects which are to render a service that the public must pay for as would prevail in the case of a successful private undertaking of the same kind." The writer denies the very essence of this statement; public works and public finance are fundamentally different from private works and private finance. Private works are for the purpose of "making money" out of the public presumably by serving it, whereas public works are for use—for service only. The private investor often—but not always—aims to liquidate his investment; but there is no reason whatever for public works to be self-liquidating and there is no need for them to be financed by bonds; in fact, it is a crime against the public to do so. The Government is the one economic entity that has complete control of its income; it can balance its budget without injury to any one in war or peace. About 150 yr ago, pioneers migrated from the Atlantic Coast into the wilderness of Western New York and settled and developed that district. They did not prepare a prospectus to show (an investor) that the roads and works they built would pay or be self-liquidating. They were the investors and the users. Making a profession of investing, separate from enterprise, from material capital in modern finance, is not healthy. Mr. Henry Ford has taken pains that that did not happen to him; the same is true of thousands of other firms and of much of agriculture. From the ethical standpoint public works, by and for the public, should be free from private investors; that is, it should be paid for when built. The investor's field is private enterprise attended with risk, where invention and enterprise are essential—a field not suited to Government management.

In so far as this Symposium contributes to prevent waste of labor and material on developments that would be of no service, or of but little service, to Man, it is constructive and meritorious. One must bear in mind that with private enterprise as well as with Government managed enterprise the public provides the labor and materials and the money, too, through its patronage. Money is never consumed or wasted or saved or made. It is only the catalytic agent of trade; so that misapplication of effort is equally a public loss in either type of enterprise; in fact, the only difference between public and private enterprise is the question of who provides the management and gets the income, if any. When Government management excels private management in vision, enterprise, efficiency, and integrity, it will not take long to displace it. It is evident that Government enterprise is rapidly displacing private enterprise quite generally throughout the world, largely alas, because of the demoralization of private enterprise—more precisely, private investment. Professor Mead's reference to Canadian "fiascos" is not a happy one for his contention. Canadian trials at public management are uniformly outstanding successes; it is rather some private promotions in Canada, as in the United States, that are not so aromatic. Although the Canadian Pacific Railway was capably managed, the constituent private railways now forming the Canadian National Railway were in such bad physical and financial condition in 1915 as to threaten the solvency of Canadian banks and, possibly, the nation. Canada assumed and carries this indebtedness (which continues to grow), but the railway management is remarkably improved and compares with the best of American railroads. The same is true of the Toronto Transportation Commission and the provincial telephone systems in the West. The Ontario Hydro-Electric Commission outdoes, by a wide margin, the accomplishments of private systems of the United States. The writer has resided in Toronto for eighteen years following eight in New York, N. Y., and can warn his fellow members in "The States" not to refer to Canada to support an aversion to public ownership. The 8 cents to 10 cents per kw-hr that he paid in New York for domestic current as against the 1 to 1.5 cents in Toronto indicates the extent to which the consumer serves the investor in New York instead of the investor serving the consumer—which is his reason for being.

Now, the very extent, in area and time, of the Tennessee Valley development is such that no private enterprise could undertake this work in order to "make money"; nor could it be tolerated in any event; and, likewise the Columbia River projects. It is impossible for one now living to establish beyond question the tolerably exact value to Man of this kind of development, which depends for its use upon continued development in many directions. Such development is similar to that of the settlement of new domain; it cannot be reduced to a simple financial statement. From the writer's viewpoint, these are very promising, brilliant conceptions, as sure as most human undertakings. Some of the other projects, like the Passamaquoddy, do not offer the same promise, at least, they have not been shown to the writer's satisfaction to do so. The economist and the engineer must not overlook the inevitable risks always attending any enterprise; no one can see the future with perfect vision.

Public work has its pitfalls, as does private work. The selfish local interest which seeks public expenditures that it may benefit from it, whether or not the public does, has its counterpart in the investment broker who markets securities without conscientious solicitude for their soundness, or which do not represent equivalent increase in real material wealth. Mr. G. C. McDonald has stated[89] that total public debts in Canada increased about $1 800 000 000 during the six years, 1932 to 1938, with little or no increase of material wealth. The purpose of money savings and investment is to put and keep people at work creating fixed wealth; turning from this duty is exactly what causes public and private deficits and unemployment. There has been a forced attempt by the Government to discharge this duty in part.

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[89] "The Plight of the Taxpayer," by George C. McDonald, *Canadian Business*, Vol. 11, No. 2 (February, 1938), pp. 16–18.

The purpose of public and private improvements is to serve Man; in short, to maintain and to raise standards of living, now or in the future. If the people do not live more fully these works are not used; and, conversely, if the nation's capacity for service is not employed, it is because its people are not spending their money. How secure, and how promising can any development be as long as the people of several rich nations decide deliberately not to spend (use) their money? Economics was formerly called Political Economy, and so it is.

K. Bert Hirashima,[90] Esq. (by letter).[90a]—The papers of this Symposium should prove of great value to all engineers whether or not they are engaged in planning work. The papers by Messrs Riggs, Mead, and Wilgus reveal some of the colossal mistakes that have resulted from hasty uneconomic planning. Through careful advance planning, it is to be hoped that past errors will not be repeated in the future.

One of the principal objects of long-range advance planning is to relieve (at least to some extent) the large volume of unemployment resulting from the fluctuations in the business cycle; but, as Mr. Fay states "a controlled public works policy should not be looked upon as a means of relieving, directly, unemployment among workers outside the construction industry by giving them employment on construction projects." To do so would only tend to upset the normal relation between the number of workers engaged in the construction industry and the number of workers engaged in other activities. The resulting unbalance may eventually prove to be as great a problem as the original one of unemployment when, with the return of better times, public works projects are "tapered off." Public works expenditures, therefore, should not be regarded as a panacea but as a kind of balance wheel to regulate the activities of one phase of the national economic picture, namely, the construction and allied industries.

As to the financing of a long-range public works program, Mr. Fay mentions two methods: The accumulation of a reserve fund during boom periods, and long-term or short-term borrowing.

The question of just how the reserve fund should be accumulated is a difficult one. In the accumulation of any reserve fund, the assumption is made that assets can be liquidated at approximately the cost price; but the greatest demands upon a public works reserve fund will fall during periods of depression when security prices are at their lowest.

The issuance of bonds for unemployment relief is unjust in that it is nothing less than asking future generations to pay for present unemployment, unless the relief projects are of a self-liquidating character. When the bonds mature, money in the form of taxes must be diverted from the channels of trade to pay for them. This means that some worker, somewhere, is deprived of a chance to work.

In boom times, money is freely exchanged for goods and services; it is then performing its legitimate function as a medium of exchange. During periods of depression, the demand for money is greater than the demand for goods.

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[90] Asst. Highway Planning Engr., Territorial Highway Dept., Honolulu, Hawaii.

[90a] Received by the Secretary May 23, 1938.

Capital, unable to earn a profit and fearing taxation, "goes into hiding" in the form of bank deposits or tax-free bonds.

The fact that Government bonds can be sold to finance unemployment relief proves that there is an abundance of idle funds—money that has ceased to perform its function as a medium of exchange. It is not asking too much that such funds be taxed accordingly. Any long-range planning, therefore, should take these factors into account; otherwise, the issuance of public works unemployment bonds will only provide opportunity for additional capital to be removed from circulation. In this connection, President Roosevelt's suggestion that future issues of Government bonds be taxed, is a step in the right direction.

Lastly, the problem of public works planning is not a static but a dynamic one; that is, the factors involved are changing continuously.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

DESIGN OF PILE FOUNDATIONS

Discussion

BY MESSRS. AUGUST E. NIEDERHOFF, A. A. EREMIN, AND JACOB FELD

AUGUST E. NIEDERHOFF,[22] ASSOC. M. AM. SOC. C. E. (by letter).[22a]—The author's development of his subject is based upon assumptions that may be at variance with facts. His analysis also appears needlessly complicated and leads to values that are different from those obtained by the writer using only the resolution of forces into components that act upon the piles axial and normal to the axis.

It is not feasible to neglect soil mechanics in investigating the probable stresses in a pile foundation. The soil in which the piles are driven, the bearing at the tip (if it exists), and the bulb of pressure about the pile, are as much a part of the foundation as the pile itself and should be considered in any analysis. The author attempts to bulwark his analysis by considering the elastic deformation of piles and in so doing he loses sight of the fact that the stress, and, consequently, the elastic deformation, of the pile itself is only pertinent in exceptional circumstances. These circumstances obtain only when the total stress in the pile is less than the amount the foundation soil can stand. In soft yielding soils in which piles are used it is the passive resistance of the soil that is critical rather than the stress in the pile. This is particularly true if the soil is prevented from bulging upward under lateral movement of the pile by an overlying concrete slab.

In developing a simple case the author assumes that the pile acts as a column and is connected to the pier by means of a frictionless hinge. It is stated (see heading, "General Considerations") that: "In many practical cases, such as bridge piers and gate piers in hydraulic structures, the error introduced by this assumption is insignificant." In the writer's experience on bridge and gate pier foundations, wooden piles have been extended into the concrete about 2 ft and were considered fixed. The value in fixing this end of the pile was

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NOTE.—The paper by C. P. Vetter, M. Am. Soc. C. E., was published in February, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. Hibbert M. Hill, and Odd Albert.

[22] Head of Structural Design, Section on Dams, U. S. Engr. Office, Los Angeles, Calif.

[22a] Received by the Secretary April 11, 1938.

demonstrated during tests conducted in the fine sand, silt, and clay deposits on the right bank of the Mississippi River about five miles up stream from Redwing, Minn. (These tests were authorized by the District Engineer, U. S. Army Engineers, at St. Paul, Minn., in 1934.) Round wooden piles, about 30 ft long and about 12 in. in diameter, were driven in a vertical position and tested by applying horizontal loads to the top of the pile at the ground line. One pile top was unrestrained and the deflection at the ground line for a lateral load of 19 000 lb was 1 in. A similar pile, tested in the same manner, but with the top restrained, showed a deflection of only 0.36 in. for a lateral load of 18 000 lb. These tests indicated that if the top of the pile was assumed free whereas it was actually fixed, the error introduced would be about 250% for the materials used in the experiment.

The author also assumes that the frictional resistance per linear foot of pile increases rectilinearly from zero at the top of the pile to a maximum value at the tip. If the resistance to driving piles can be taken as an indication of the frictional resistance per linear foot of pile, personal observation would lead the writer to assume a parabolic variation rather than a rectilinear variation. However, in the case of vertical piles, driven in soil and supporting a concrete footing at the ground surface, the effect of lateral loads is believed to produce entirely different distribution of resistive forces. With the soil confined by the overlying slab the maximum resistance to horizontal movement is the passive pressure of the soil and is directly proportional to the pile deflection. The maximum pile deflection in this case occurs at the ground surface near the top of the pile and becomes smaller toward the tip of the pile. The distribution of resistance to horizontal loads, therefore, can be assumed as a maximum value at the top of the pile and a minimum value somewhere near the tip.

The depth in the foundation soil at which a pile can be considered fixed or hinged depends upon the material and dimensions of the pile and the character of the soil in which it is driven. Discussions[23] of a paper by Lawrence B. Feagin, M. Am. Soc. C. E., fix this point for these tests mathematically, and the paper substantiates the tests with recorded measurements. It is not necessary to assume fixation or hinge action at a point two-thirds of the pile length, as the author has done, when a series of tests will fix this point definitely.

The use of dummy piles to aid in the analysis of a complicated pile foundation with restrained pile ends is recognized as a convenient tool. However, for the more simple systems it is believed that dummy piles add to the difficulty in determining actual pile loads. The numerical example given shows that the computations and procedure are involved. Using Fig. 9, the writer computed the total vertical force acting on the pile bent at 142 kips. The net moment of all external forces above Plane Y and about the top of Pile No. 4 has been computed as 844.165 ft-kips. The eccentricity, e, of the point of application of the resultant force from the center of gravity of the system of pile tops is $\frac{844.165}{142} - 5.5 = 0.44$ ft. The vertical load taken by any one of the four piles can be obtained from the formula:

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[23] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 255.

$$P = \frac{V'}{n} \pm \frac{V' e c}{I_g} \qquad (59)$$

in which c = the distance from the center of gravity of the system to the center of the pile; and I_g = the moment of inertia of the pile system about its center of gravity.

Since the piles in the example are of the same cross-section, each value of A can be considered as unity and the moment of inertia, I_g, of the pile system about Plane X equals $2\,(5.5)^2 + 2\,(1.5)^2 = 64.8$. Substituting these values in Equation (59), the vertical loads are found to be as listed in Column (2), Table 15.

TABLE 15.—PILE LOADS, IN THOUSANDS OF POUNDS OR KIPS

Pile No. (1)	Vertical load (see Equation (59)) (2)	Axial load (see Equation (60*a*)) (3)	Normal load (see Equation (60*b*)) (4)
1....	40.80	39.8	8.4
2....	36.51	35.7	7.5
3....	34.49	34.2	4.2
4....	30.20	30.0	3.7

For comparative purposes, the loads on the piles are axial (P_a) and normal (P_n) to the axis, expressed, respectively, as:

$$P_a = P \cos(\alpha \pm \theta) \qquad (60a)$$

and,

$$P_n = P \sin(\alpha \pm \theta) \qquad (60b)$$

The angle the resultant force makes with the vertical is $\frac{6.1}{142} = \text{arc} \tan \theta$. The value of θ in the example is 2° 28′ and the value of $\cos \theta$ is so close to 1 that no correction of the vertical pile load need be made for the effect of the small horizontal force. The angle of batter of the piles is given as α and is equal to 9° 28′. Values of P_a and P_n, as computed from Equations (60), are given in Columns (3) and (4), Table 15. These values differ materially from those obtained by the author, and it is believed that the introduction of a dummy pile into the analysis obscured the author's computations.

Mr. Vetter is to be commended for a refreshing review of pertinent factors entering into the design of a pile foundation as recorded in the "Synopsis" of his paper.

A. A. EREMIN,[24] ASSOC. M. AM. SOC. C. E. (by letter).[24a]—Formulas for computing stresses in the piles of a pile foundation are developed in this paper. Mr. Vetter has simplified the computations by combining them with graphical construction, and the resulting process is simple if applied to piles in symmetrical pile footings. However, with the present application of mechanical calculating machines, a more direct method of computing stresses in more complicated cases is often desirable.

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[24] Assoc. Bridge Designing Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[24a] Received by the Secretary April 18, 1938.

Stresses may be computed by three equations derived from statics. Assuming the origin of the co-ordinates at a distance, y_1, above the bottom of a pile footing, the equations may be written (see Fig. 16),

$$H = \Sigma P \sin \alpha \quad \text{.......}(61a)$$

$$V = \Sigma P \cos \alpha \quad \text{.......}(61b)$$

and,

$$M = \Sigma P x \cos \alpha + y_1 \Sigma P \sin \alpha \quad \text{..}(61c)$$

in which,

$$P = \frac{E A}{L} r \Delta \phi \quad \text{.......}(62a)$$

and,

$$r = (x_0 - x) \cos \alpha - (y_0 - y_1) \sin \alpha \quad \text{..}(62b)$$

and x_0 and y_0 are co-ordinates of the center of rotation. Solving Equations (61) simultaneously, unknown co-ordinates of the center of rotation and stresses in piles may be determined. Forces at the restrained ends of piles may be treated as explained in the paper.

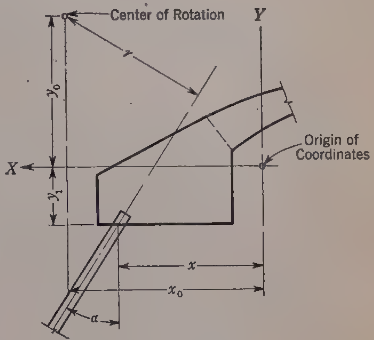

FIG. 16

Designers are often required to compute the elastic deformation in structures. Therefore, Mr. Vetter's paper is timely and creditable.

JACOB FELD,[25] M. AM. SOC. C. E. (by letter).[25a]—The purpose of piles is to transfer the base reactions of a footing to a soil that can carry such loads with settlements not exceeding the permissible values. Any resultant load can be resolved into a moment, and vertical and horizontal forces. The requirements of statics are the prime condition for any determination of what loads each pile (or group of piles) transfers to the carrying soil layers.

If all loads are vertical and the pile group is symmetrically spaced about the center of gravity of the loading, and if the footing may be considered a rigid body, the distribution of loads is uniform among all the piles. If the center of the pile group does not correspond with the center of the loading, the true distribution must take into account the eccentricity of loading, and also the compressibility of the piles and the settlement of each pile tip under the unequal loadings. The problem now falls into the class of indeterminate structures.

If all loads are vertical and if some or all of the piles are not vertical (battered) either the vertical load is transferred by skin friction or the pile is affected by a vertical couple. If the soil is sufficiently resistant to provide the skin friction in the immediate depths below the footing, no piles would be needed. In the other case, the lower end of the piles are embedded for a comparatively short distance in a soil much more resistant than that directly below the footing. A vertical couple acting on a pile can only be balanced by a horizontal couple resulting from a reverse type of deflection curve in the distance of resistant embedment.

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[25] Cons. Engr., New York, N. Y.

[25a] Received by the Secretary May 17, 1938.

Unless special details are insisted upon, the usual short pile embedment into a footing is not a rigid, moment-transferring (restrained) joint, nor is it a hinged joint. Such a connection is sufficient to transfer the horizontal loads from footing to piles. A horizontal load acting on a vertical pile can only be balanced by two unequal horizontal reactions with opposite directions. In no other way can the requirements of statics be satisfied, namely, equality of forces and of moments.

A horizontal load acting on a batter pile has the same effect as on a vertical pile with the exception of the unusual case in which the resultant load is axial with the sloping pile center. As has been well proved by tests on telegraph poles, as well as full-sized piles and sheet-piling, even short embedment of piles provides large horizontal strength. The greater cost of batter piles, together with the uncertainty of the fixed direction of the resultant loads, makes it advisable to consider such horizontal strength of piles before deciding on the use of batter piles.

Driving piles in a weak soil, without being certain that there is sufficient embedment in a resistant soil layer to take care of horizontal loads, is a common error. The result is a "creep" of viaducts, tipping of abutments, twist, and tip of towers.

The author carefully enumerates a number of the fundamental ideas in pile foundations, but his paper is really a mathematical study of the design of piles as elastic and static members. The final solution of a pile-foundation problem must take into account the fact that resistance is a function of the deformation resulting from load. The assumption (see heading, "General Considerations") that the effective length of a pile is independent of the load imposed, does not agree with the known action of soils. If sufficient resistance can be developed at the upper length of a pile, the lower part will be dormant. Distinction must be made between the actual soil resistance or friction and the latent maximum values. The distribution of one is not the same as that of the other. In this respect, soils are not elastic bodies.

The author's paper is excellent if restricted to the design of trestle or tower legs, or even for that part of piles above a resistant soil layer; for sub-surface design problems, the solution is only an approximation.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

PRACTICAL METHODS OF RE-ZONING URBAN AREAS

Discussion

BY MESSRS. ROBERT D. KOHN, MYRON D. DOWNS, HARLAND BARTHOLOMEW, AND BENJAMIN SALTZMAN

Robert D. Kohn,[4] Esq. (by letter).[4a]—In 1936, the writer came to the realization that no adequate change had been made in the zoning laws applying to the City of New York, N. Y., especially as to the density of use, although a building boom of a kind seemed to be on its way. It has not actually eventuated but much construction of a purely speculative nature was done in 1937. Such building, which planners have feared and tried to prevent, is going on with the same density that was permitted before.

The fact is that the very basis of zoning in New York City is absurd. That is, the absurdity of any scheme based on a scarcity of price placed on land when no scarcity exists. Before planners can get started on any worth while change they must make the difference between price and real value known to the public and faced by the real estate owner. For the difference is colossal. The high price is based in some measure on this very unrealistic zoning which has been accepted. The Housing Authority of New York found that districts zoned for residential purposes in New York City, if built up to the extent permitted by the zoning law, would accommodate 77 million people, and that areas provided for business and manufacturing would accommodate, if built up to the extent they were permitted by the existing zoning law, 340 million people, or, all the people in North America and South America combined.

Whenever a piece of property in a blighted area is purchased for housing purposes, the price the buyer must pay is based on the assumption that the land might be used for some purpose that the zoning law permits and to the limit permitted by the zoning law. Even if the idea with regard to trustee plans for combining properties in blighted areas were adopted they will not

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NOTE.—The paper by Hugh E. Young, M. Am. Soc. C. E., was presented at the meeting of the City Planning Division, New York, N. Y., January 21, 1937, and published in March, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] Archt., New York, N. Y.

[4a] Received by the Secretary April 2, 1938.

work as long as either the trustees have to base their plan on prices that have no relation to real values—earning values—or if the co-operative body is the State, the city, or the nation, and is forced to pay those prices. Another matter to which Mr. Young referred suggests that if the principle of earning value for land were really established, instead of scarcity price, the city would be bankrupt within one year because its tax base would disappear.

Those who are interested in this subject must begin by educating the public to the fact that there is this colossal spread between price and value; that owners can never really get such prices for their land, absurdly based on the idea that all the people in North America and South America will be working in an area such as the New York district, or the Chicago district. Planners will never accomplish anything with housing or better city planning until they realize that the entire present structure of real estate prices is built on a false basis.

MYRON D. DOWNS,[5] ESQ. (by letter).[5a]—Why is there a need for re-zoning cities zoned ten to twenty years ago? Most of the older ordinances were prepared by capable consultants, although many of them were written by city solicitors or city engineers' assistants under the direction of city councils seeking an economical method of satisfying a persistent group of citizens. Unfortunately, many consultants and the non-professional planners did not consider it essential to study other elements of the city plan prior to the recommendation of a zone plan. Too many zone plans already prepared and adopted, constitute merely maps of existing uses, no effort having been made to proportion the several types of use scientifically. It is well to bear in mind that the United States Supreme Court, in its decision in favor of the Village of Euclid, Ohio, in 1926, found zoning a constitutional exercise of the police power when based on comprehensive planning. It is not to be inferred carelessly that any map of an urban area, divided into three or four types of use districts, will withstand successfully the test of time, either in the Courts or as a medium for furthering the elements of a comprehensive and complete city plan, which should be the planner's test of a good zone plan. It would seem that the greatest need for re-zoning urban areas exists by reason of failure to plan first.

Mr. Young states (see heading "Need for Zoning: Why Zone") that "city planners have come to sense the great social need of transforming the physical structure of urban areas." It is to be hoped that his observations are correct. If so, the larger cities which, twenty years ago, followed New York's leadership in zoning, will again play "follow the leader" and provide workable public agencies in the form of public housing authorities to do the work. Much of the practical difficulty involved in the operation of housing projects by the Federal Government had its beginning in the form of bad land sub-division practice and the speculative rather than investment campaigns to "own your own home."

The author properly emphasizes the need of a strong administrative branch of government if good zoning is to succeed. Liberal or inadequate enforcement is very quickly resorted to, if a poor piece of zoning has been done by the planner, or if municipal officials have not provided for adequate planning as a foundation upon which to base the zone plan. Many a community has turned to the supposed economy of charging the assistant sewerage engineer and the bridge engineer's draftsman with the task of supplying the local planning commission with a zone plan during idle winter months. Naturally, the product of their efforts is comparable with that of a planning engineer, suddenly and without special technical training, assigned to the task of designing a sewage treatment plant for a community of unknown population, or detailing the shop drawings for a bridge of unknown length, width, and loading. Use of the resulting structures, in either case, is perilous.

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[5] Secy., Cincinnati City Planning Comm., Cincinnati, Ohio.

[5a] Received by the Secretary April 8, 1938.

Advances in the quality of public planning must be gradual and ideals must be compromised. Cities with too few use districts for proper transition, can amend the use district map. In 1932 the City of Cincinnati increased its use districts from eight to ten, providing thereby an intermediate residence zone between the one-family house district and the three-story, multiple-family building district, and a new business zone restricted to local services and goods establishments, with height limitations in keeping with those of surrounding residential areas.

There is still room for improvement, in most large communities, in the public control of population density. Height, area, and bulk regulations are of relatively little assistance in regulating population to the extent that school buildings and pavements can be built with any degree of certainty that the estimated capacity will not prove inadequate long before the bonds issued for the improvement are retired. There are few large cities which do not have some area congested with a density of more than 100 persons per net acre of land, and, what is more serious, they have no legal control of density in other areas, commercial or residential. Both Chicago and Cincinnati have certain areas with densities in excess of 120 persons per acre.

High buildings always present a difficult problem to the planner, in so far as the regulation of bulk is concerned. Cincinnati has improved the bulk regulation of the central business district approximately 25% by the recent amendment (1932). However, as emphasized in 1936 by Mr. Harold S. Buttenheim,[6] the provisions are still most liberal. (The total cubical contents above the established grade must not exceed the contents of a prism having a base equal to the area of the lot and a height equal to two and one-half times the width of the street—maximum, 250 ft.)

Good zoning, and the ability to place it on the ordinance books of any city, can only result from a good quality and an ample amount of technical work by competent planners supported by community endorsement of a planning commission of proved integrity.

Harland Bartholomew,[7] M. Am. Soc. C. E. (by letter).[7a]—A most convincing case for the re-zoning of urban areas is presented in this paper. Surely, no one who has studied this problem in large American cities can deny its necessity. Apparently, there is now a wide realization of this need, which is indicated by the large number of cities which express themselves in favor of re-zoning in answer to Mr. Young's inquiry.

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[6] "Land Overcrowding Allowed," *American City Magazine*, June, 1936, p. 81.

[7] (Harland Bartholomew & Associates), St. Louis, Mo.

[7a] Received by the Secretary April 11, 1938.

Cities will be re-zoned to best advantage only when the plan is based upon an accurate knowledge of the total land in urban use for various purposes and the annual rate of absorption. It is interesting to note how closely Mr. Young's data for the total area of land used for urban purposes in Chicago and certain suburban areas corresponds with similar values for about sixteen self-contained cities cited by the writer.[8] Had Mr. Young been able to secure data for all land in urban use in all the incorporated and unincorporated areas in the Metropolitan District of Chicago, there is no doubt but that the data would have corresponded even more closely with the average in other cities. There are well-defined laws of land use which, when thoroughly understood, will make possible far more scientific zoning practices.

In 1937, the St. Louis (Mo.) Regional Planning Commission made a land-use survey of the entire 840 sq miles within the St. Louis Metropolitan District, with the aid of the Federal Emergency Relief Administration (FERA) and the Works Progress Administration (WPA). The percentage of land used for various purposes in this Metropolitan District (which is really nothing more than a very large self-contained city) corresponds very closely with the average percentage used for various purposes in the sixteen self-contained cities (see Table 6).

TABLE 6.—LAND USED FOR VARIOUS PURPOSES; PERCENTAGES

Purpose	Average of sixteen cities*	St. Louis Metropolitan District	City of St. Louis
Single-family dwellings	36.10	29.58	20.4
Two-family dwellings	2.10		7.7
Multiple family dwellings	1.09	2.38	7.5
Commercial areas	2.38	4.27	5.3
Industrial areas	10.79	17.60	14.5
Streets	33.60	25.57	26.2
Parks and playgrounds	6.33	4.38	8.0
Public and semi-public	7.61	16.22	10.4
Total	100.00	100.00	100.00

* "Urban Land Uses," by Harland Bartholomew.

The importance of including all the area in urban land-use surveys, regardless of municipal boundaries, is revealed by Table 6. Although the percentages of land used for various purposes in the St. Louis Metropolitan District correspond fairly closely with those of the sixteen self-contained cities, it will be noted that those for St. Louis (which is only part of a self-contained city) show certain marked variations from the normal averages. Re-zoning of partial urban areas should be undertaken only after a full knowledge of total urban land-use characteristics based on comprehensive surveys.

Most city planners will agree with all Mr. Young's fourteen conclusions. It is particularly interesting to note that he concludes that both public and private enterprise will have to unite in the rebuilding of blighted districts—a conclusion that should be closely noted by the extremists who advocate one or the other of these methods of approach as the only possible means of solving these problems.

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[8] "Urban Land Uses," by Harland Bartholomew, Harvard Univ. Press, 1932.

Mr. Young advocates the creation of a strong citizens' council to promote the re-zoning plan which has been prepared as the result of the land-use survey. With this the writer is in full sympathy, but the success of this council depends upon many factors. The desires of a small but influential group of property owners have caused bad distortions in original zoning ordinances. These same speculative interests have been responsible for many unwarranted changes in zoning ordinances after their adoption in many cities. What assurance is there that any sound re-zoning ordinance adopted as the result of the initiative of a strong citizens' council will not gradually be broken down by these same speculative interests? Some kind of a permanent agency is needed to insure that all zoning is soundly conceived and wisely maintained. It is for this reason that the writer advocates one step farther than Mr. Young in his approach to this problem.

A real objective view of the present American city reveals it to be divided into three different types of residential areas: (1) Slums; (2) blighted districts; (3) new sub-divisions and suburban areas.

The actual slum areas are found generally in the oldest residential districts. They are relatively small in size, constituting probably not more than 5% of the total land in urban use. There is nothing that can be done about them except tear them down completely and rebuild them upon a totally new plan. The new sub-divisions and suburban areas are also relatively small in size, constituting perhaps not more than 20% of the total land in urban use. There is no immediate problem in these new areas other than that of protecting and preserving their present character. This is fairly well provided for by deed restrictions and by zoning regulations. The greatest success with zoning has been in preserving the character of new development.

Between the slums and the new suburban developments lies a vast area of residential development in various stages of decay. These are the blighted districts. They are all potential slums. Probably the most severe indictment that could be brought against the American city of to-day is the failure to give adequate protection to residential areas, from which more taxes are collected than from all other urban land uses, commerce and industry included. Residential development, particularly of the one-family and two-family types, once developed in an American city, soon starts either a gradual or a rapid process of depreciation. These large blighted districts cannot and need not be rebuilt. They are too large and the individual buildings within them are not yet obsolete. The problem then is one of rehabilitation rather than reconstruction. Sound rehabilitation involves the development of constructive urban land policies heretofore almost completely absent in American cities.

Thus, there are three different problems within the urban residential area: (1) Reconstruction of slums; (2) rehabilitation of blighted districts; and (3) protection for new suburban development. Zoning, in itself, or re-zoning, will not accomplish either of these three objectives. It will do very much to facilitate each of these objectives and, in fact, is indispensable to each of them. Zoning, or re-zoning, however, is merely one phase of urban land policy. To accomplish the desired objectives in all three areas involves a long-time program and continuous study of relatively small unit areas of land within each of the three broad areas. The writer, therefore, favors the division of all the residential areas, slums, blighted districts, and newly developed areas, into a comprehensive plan of neighborhood units, and the formation within each unit of a citizens' organization for permanent study and planning, as well as for concerted action in all matters pertaining to the welfare of the neighborhood unit which it represents.

Stability of home ownership and satisfactory living conditions have been destroyed by uncontrolled speculation in urban real estate. One can never establish stability and desirable living conditions without control of real estate, which is consistent with normal land use. Where there is a city plan commission capable of determining broad principles of land policy, and a well organized neighborhood unit in each part of the residential area, planners can soon determine what is the proper zoning and policy of land development within each neighborhood; and when the citizens of each neighborhood realize and understand what the zoning and land policy should be, they will have a far better chance of adopting, enforcing, and maintaining appropriate zoning and other parts of a sound land policy and program.

This comment, with respect to the establishment of permanent agencies in the form of neighborhood units and neighborhood unit organizations in the interest of slum clearance, rehabilitation of blighted districts, and protection of new residential areas, as previously stated, is merely supplementary to, and in no sense believed to conflict with, Mr. Young's recommendations. The writer is in full sympathy with his recommendations, and he should be congratulated upon the preparation of so thorough and so constructive a paper.

BENJAMIN SALTZMAN,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—The increasingly large number of municipalities which have adopted ordinances regulating the use to which land and buildings may be put, and limiting the height and bulk of buildings, is evidence that the benefits of a sound zoning system have been recognized everywhere. The private restrictions that constituted the sole regulations prior to zoning, were haphazard and sporadic, covered only a small area, and were formulated and often distorted by the developer to secure him a maximum financial return. These private restrictions were inflexible and, under changing conditions, frequently impeded the logical growth of the neighborhood, thereby making application to the Courts necessary for their removal. As cities became larger the need for city-wide regulation became more and more essential. It was apparent that city planning and zoning are inextricably intertwined. It was difficult for a community to plan street and block layout, park and recreational facilities, water supply and sewerage systems, transit and transportation systems, port and terminal facilities, and to locate schools and hospitals when the control of the city extended only over city-owned property, and when private property, covering about two-thirds of the city's area, was permitted to develop without any control, although the manner in which private property was developed strongly influenced the direction of city planning. No plan for the development of public facilities can be complete or effective which is not based upon a comprehensive plan of control of building development on private property.

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[9] Asst. Engr., Dept. of Housing and Buildings, Borough of Brooklyn, Brooklyn, N. Y.

[9a] Received by the Secretary April 9, 1938.

It is comparatively simple to map a proposed city along logical zoning lines. It is extremely difficult to zone land which has developed haphazardly, especially in a large city created by combining many smaller municipalities, each of which developed independently. The average zoning ordinance, as first enacted, represents a series of compromises between the city planner working along established principles of zoning and the real estate interests desirous of diverting the ordinance to those directions which might enhance the value of their property, which explains the unusually high percentage of land zoned for industry.

Almost every zoning ordinance contains "congenital" defects which prevent the fullest realization of the benefits of zoning and which contribute toward the creation of blighted areas. As time elapses the defects become glaringly apparent and real estate interests become reconciled to the idea that re-zoning is necessary. To determine the nature and the extent of re-zoning requires an examination of the ordinance, a study of what it sought to achieve, where it failed of achievement, and the factors contributing to that failure. All the symptoms must be studied, and a diagnosis made before any remedy can be prescribed.

Zoning is designed primarily to reduce congestion, provide adequate light and air, keep obnoxious uses out of residential areas, lighten the burdens of traffic regulation and fire protection, ease the strain on transportation systems, reclaim blighted areas, and finally, to stabilize real estate values. Haphazard building tends to reduce taxable values, as entire districts may become depreciated, while orderly development aids in the conservation of property values.

An area that has been properly zoned will not become blighted. A residential area that has been protected against invasion by industry by a buffer zone devoted to local business use, will retain its essential character for many years. An area devoted to one-family or two-family dwellings must be protected from the congestion caused by the erection of multi-family dwellings. In New York City, the ordinance needs revision in this respect, as the only restrictions designed to keep the multi-family dwelling out of the single-family area are limitations as to height and bulk, which are, more and more frequently, legally and successfully evaded. Where such areas, even if well shielded, do decay and become blighted through obsolescence and gradual occupation by lower-income families, the remedy is not to re-zone so as to permit business or industry to enter, but rather to re-build, either with private capital or Government housing, or both.

Most blighted areas have either been improperly zoned in the first instance or have been destroyed by an invading industry which later departed, leaving the empty shells of buildings as a reminder of their existence. These are the areas to reclaim. Instead of encouraging the development of out-lying districts, which induces abandonment of the old, and removal to the new, by extension of rapid transit facilities and by the creation of other desirable features, this emphasis should be placed upon the redemption of the old. It is illogical as well as uneconomical to permit an area provided with schools, streets, fire-fighting equipment, sewers, water supply lines, and transportation facilities, in fine geographical locations, to decay and become more and more uninhabitable, with real estate values wasting away, and proceed to duplicate all this expensive development in some outlying part of the city.

Decentralization of industry may become a cause of blight, but it may also be made to contribute to progress rather than to retrogression. If industry could be confined to small areas, each surrounded by local business uses and then by residential area devoted to the homes of the workers, the strain on transportation systems would be eased and street congestion materially reduced. Unquestionably, too much area has been zoned for industrial use; segregation and the reduction of these areas to reasonable limits would fix real estate values and diminish the area subject to blight.

Re-zoning alone, however, is insufficient to correct all the negative factors that contribute to the ineffectiveness of a zoning ordinance. The defects in the ordinance may be noted and noted, and the areas to be devoted to different uses scientifically apportioned, and yet the entire system may break down in the enforcement of the law. The zoning ordinance must be administered honestly and capably to be successful. Political expediency or personal favoritism have no place either in the administration of the law, in the granting of variations, or in the framing of amendments to the ordinance or the maps.

The burden of administration and enforcement is lightened by the requirement of a certificate of occupancy for every new or altered building. The issuance of this certificate is an assurance that the building complies with all the laws applicable thereto, including the zoning ordinance, and occupancy contrary to the terms of the certificate is in violation of the law. The certificate of occupancy is a public record, and any person interested can easily determine the contents of any certificate. Illegal occupancies may be noted and corrected. The department charged with the enforcement of the ordinance should issue the certificate; any other method results in a shifting of responsibilities and bickering between the agency enforcing the law and the one issuing the certificate. In New York City, the Department of Housing and Buildings administers the zoning ordinance, with a Borough Superintendent in charge of each Borough. This Department, or its predecessors in duties, has issued certificates of occupancy since they were first required in 1914. It is only on rare occasions, and then in border-line cases of interpretation of the law, that these certificates have been challenged by any request for revocation.

In New York City the City Planning Commission has the authority to amend the zoning ordinance and the maps; the right to grant variations from the strict requirements of the law, in stipulated conditions, is vested with the Board of Standards and Appeals. Both these bodies act only after public hearing. Variations in interpretations of the zoning law are granted only after thorough investigation and upon a sound legal basis, only where the development along conforming lines would inflict unnecessary hardship or where practical difficulties intervene; and then only under such conditions as will safeguard the character of the neighborhood. Furthermore, the relief granted to an applicant must not inflict any hardship on neighboring property owners. The Courts have consistently upheld the Board in their decisions along these lines.

The full benefits of zoning or re-zoning cannot be secured unless the concomitant problem of the non-conforming use is solved. In any area previously zoned for business or industry and re-zoned for residence use, or in any area previously zoned for industry and re-zoned to permit only small local business use, there will be numerous buildings whose use does not conform to the regulations for the new district. It is the hope of the zoning engineer that eventually obsolescence and natural decay will force the demolition of these structures. Where the Board of Appeals grants indiscriminate variances permitting the modernization or enlargement of such structures, the effectiveness of the ordinance is impaired and the restrictions become valueless. The Board, therefore, at all times, must regard the interests of the community as paramount, and allow variances only where the granting of the appeal is imperative and where denial would be equivalent to confiscation of property. Unless such an attitude is maintained re-zoning is valueless. Honest and capable administration of the ordinance is the keystone of the structure of zoning; without it any zoning or re-zoning is an idle gesture.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

FLOOD-PROTECTION DATA
PROGRESS REPORT OF THE COMMITTEE
Discussion

BY MESSRS. CHARLES F. RUFF, SAMUEL A. WEAKLEY,
AND HOWARD M. TURNER

CHARLES F. RUFF,[17] M. AM. SOC. C. E. (by letter).[17a]—The warning to exercise caution in storm transposition embodied in this report is a timely one. The procedure of taking the largest storm that has occurred anywhere in the same part of the country, and fitting its isohyetals as closely as possible to the water-shed must often lead to over-design; and in some cases it will cause rejection of flood-control projects, which, on a more reasonable basis, might be feasible and desirable. However, where records are short, as is often the case, or when there has been no notable storm over the area under study, some form of storm transposition is necessary to develop the full potentiality of the water-shed. The envelope curves of flood flow plotted against drainage area, and most of the flood flow formulas, are based on the tacit assumption that storm transposition over a very wide area is permissible. It is to be hoped that the U. S. Weather Bureau can furnish some meteorological basis to engineers to help decide the propriety of storm transposition in individual cases.

As a tentative guide in deciding to what extent storm transposition might be used in the Atlantic Coastal region, Fig. 1 and Table 1 were prepared. The greatest storms in this region occur in summer. The Miami Conservancy District studies show that, during the forty odd years covered, only a few winter storms (and those mostly in the South) were large enough to meet their criterion of 6-in. rainfall in 3 days at five or more stations. The paths of the storms considered run generally northwest to the coast, or into the Gulf of Mexico, where they turn and run northeastward, parallel to the coast line, the center of most intense precipitation occurring somewhere along this

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NOTE.—This Progress Report of the Committee on Flood-Protection Data was presented at the Annual Meeting, New York, N. Y., January 19, 1938, and published in February, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[17] Hydr. Engr., Pennsylvania Flood Control Bureau, Harrisburg, Pa.

[17a] Received by the Secretary April 19, 1938.

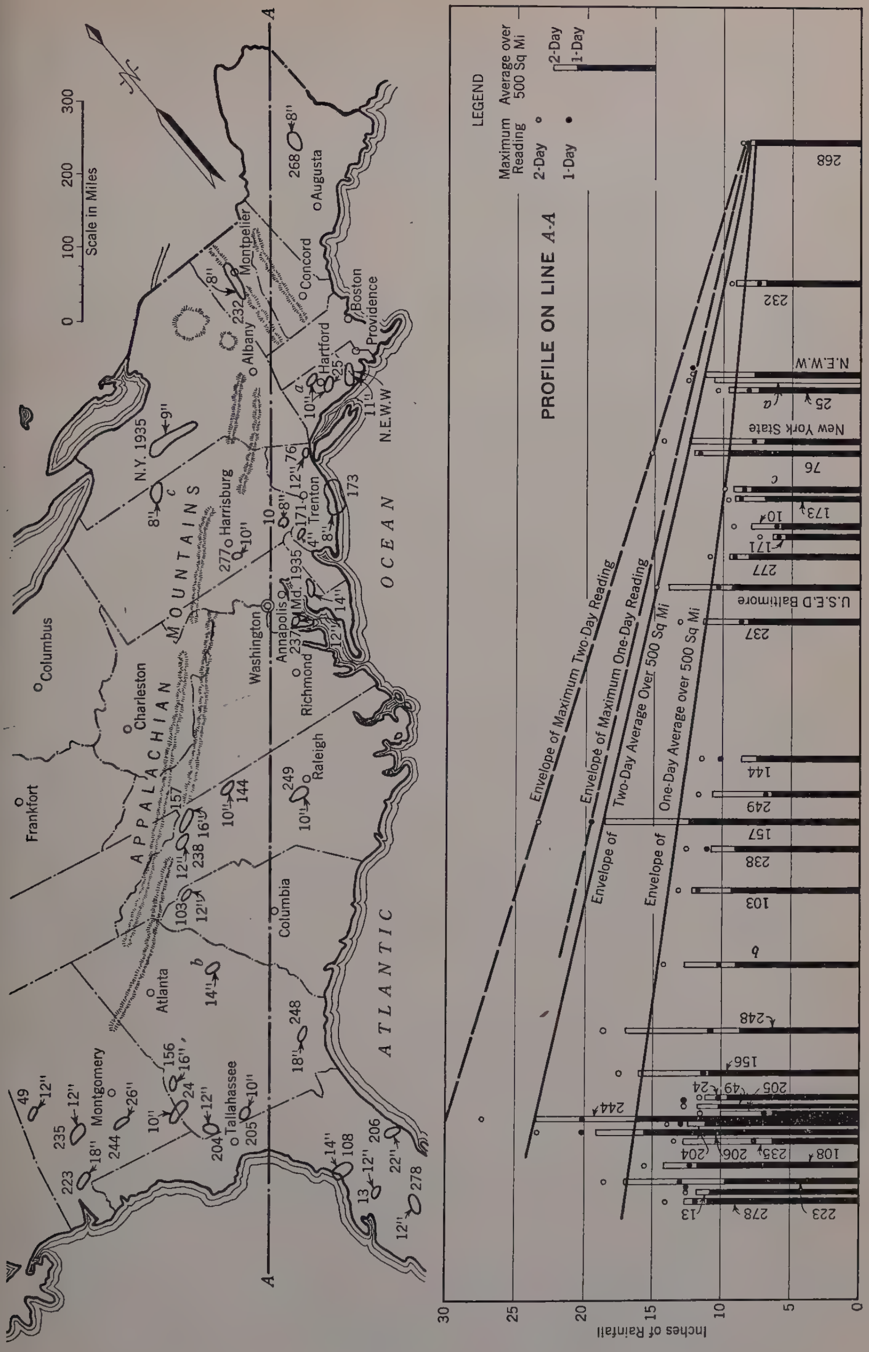

FIG. 1.—RELATION OF STORM RAINFALL TO LATITUDE ALONG ATLANTIC COASTAL PLAIN

course. The location of these rainfall centers was plotted on a map of the Eastern United States, the position being indicated by the approximate shape of the highest 2-day isohyetal. All the storms used (see Table 1) were those for which the time-area-depth curves were plotted in "Storm Rainfall of Eastern United States," 1936 Edition, by the Miami Conservancy District, and which occurred in the area between the mountains and the shore. This includes the largest storms of record up to 1933. A storm in Maryland in August, 1935, data for which were secured from the U. S. Engineer Office at Baltimore, Md., and the storm of July, 1935, in New York, N. Y.[18] were added. Data on the Connecticut center of Storm No. 268, September, 1932, have also been published.[19] Data for the storm of March, 1936, were not available, and they are not included. This was a storm of a different type, and despite the great floods it caused, it was not large enough, on a 2-day basis, to affect the diagram.

TABLE 1.—List of Storms Occurring Along the Atlantic Coastal Plain

No.* (1)	Location (2)	Date of observation (maximum, two days) (3)	No.* (1)	Location (2)	Date of observation (maximum, two days) (3)
278	Florida	September 5, 6, 1933	249	North Carolina	October 1, 2, 1929
13	Florida	September 25, 26, 1894	144	North Carolina	October 14, 15, 1914
223	Alabama	September 20, 21, 1926	237	Maryland	August 11, 12, 1928
108	Florida	July 2, 3, 1909	....	Maryland‡	September 5, 6, 1935
235	Alabama	June 4, 5, 1928	277	Pennsylvania	August 23, 24, 1933
206	Florida	October 10, 11, 1924	171	New Jersey	July 19, 20, 1919
204	Georgia	September 14, 15, 1924	10	Pennsylvania	May 20, 21, 1894
224	Alabama	March 14, 15, 1929	173	New Jersey	August 13, 14, 1919
205	Georgia	September 29, 30, 1924	*c*†	Pennsylvania	May 31, June 1, 1889
49	Alabama	April 16, 17, 1900	76	New Jersey	October 8, 9, 1903
24	Alabama	March 22, 23, 1897	....	New York§	July 7, 8, 1935
156	Georgia	July 7, 8, 1916	25	Connecticut	July 13, 14, 1897
248	Georgia	September 26, 27, 1929	*a*†	Connecticut	October 3, 4, 1869
b†	Georgia	July 29, 30, 1887	....	Connecticut‖	September 16, 17, 1932
103	South Carolina	August 25, 26, 1908	232	Vermont	November 3, 4, 1927
238	North Carolina	August 15, 16, 1928	268	Maine	September 16, 17, 1932
157	North Carolina	July 15, 16, 1916	....	....	....

* "Storm Rainfall of Eastern United States," Miami Conservancy Dist., Revised, 1936.

† Miami Conservancy Dist. uses the letters, *a*, *b*, and *c*, as numbers for early storms, before 1890, the same as the numbers, 278, 13, 223, etc.

‡ Baltimore Office, U. S. Corps of Engrs.

§ "New York State Flood," *Water Supply Paper 773-E*, U. S. Geological Survey.

‖ *Journal*, New England Water-Works Assoc.

The positions of the storms so plotted were then projected on a line parallel to the coast, and their magnitude shown on a profile by plotting the rainfall, in inches, for the following conditions: Maximum 1-day reading; maximum 2-day reading; 1-day average over 500 sq miles; and 2-day average over 500 sq miles. Of course, other conditions of duration and area covered might be selected in accordance with the size and area of the water-shed under consideration. Enveloping curves, which are nearly straight lines, were drawn to include the highest points for each of the four conditions. A decided and uniform trend is evident, the rainfall growing less as the storms are located farther to the northeast.

The envelope lines furnish a guide to the relative size of the different storms. It seems reasonable to suppose that the storms that determine the envelope represent the maximum for their latitude over the 40-yr period. Thus, Storm 277, near Harrisburg, Pa., although practically the same size as Storm 232, in Vermont, in actual rainfall, is, relative to the latitude, a much smaller storm, and one which is more likely to be exceeded. The Maryland, 1935, storm would be about the right size to transpose to the neighborhood of Harrisburg. The envelope lines may also be used as a means of estimating a storm of a size comparable to the envelope storms, by reading, opposite the point in question, the 1-day and 2-day maximum and 500-sq mile average rainfall.

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[18] *Water Supply Paper No. 773 E*, U. S. Geological Survey.

[19] *Journal*, New England Waterworks Assoc., June, 1933.

Attempts to correlate the storm size with the elevation or the distance from the coast were not successful. The largest storms (those nearest the envelope) seem to occur most frequently either on the coast or at the beginning of the mountains.

Some idea of the probability, or frequency of the envelope storm occurring at any point may be gained by the following reasoning: A storm reaches the 2-day 500-sq mile envelope no oftener than once every 5 yr—somewhere within the 300 000-sq mile region in which such storms occur. A total of 600 storms could occur without any overlapping of their 500-sq mile area of maximum rainfall. Assuming that the point in question is no more likely to experience such a storm than any other part of the area, there would be less than 1 chance in 600 of an envelope storm occurring there in a 5-yr period. Although this assumption is invalidated to an unknown extent by the effect of local topography, and probably other factors, it indicates that, for a given point, envelope storms must be extremely rare. It should be remembered, however, that all these storms covered much larger areas, in some cases, with only slightly less rainfall, and the chance of any given water-shed being included in these larger areas is correspondingly greater.

The seasonal distribution of the storms considered herein is shown in Table 1. They are all summer storms. Except for three storms in Alabama (two in March and one in April) the storms are limited to the period from the middle of May to early November. Expressing the size of each storm as the percentage of a 2-day, 500-sq mile storm which would equal the envelope, the following limits apply:

100% of envelope occurs from July 6 to September 16
90% of envelope occurs from July 1 to November 6
70% of envelope occurs from June 1 to November 6
50% of envelope occurs from May 15 to November 6

A rather generalized explanation of the foregoing variation in size, location, and time of occurrence of the storms, is as follows: The storms are caused by the upsetting of masses of warm, moist, and unstable air from the tropical part of the Atlantic Ocean or the Gulf of Mexico. The instability is due to the high temperature of the lower layers of the air relative to the temperature of the upper air, so that, when the lower layers are slightly elevated they are surrounded by cooler and, therefore, heavier air, and continue to rise at a rapid rate. As air rises it expands and cools, the moisture condenses, and precipitation results. The warmer the lower layer, the faster it rises and the heavier the resulting rainfall. The initial upset may be caused by additional heating of the bottom layer by contact with the land surface (generally hotter in summer than the sea) or by the air being forced up the slope of a mountain, or by over-riding a mass of cold, heavy air. As the storms strike farther north they encounter cooler land surfaces, and those that come in from the sea cross a belt of cooler water between the Gulf Stream and the shore, both of which tend to cool the lower layers and reduce the violence of the upset, when it occurs. It is probably only the stable air masses that travel very far north without being upset, and the rainfall is finally caused by meeting colder air from the north. The Vermont, 1927, and the New York, 1935, storms were both of this type. Frozen ground and snow also have a stabilizing effect on these storms, and for this reason they are unlikely to occur under conditions associated with maximum run-off.

SAMUEL A. WEAKLEY,[20] M. AM. SOC. C. E. (by letter).[20a]—There is no doubt but that the Committee is doing a great work in stressing the needs for more and better basic data on floods. The recent "disastrous floods * * * have proved a blessing in disguise in stimulating much needed interest in flood data" (as stated in the opening paragraph of the report) but they have not been wholly responsible for the flood records of to-day being more detailed and trustworthy. A considerable part of the credit is due to the fine work of the Committee.

For several years the U. S. Engineer District Office, at Nashville, Tenn. has preserved the records of flood crests along the Cumberland River and its tributaries. In many instances the field records are in the form of tablets or other permanent markers placed at the flood-crest line and referred to sea-level elevation, whereas, in other instances, the crests are referred to convenient bench-marks.

The writer is in hearty accord with the suggestion that a central agency is needed to act as a permanent source of information for flood data. Such an agency should collect, compile, and catalog data in such form that they would be readily available to practicing engineers and others. Areas or river basins could be spotted where basic data were meager or lacking, and the agency could foster or stimulate special interest in those sections toward the end that the maximum of information would be collected. This agency could also prepare standard forms and directives covering all kinds of data desired, thus standardizing information from different sections for uniform study and comparison.

Nashville, perhaps of all the Southern cities, has the best and most complete files of local newspapers well preserved in the libraries, historical societies, and colleges, from which innumerable records of river and climatic conditions may be gleaned. Newspapers have been published in that city for more than 125 yr and have been preserved in remarkable numbers. Descriptions of floods and droughts dating back to the early days of settlement are available to show that "there is nothing new under the sun" because, apparently, there were always "unprecedented" floods and droughts.

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[20] Engr., U. S. Engr. Dept., Nashville, Tenn.

[20a] Received by the Secretary April 25, 1938.

The theory of air currents and their effect on rainfalls over both large and small areas is coming into much prominence. The writer has observed air masses that produce great rainfalls and local air currents and columns of air that produce cloudburst rains. An outstanding example of the first case occurred when he left Nashville by audo-sedan about midnight on the balmy night of January 27, 1937, in shirt sleeves. Before he reached Hopkinsville, Ky. (65 miles to the northwest), he had on an overcoat and a blanket with the car closed tight as he passed into the section covered with ice and snow. He was entering the southern edge of that "cold front" that persisted during the period of the great rain and flood. This transition occurred within a distance of probably not more than 30 miles.

In the second case, a heavier-than-air glider recently soared in the air currents around the Nashville and the Monteagle sections of Tennessee for hours, being repeatedly carried aloft by the vertical currents after the miles of gliding had wasted away its altitude. If vertical columns of air are present in such force one can readily understand why occasionally conditions develop such that warm moist air is "shot" upward into cold air and precipitates a phenomenal quantity of rain in a short time.

HOWARD M. TURNER,[21] M. AM. SOC. C. E. (by letter).[21a]—The large number of technical reports and papers dealing with floods and flood control, especially those published since the great floods of 1936 and 1937, are described in this report. These publications are from various sources, such as Federal and State departments, technical society publications and magazines, and the universities. Most of the basic flood data are now published by the United States Geological Survey, except for special local and detailed reports on various specific flood projects, or localities, with the United States Weather Bureau publishing the meteorological data.

The Committee recommends that some central bureau be established in Washington to act as a clearing house for factual flood data and to publish such data. It suggests that consideration be given to the Geological Survey or the Weather Bureau, and that Federal funds be allotted for this purpose. The writer is not certain just what it is proposed that this bureau is to do. With the idea that all the various flood data published be collected at some convenient place, which might well be the Library of the U. S. Geological Survey (and also, preferably, the Engineering Societies Library, in New York City), the writer is in agreement. However, if the recommendation means (as it appears to imply) that an attempt should be made to confine the publication of all flood data for the entire country to a Federal Bureau in Washington, it seems not only unnecessary and expensive, but entirely unwise. The Committee complains because it has to consult various sources of information among which it mentions four U. S. Government Bureaus, State agencies, universities, engineering societies, engineering magazines, and special organizations such as the Mississippi River Commission, the International Boundary Commission, and the Massachusetts Geodetic Survey. It seems to be implied that all the data and reports published by these various organizations should be published by this central bureau. Is there to be no independent engineering analysis at various universities, or by State or local agencies, and by the technical societies? Many of these publications are not entirely factual but analytical in nature. Even purely factual information depends for its usefulness on its selection and presentation and often should be accompanied by analysis and interpretation. Frequently, the data for a local flood can best be obtained, and its method of presentation determined, by local engineers. If the nation is to secure the full benefit of the best engineering effort in the country, these publications, from as many sources as possible, should be continued and encouraged. Instead of there being fewer, it would be better if there were more.

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[21] Cons. Engr., Boston, Mass.

[21a] Received by the Secretary May 12, 1938.

The Committee cites, as an example, U. S. Geological Survey *Water Supply Paper 771*. This is not entirely factual data. Practically all the data available are selected from sources already published. A considerable part of the paper is a summary of various methods of analyzing flood flows and frequencies taken chiefly from publications of the technical societies or engineering magazines. This is a valuable publication but it does not seem to the writer advisable that all publications of this kind should be confined to a central flood bureau. Another case cited is the flood-height data published by the Massachusetts Geodetic Survey for the 1936 flood. To be effective this study had to be done at once. If it had not been done by the State Geodetic Survey it probably could not have been done at all. Clearly, the data could not have been collected at a central bureau; nor, the writer believes, could their publication have been so well done. The selection of the material and the decision as to how it should be presented were matters which required a familiarity with the local situation. The information is of such detailed local nature that its publication by a Federal bureau would scarcely have been justified. Another case in point would be the work of the Miami Conservancy District, which was a local problem, financed locally. Certainly, the publication of these reports, even the factual data, was much more logically done by that organization than it would have been by any central flood bureau. Other similar cases might be mentioned of reports, articles, and papers containing factual data published in recent years from different sources.

In the case of a single publication bureau, as recommended, there is always the question of expense. This means selection of material, and selection means limitation. The United States is a large country of varied climate, with thousands of rivers of varying size and characteristics. It seems unwise to attempt to have a central bureau that will select and publish all the factual data on all floods, because this will not only tend to limit the amount of data published but will also limit the originality of the methods used to present such data. What is more, it cannot be effective unless the profession intends to have all its flood engineering standardized and "collectivized" in Washington.

The writer is heartily in agreement with any suggestion that will permit the U. S. Geological Survey to continue and expand, as may be necessary, its excellent work in the collection and publication of flood data with the remainder of its stream flow work. The admirable publications of the data of the floods of 1936 (as, for example, *Water Supply Paper 798* for New England) leave little to be desired for completeness and usefulness. The Survey should be amply provided with funds to continue this type of work and preferably to expand it. If a "clearing house" of any kind, for flood data, is to be created, the writer considers that the Geological Survey is clearly the place for it, as it is the chief flood-data collecting agency now in existence.

The writer is in complete agreement with the sections of the report regarding the meteorology of floods, and particularly the caution against the transposition of flood-producing storms from one region to another. The publication of the Weather Bureau's study of maximum storms to be expected in different regions should give light on the flood problem.

Historic floods present a very interesting and profitable field for investigation which might well be undertaken by the various offices of the Geological Survey, or by other organizations and individuals familiar with the local history and sources of information available.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

MOMENTS IN FLAT SLABS OF VARIOUS TYPES

PROGRESS REPORT OF SUB-COMMITTEE (F) OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON MASONRY AND REINFORCED CONCRETE

Discussion

BY HENRY D. DEWELL, M. AM. SOC. C. E.

HENRY D. DEWELL,[9] M. AM. SOC. C. E. (by letter).[9a]—The moment coefficients for special cases of flat slabs, as presented by the Sub-Committee, are helpful in that they extend the field for the design of flat slabs beyond the limits of the standard case which, according to the Tentative Building Regulations for Reinforced Concrete of the American Concrete Institute,[10] is "for a series of slabs of approximately uniform size arranged in three or more rows of panels in each direction, and in which the ratio of length to width of panel does not exceed 1.33." By the provisions of this code, which are largely of empirical origin, each special case must necessarily be treated by itself. Obviously, some method of design, which would include the special, as well as the general, cases would be advantageous.

Harold B. Hammill, M. Am. Soc. C. E., and the writer, in connection with editorial work for the Uniform Building Code (California Edition), and at the suggestion of L. H. Nishkian, M. Am. Soc. C. E., another member of the Building Code Committee, made and reported an extensive study[11] of flat-slab construction, considering the slabs and their supporting columns of a typical bay as an indeterminate frame. The investigation covered a wide range of spans and live loads, both with and without dropped panels. It was found that this treatment gave positive slab bending moments comparable to those of the 1928 Joint Standard Building Code.

NOTE.—The Progress Report of the Committee on Moments in Flat Slabs of Various Types was presented at the meeting of the Structural Division, New York, N. Y., January 20, 1938, and published in March, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[9] Cons. Civ. Engr. (Henry D. Dewell & Austin W. Earl), San Francisco, Calif.

[9a] Received by the Secretary April 19, 1938.

[10] A. C. I. 501–36T.

[11] *Journal*, Am. Concrete Inst., January–February, 1938.

The method proposed by the writers frees the designer of flat slabs from all the arbitrary and restrictive limitations of the A. C. I. Regulations, such as definite dimensions of dropped panels, slope of column capitals, and number and shape of panels. More important, it takes account of the intensity and distribution of live loading which the A. C. I. Regulations[10] treat in an unsatisfactory manner. For example, these regulations[10] tacitly assume that the live load is always of uniform distribution; for one set of slab moments is made to suffice for all live loads. Obviously, the floor-slabs in the lower stories of a multi-storied building of flat-slab construction will be much stronger for conditions of unbalanced live loading than the floor-slabs of the upper stories. The only reference to bending moments in columns, such as those due to non-uniform loading of floors or to unequal floor panels, is the general statement,[12] applying to all types of construction, that:

> "The bending moments in the columns of all reinforced concrete structures shall be determined on the basis of loading conditions and restraint and shall be provided for in the design. When the stiffness and strength of the columns are utilized to reduce moments in beams, girders, or slabs, as in the case of rigid frames, or in other forms of continuous construction wherein column moments are unavoidable, they shall be provided for in the design. In building frames, particular attention shall be given to cases of unbalanced floor loads on both exterior and interior columns and of eccentric loading due to other causes."

This provision supplants the clause in the 1928 Joint Standard Building Code which specifically refers to flat-slab construction and states: "In flat slab construction * * * for known eccentric loads or unequal spacing of columns, computation of moments shall be made accordingly."

It is to be noted that the foregoing paragraph from the 1936 regulations[12] makes no reference to flat-slab construction. Although flat-slab construction falls under the general classification, "when the stiffness and strength of the columns are utilized to reduce moments in beams, girders, or slabs, etc.," it seems doubtful whether the designer of flat slabs will realize this; indeed, it seems doubtful whether it is intended that he should.

The German specifications for flat-slab construction differ materially from American standards. For example, the German specifications of 1929 state that, "bending moments in flat slabs and their supporting columns are to be computed by an exact theory in which the secondary moments are provided for." The determination of bending moments is made in accordance with a theoretical treatment by Dr. H. Marcus and the slab moments, both negative and positive, vary considerably from those of American practice. For example:

> "Secondary Moments: In German these secondary moments are called 'Drillungs Momente.' By way of explanation as to what these moments are: If a slab supported on four sides is considered as made up of a number of strips in each direction; when loaded, each strip will have a different deflection. Actually the deflections of adjacent strips must be the same. This sets up torsional moments in the strips as well as other moments due to the interconnection. For want of a better term they have been here called 'secondary moments.' The exact theory developed by Marcus and others evaluated these secondary moments which add greatly to the stiffness and strength of the slab.

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[12] Building Regulations for Reinforced Concrete, A. C. I. 501–36T, Section 1108.

"According to the older method of analysis by considering two sets of separated strips whose deflections at the points of intersection are equal, the corners of the slab would remain in contact with the supports. Actual test shows that the corners lift off if not held down. This lifting off is due to the secondary moments."

Finally, the method of analyzing flat-slab structures, which treats them as indeterminate frames, although not a rigorous theoretical treatment, is approximately correct and has the advantage that it can be applied equally well to special, as well as to standard, construction.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ECONOMICS OF SEWAGE TREATMENT

Discussion

BY MESSRS. CHARLES GILMAN HYDE, AND FREDERIC BASS

CHARLES GILMAN HYDE,[5] M. AM. SOC. C. E. (by letter).[5a]—Referring to the economics problem, in general, practically every engineering project demands consideration from three economic standpoints: Justification, selection, and proportioning.

In the usual case, private and quasi-public undertakings, such as manufacturing enterprises, office buildings, hotels, railway, water, gas, and electric systems, etc., must eventually justify themselves, in so far as their owners are concerned, on the basis of financial returns only. Enterprises undertaken by the public, such as water supply and highway systems, should also justify themselves from the economic standpoint, in considerable part, at least, if not wholly. However, many private, quasi-public, and public enterprises of the type in question yield intangible returns of distinct social and esthetic value that cannot be expressed in monetary terms. In those instances, where such dividends are relatively large, expenditures on the part of the public are justifiable either in the form of subsidies or of direct investment, as the situation may suggest. Rarely, however, does it appear that public monies are thus contributed to private undertakings even if the returns therefrom are of great social benefit.

The Economic Justification of Sewerage and Sewage Treatment.—It so happens that investments in sewerage and sewage treatment works can seldom be economically justified because their benefits are not to be reckoned in dollars. It is unquestionable that public health and comfort are immeasurably and directly improved by sewerage and to a less degree by sewage treatment. However, there seems to be no financial "yardstick" (for instance, in terms of the value of the human lives which might be saved by any particular system of works) by which the advantages of this indispensable public service can be

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NOTE.—The paper by George J. Schroepfer, Assoc. M. Am. Soc. C. E., was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 21, 1937, and published in April, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion on the paper.

[5] Prof. of San. Eng., Univ. of California, Berkeley, Calif.

[5a] Received by the Secretary May 16, 1938.

measured. Occasionally, the treatment of sewage may lessen the cost of treatment of water taken from a source which would otherwise be more grossly polluted or possibly too polluted to permit of its use for water supply purposes. Occasionally, also, fish and shell-fish industries of commercial value may become possible where, except for such treatment, they would not. In such cases some form of estimate can be made of the economic value of sewage treatment.

It is believed, however, that in most cases and for the most part sewage treatment must be justified upon social and esthetic, rather than upon economic, grounds. Thereby odors are eliminated, unsightliness avoided, recreation in the forms of boating, bathing, fishing, and picnicking made possible, water sources rendered more available and amenable to successful treatment for industrial, domestic, and other public purposes, and, in general, a better and more satisfactory environment vouchsafed.

Problems which, perhaps, may be looked upon as coming within the category of economic justification sometimes present themselves in connection with the design of sewage treatment works. For instance, the question may arise: Can the dehydration of sludge and its preparation as a fertilizer be justified on the basis of the expected sales? Since the sludge must be disposed of in some fashion, however, this problem may also be regarded, and perhaps more properly, as one of economic selection—selection as between different methods of securing a final or end result.

Economic Selection in Sewage Treatment.—The paper deals only with problems of economic selection in sewage treatment planning and design. It is a noteworthy contribution of extreme value because of the truly enormous mass of worthy fundamental data that have been brought together and critically examined, because it is virtually a pioneer discussion of the major phases of this important matter, and because in no other single place, known to the writer, can the material or any substantial part of it be found. It appears that much of this information has never been published, at any rate in available form. Mr. Schroepfer deserves the gratitude of the profession for the faithful, untiring effort that he has put forth over a long period of time to bring these data together. He is to be congratulated upon their careful analysis and excellent presentation. His discussion of the many factors to be considered in this particular group of problems in economic selection is so comprehensive that little need be said further on that score.

Economic selection implies a choice, on the basis of involved costs, between different methods or machines for accomplishing a given, pre-determined result. In sewage treatment the desired character and condition of the effluent must determine the processes and mechanisms which are available and which can be utilized. The selection as between two or more possibilities will be made, wholly or in part, as the case may be, upon an economic basis. The most significant criteria of performance will ordinarily be the removal of suspended solids and bio-chemical oxygen demand, as suggested and used by the author. There will usually be other factors, not readily expressible in monetary units, which should be given due consideration, and these may be controlling in some instances. The author has wisely stated that such factors demand attention and that each local problem should be analyzed independently with respect to all influencing circumstances and conditions.

The total annual cost in every case should constitute the criterion of economic selection. This cost must comprehend all fixed and operating charges. The fixed charges must include interest, insurance, taxes (if any), depreciation, and maintenance. The operating charges must comprise all labor and overhead, and supplies of every kind not overlooking power, heating, and lighting costs.

Unfortunately, published data on sewage treatment costs too frequently present operating costs only, and omit the fixed charges. Both groups of charges should be presented, properly segregated. The total investment in the works covered by the operation cost data should be stated, together with the rate of interest thereon and the depreciation allowances. Maintenance expenses should be shown as a separate item. The author is to be complimented on the fact that he has taken both main categories of expense into due account.

It frequently happens that the investment costs of projects, selected to yield comparable effluents, are widely different. It is not always an easy matter to convince administrative authorities and the public that works requiring the larger initial investment may actually be the cheaper in the long run, all factors considered. It happens, also, that funds for the economically more advantageous scheme are often unavailable, or are assumed to be.

Different mechanical equipments may involve different types and shapes of structures which, in turn, may represent different investment and perhaps different annual costs. In such cases, if the performances are equivalent, the selection should be made upon the economic basis, unless local conditions dictate to the contrary.

It must be obvious that problems of economic selection present themselves not only with respect to entire projects, but also with respect to numerous features of these projects. For example, such problems arise in connection with the selection of screening, pumping, and aeration equipment, of types and shapes of settling basins and their mechanisms, and of sludge-handling and disposal procedures and equipments.

Economic Proportioning or Dimensioning in Sewage Treatment.—So much has been spoken or written upon the general subject of economic proportioning that designing engineers should be well aware of its importance and thoroughly acquainted with the calculation procedures. As in the domain of economic selection, however, there are many factors, aside from those of cost, which must be considered in determining the proper shapes, proportions, and dimensions of sewage treatment plant structures and equipments. The characteristics of sewage and sludge, of course, must always be borne in mind. Dependability is frequently of more importance than mechanical efficiency. Features and works that might appear to be most advantageous from an economic standpoint might actually perform so badly and demand so much operating attention that they might prove in reality (that is, in service) to be the more costly. The primary effort of the designer must be directed toward securing, with ease and assurance, the treatment effects that are sought.

Economic proportioning must be a secondary, although still an important, consideration. There are many features of sewage treatment plants, nevertheless, which may be subjected to analysis to determine the most economic proportions. These features may be both structural and hydraulic.

Among other functions economic proportioning has to do with the determination of the most advantageous plant elevations. In this single item of planning there is frequently to be found a signal opportunity for cost saving. There may be one controlling elevation which will represent the greatest overall economy for the project. That elevation should be sought. Any other might involve more pumping, more excavation, more difficult and expensive foundations, less flexibility in operation, or some other costly or undesirable feature.

Effective sedimentation imposes severe depth limitation. Sludge-removal equipment may control the width of settling basins. Lengths must be sufficient to prevent excessive short circuiting. Certain shapes of basins require much more area than others. The economic dimensions of two basins in a single structural unit will usually differ from those of a single unit; and the economically most advantageous dimensions of several basins in a single structural unit will vary from those of a smaller number of basins. Unfortunately, however, the dimensions that will produce the best effluents may not vary in such manner.

One of the most common, and perhaps the best understood, problems of economic proportioning in sewage works design, as well as elsewhere, is that of the determination of the most economic diameter of force mains. In this case, where the volume of sewage or sludge will increase with the growth of the contributing district, the annual costs of pumping should be calculated for various pipe sizes and for various dates throughout the anticipated life of the pipe. The mean minimum cost will define the most economically advantageous diameter. Here again, however, the characteristics of the sewage or sludge must be considered. Pumps and pipe lines most economically proportioned for water may be so small as to become clogged with sewage or sludge. Moreover, the friction factor, due to grease accumulation, may increase with age. The entire subject is assuredly one of great interest and importance.

FREDERIC BASS,[6] M. AM. SOC. C. E. (by letter).[6a]—In the voluminous literature on sewage treatment relatively little is found on the economic phase, and for that reason alone this admirable paper is welcome. The author has industriously and painstakingly accumulated a great quantity of data on costs and performances of the larger plants of the United States, and has arranged and correlated them in a manner adaptable to various situations under average conditions.

As the obvious purpose of this paper is to present data that will be of assistance to the engineer in selecting the degree and type of treatment for a particular case, the writer believes that emphasis should be placed on: (1) The need of more extended study of the requirements of each situation; and (2) the effect of local factors that may alter the relative economy of types of treatment, as indicated by the average costs given. The author states (see "Comparison of Various Processes: General"): "Having arrived at what seem to be reasonable basic cost and reduction data for normal conditions, this section of the paper is devoted to a comparison of the various processes, and a discussion of certain factors that affect their relative economy and thus serve to alter the general indications presented in the comparisons." Furthermore (see heading, "Comparison of Various Processes: Miscellaneous Factors Affecting the Comparison"): " * * * 75% [of the plants] are within a range of 20% above or below the estimated curves of costs shown in the curves"; and 61% are within the same range for costs of operation and maintenance. The specific causes of variation from the average are mentioned: Fixed charges, sewage characteristics, seasonal variations in degree of treatment, commodity costs, head available, and design features. However necessary it may be for the designer to be in possession of average over-all costs of construction and operation, it is of prime importance for him to keep in mind the possible variations from the average due to local conditions.

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[6] Cons. Engr.; Prof. of Municipal and San. Eng., Univ. of Minnesota, Minneapolis, Minn.

[6a] Received by the Secretary May 20, 1938.

The author recognizes that the fundamental economic consideration is the degree of treatment necessary. This most important factor has quite often been given scant attention, particularly in smaller plants. In many cases it is apparent to the engineer that primary treatment only is required for all conditions, or that, on the other hand, continuous secondary treatment is necessary, but there are many cases in which only extended laboratory-controlled observation can determine the degree of treatment required to maintain a given standard in the receiving body of water and the varying degrees of treatment required in a stream of widely fluctuating discharge. The standard may be that of a minimum dissolved oxygen content in the stream alone, or in combination with suspended solids—or, in some cases, in addition, a bacteriological standard. At present there appears to be difference of opinion with respect to dissolved oxygen content standards in the receiving body of water, and until this question is fully answered in each doubtful case, the economic degree and type of treatment cannot be determined. Relatively few designing engineers possess the equipment for making the investigations necessary; the expansion of the services of the U. S. Public Health Service and of State Health Departments in making continuous surveys of sewage-polluted streams and from them establishing definite conditions to be met by treatment plants would, in the end, contribute very greatly to the economic design and operation of all treatment plants. At present (1938) such investigations are limited to the larger cities. Investigations by Federal, State, or by inter-State authorities have the further advantage of treating natural drainage districts as a whole, allotting to each community its proper share of the burden of cost.

As such investigations multiply and are made generally available the primary economic factor of degree of treatment required will tend to become limited within narrower ranges. In the case of the Minneapolis-St Paul plant (to which the author calls particular attention) long and detailed studies of the Mississippi River were made under the direction of the late J. A. Childs, M. Am. Soc. C. E., but the uncertainties in predicting the effect of some of the factors, particularly the rate of oxygen absorption by sludge deposits, required a considerable factor of safety to assure the required dissolved oxygen content in the river after the plant was in operation. If more extended investigation resulting in more firmly established and generally accepted technique had been available at that time, more precise interpretation of the situation would have been possible. From the economic standpoint, the decision as to the minimum dissolved oxygen content to be maintained in the receiving stream, whether it be 2 ppm or 4 ppm is vitally important. Accumulation of further data that will throw light on this factor will be of great value. Having such data, all the information in this paper may be used to its fullest extent.

One of the noticeable characteristics of the data presented is that practically all of them are of the last decade (1927–38) and many of the last two or three years. Even during the period when the Minneapolis-St. Paul plant was being designed, new and important information concerning developments in all types of treatment was forthcoming. In 1928, a trickling filter plant would have been designed and, in 1933, an activated sludge plant. The development of the chemical treatment process and effluent filter in this period contributed to the principle of flexibility in degree of treatment as required by the fluctuations of stream flow.

The feature of this paper showing the costs of operating secondary processes for fractions of the year suggests emphasis upon the economic values of this principle and that other combinations than those mentioned may be suitable in certain cases. In Table 16 the cost of effluent filters (if regarded as a secondary process) might be added, as such filters appear to be available in connection with plain sedimentation when that is not at all times sufficient, and when higher degrees of treatment attainable by other secondary processes are not required. From the data given of such filters, it appears that with plain sedimentation, 31 ppm suspended solids and 21 ppm B.O.D. would be removed by filtration. These values must be added to the 56% of suspended solids and 36% B.O.D. removed by plain sedimentation to obtain the over-all removal. From the costs of filtration shown in Table 18, on a plant of 100 mgd capacity, the fixed charges at 6% would be $24 000 and the operation and maintenance cost, $25 550, a total of $49 550.

TABLE 18.—ADJUSTED COSTS OF FILTRATION

Sewage treated, in million gallons per day	Construction cost, in dollars per million gallons per day	Operation and maintenance costs, in dollars per million gallons
20	4 500	0.95
50	4 200	0.80
100	4 000	0.70

Another combination that might be considered is that of short-period aeration (perhaps 2.5 or 3 hr) followed by effluent filters, if and when data are available to show the reductions by filtration of the suspended solids and biochemical oxygen demand from the effluent of the secondary settling basins of the activated sludge process.

In the section on trickling filters, the author has given as a basis the costs of construction and operation-maintenance at a rate of 2.0 mgd per acre. He has mentioned the fact that large-scale experiments (at Minneapolis, Minn., and Chicago, Ill.) have used rates as high as 20 and 25 mgd per acre. These experiments have resulted in securing bio-chemical oxygen demand removals from 65% to 75% and, in some cases, more. If, in practice, an average of 70% bio-chemical oxygen demand removal is attained (which seems likely) and a total construction cost of from 50% to 60% of a plant having a capacity of 2 mgd per acre with an operating cost perhaps 5% greater than that of the slower rate plant, values in Table 19, based on costs given in the paper for trickling filters at the lower rate, would apply. A curve might be added to Fig. 1(*b*) based on Table 19 which would appear quite closely parallel to the trickling filter curves, but to the left of them.

TABLE 19.—COST OF HIGH RATE TRICKLING FILTERS

Plant capacity, in million gallons per day	Estimated construction cost	Estimated operation and maintenance	Fixed charges	Total annual cost	Annual cost, in million gallons	Cost; 1% per million gallons per 5-day B.O.D. reduction
10	$ 660 000	$ 31 500	$ 39 600	$ 71 100	$7 110	$0.278
20	1 265 000	52 500	75 900	128 400	6 420	0.251
50	3 025 000	115 500	181 500	297 000	5 940	0.232
100	5 885 000	226 300	353 100	579 400	5 794	0.227

Although these last two illustrations of possible combinations cannot be substantiated by costs from actual large-sized installations (in view of the rapid rate at which new processes and combinations of processes are developing under adequate laboratory control) it is in that direction that sanitary engineers may look for advances in the near future.

In treatment plants with capacities of approximately 5 mgd and more, it is reasonably certain that careful and continuous laboratory control will be secured and that the performances cited in this paper will be achieved at the costs indicated. At present, information is necessarily obtained from them. Unfortunately, in considering the smaller plants, which obviously are greater in number, such control and information are lacking in many cases. In these plants the actual performances of the various types are likely to be so variable and to differ so much from their potential performances that not only is economic comparison more difficult, but certain types, because of the need of their more careful operation, due to their sensitiveness to varying conditions, should be eliminated altogether from consideration. In order to secure true economy in sewage treatment in the smaller cities, generally, the engineer must recognize the need for educating the public and its representatives in the value of acquiring necessary data for the correct solution of engineering projects.

This paper suggests that a great mass of unpublished data on the economic aspect of sewage treatment is existent and that this rapidly advancing science would benefit by its publication.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

DEOXYGENATION AND REOXYGENATION

Discussion

BY EARLE B. PHELPS, ESQ.

EARLE B. PHELPS, ESQ.[8] (by letter).[8a]—In two ways, this paper is a distinct contribution to the practical art of stream pollution control. By an ingenious development of graphic procedures, Mr. Velz has simplified the application of the complicated formulas of re-aeration and de-aeration and has rendered them directly applicable to any given situation. It is especially important that this simplification has been introduced without altering the form of the basic equations. The latter are rational in form and their constants are capable of experimental determination. Their conversion to simpler empirical forms leads too readily to loss of significance and to faulty applications. This, the author has happily avoided.

Of equal importance is the author's insistence upon the study of stream loadings as a factor in design. If a highway engineer were to build a dirt road and utilize it until traffic became impossible because of stalled cars, and then replace it with a six-lane concrete highway when two lanes would suffice, he would be subject to legitimate criticism; and yet, raw sewage is often discharged in increasing loads until the resulting stream conditions become intolerable. Then treatment is undertaken upon the basis of some successful procedure elsewhere, or under some general specifications of a State authority, and wholly apart from any but the most casual reference to stream capacity.

In his paper entitled "Economics of Sewage Treatment,"[9] George J. Schroepfer, Assoc. M. Am. Soc. C. E., has emphasized the fact that the first point of attack is a determination of the required degree of treatment. Mr. Velz' contribution fits exactly into this vacant niche and, in a sense, "completes the picture." It may quite reasonably be stated, that, in sewage treatment of any kind, the economic determination of the extent of treatment required is at

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NOTE.—The paper by C. J. Velz, Assoc. M. Am. Soc. C. E., was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 21, 1937, and published in April, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[8] Prof. of San. Science, Coll. of Physicians and Surgeons, Columbia Univ., New York, N. Y.

[8a] Received by the Secretary May 10, 1938.

[9] *Proceedings*, Am. Soc. C. E., April, 1938, pp. 737–765.

least of correlative importance with the complementary discussion of the economics of design.

Perhaps the first systematic attempt at establishing stream capacities was that made by the late Frederic P. Stearns, Past-President, Am. Soc. C. E., who, in 1890, studied Massachusetts streams carrying varying sewage loadings upon the basis of dilution per contributing capita. He suggested,[10] as tentative limits, that a dilution of less than 2.5 cu ft per sec per 1 000 persons leads to inadmissible pollution and that a corresponding dilution of 7 cu ft leads to results that are clearly inoffensive. The late Allen Hazen, M. Am. Soc. C. E., in his 1898 report on the Ohio River,[11] set the limits of necessary dilution at 1.5 to 4.0 cu ft, noting, however, that in the case of sluggish streams the dilution might have to be increased even to 10 cu ft. In 1902, the late X. H. Goodnough, M. Am. Soc. C. E., on the basis of more extensive Massachusetts data agreed[12] essentially with Mr. Stearns but set the upper safe limit at 6 cu ft, and the lower limit of necessary dilution at 3.5 cu ft. The late Rudolph Hering, M. Am. Soc. C. E., based his Chicago Drainage Canal design upon a dilution ratio of 3.33 cu ft. It will be particularly noticed that throughout all this period only Mr. Hazen made any allowance for the influence of velocity of flow although this influence is traditional. With present-day knowledge, a simple computation will readily show that, in any of these minimum dilution volumes that have been set up, the stream must rely largely upon re-aeration to preserve any semblance of purity.

The British Royal Sewerage Commission, in its 8th Report (1912), recognized re-aeration as a factor in stream pollution standards and made an attempt to evaluate it.

The Engineering Board of Review, of the Sanitary District of Chicago (1925), clearly recognized the essential relation existing between dilution and the oxygen demand of sewage but the most recent textbooks either omit all reference to re-aeration as a stream asset or merely note the lack of quantitative knowledge concerning it.

In 1915 the writer made a report to the International Joint Commission in which the principles of permissible loading and of allocation of stream assets among the users of the boundary waters of the Great Lakes system were laid down as bases for a remedial program.[13]

Under the direction of Mr. C. A. Holmquist, Director of the Division of Sanitation, New York State Department of Health, a similar study was made in 1932 of the capacity of the Hudson River below Albany, N. Y. Mr. Velz was associated with the writer in that study. The oxygen profile was determined under summer conditions, and estimates were made of the rate of de-oxygenation and re-aeration, and of the relation of the contribution of each community to the total pollution of the stream. A similar treatment of the pollution and self-purification of New York Harbor, as influenced by the ebb and flow of the tidal prism as well as by the flow of the Hudson, has been published.[14]

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[10] "Examination of Water Supplies," Special Rept., Mass. State Board of Health, 1890.

[11] "Investigation of Rivers," Special Rept., Ohio State Board of Health, 1897.

[12] Rept., Mass. State Board of Health, 1902.

[13] Rept. of the Cons. Engr. on Pollution of Boundary Waters, International Joint Comm., Washington, 1918.

[14] "Pollution of New York Harbor," by Earle B. Phelps and C. J. Velz, *Sewage Works Journal*, Vol. 5, 117, 1933.

In making plain the necessity for such considerations and in developing and presenting a simplified and workable method for their application the author has rendered the profession a distinct service.

Briefly stated, de-aeration is a function of concentration of oxidizable organic matter, temperature, and time. The form of this function, and its time and temperature constants, have been adequately investigated and are comparable in their accuracy and applicability to weir formulas and others of similar nature upon which engineers rely in many of their design problems. Re-aeration is a function of the initial state of de-aeration or saturation deficit, of time and temperature, and of stream turbulence. The basic constants of this relation are also known with sufficient accuracy, except that, for the present, the item of turbulence must be determined in a more or less empirical manner. As shown by H. W. Streeter, M. Am. Soc. C. E., and the writer[15] it may be developed in streams as a somewhat complicated function of depth, velocity, and wetted perimeter. In larger areas of more nearly quiescent water, the late William Murray Black, M. Am. Soc. C. E., and the writer have shown that it may likewise be developed as a function of depth, wind velocity, agitation by passing boats, and the movement of tidal flow through constricted sections.[16] In each case the turbulence factor appears in the basic formula as a time relation expressed in the form "time between mixings." The author's experience, indicating that under many conditions occurring in actual practice on small streams this time may be taken at from $\frac{1}{8}$ hr to $\frac{1}{2}$ hr for streams ranging from shallow, rapid-flowing, to deep slow-flowing, probably represents satisfactory approximations. The British Royal Sewerage Commission, on the basis of direct observations with dye solutions, estimated for some of the larger streams times of mixing ranging from 1 hr in the case of moderately rapid flows to 6 hr in the case of very sluggish flows. The results, computed on this basis, were found to be in practical agreement with those of field investigation and of laboratory experiments in tanks.[17]

It is believed that the procedures advocated by the author are worthy of much wider use on the part of those charged with the responsibility of stream control and that such use will result in economy of effort and conservation of resources.

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[15] "Factors Concerned in the Phenomena of Oxidation and Re-aeration," by H. W. Streeter and Earle B. Phelps, *Public Health Bulletin 146,* Washington, D. C., 1925.

[16] Rept. on Discharge of Sewage into New York Harbor, by William Murray Black and Earle B. Phelps, Board of Estimate and Apportionment, New York City, March, 1911.

[17] Royal Comm. on Sewage Disposal (Great Britain), 8th Rept., Vol. 1, p. 10, 1912.